1. Probability

AS Recap – the theory for the A2 unit builds upon the following Y12 work

Venn Diagrams and Set Notation

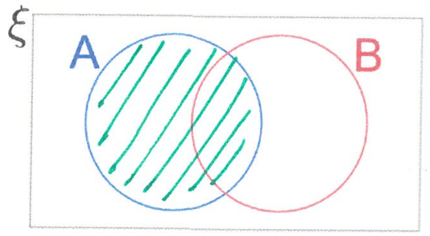
For two events A and B

- the event that A or B or both occur is called the UNION of A and B, which is written $A \cup B$
- the event that A and B occur is called the INTERSECTION of A and B, which is written $A \cap B$
- the event that A does not occur is called the COMPLEMENT of A, which is written A'
- the event that A is a SUBSET of B, for example A = {Hearts} B = {red cards} is written $A \supset B$

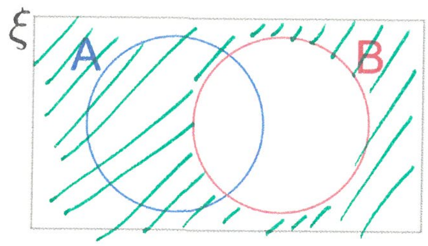
Representing information on Venn diagrams can often help when solving some probability problems.

Eg1 Shade the following Venn diagrams to correctly represent the given events, A and B:

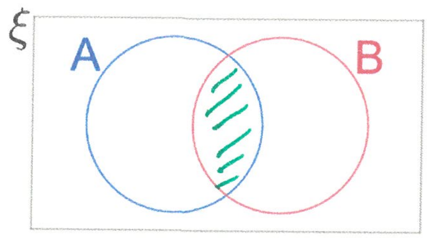
(a) A



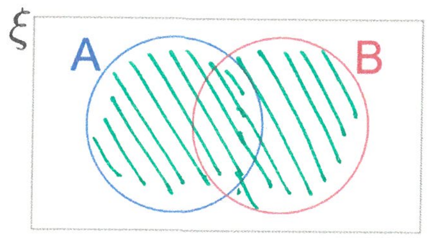
(b) B'



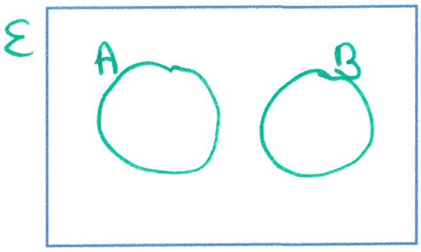
(c) A ∩ B



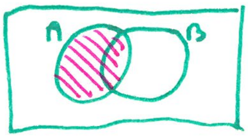
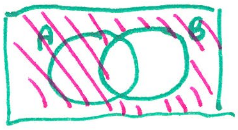
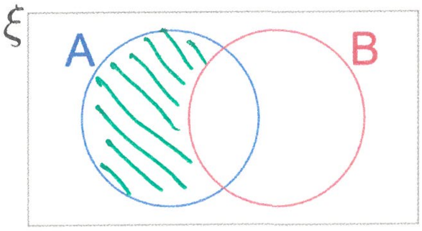
(d) A ∪ B



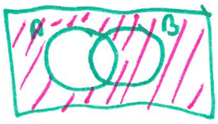
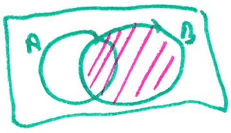
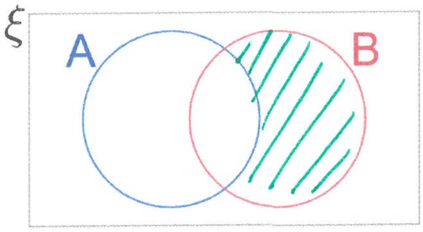
(e) $A \cap B = \varnothing \leftarrow$ NULL SET



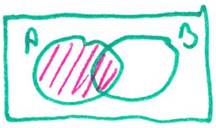
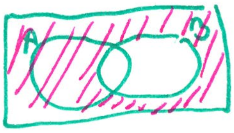
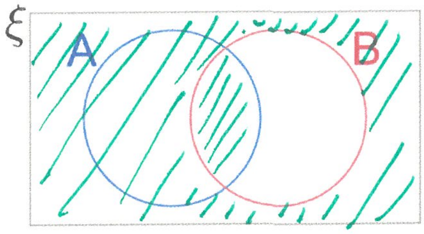
(f) $A \cap B'$

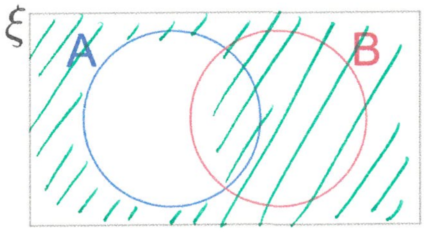
(g) $A' \cap B$

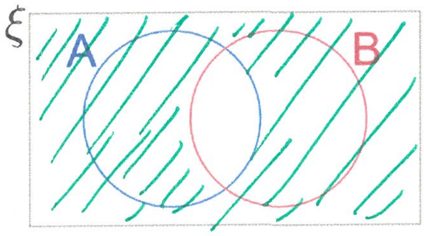
(h) $A \cup B'$

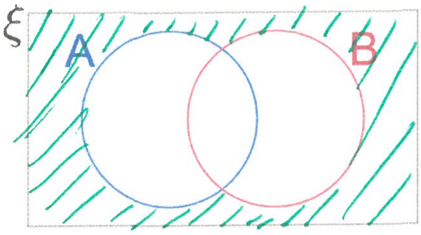
(i) $A' \cup B$



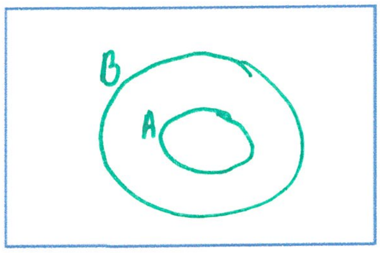
(j) $(A \cap B)'$



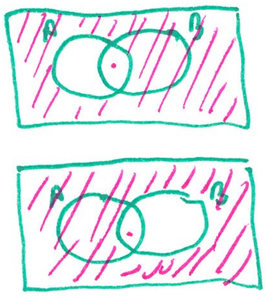
(k) $(A \cup B)'$

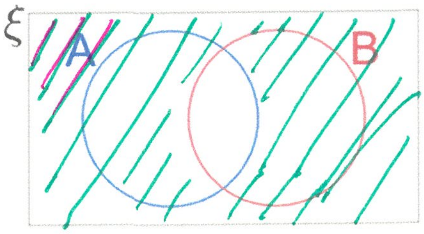


(l) $(A \supset B)$

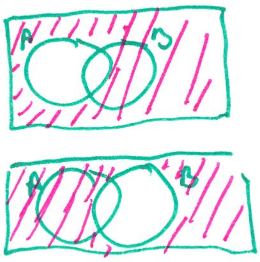


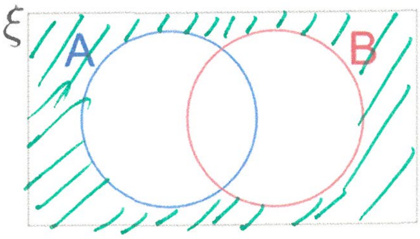
(m) $A' \cup B'$





(n) $A' \cap B'$





Using some of the Venn diagrams above, identify two set relationships which are equivalent:

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

The Addition Law for Mutually Exclusive Events

If events A and B are MUTUALLY EXCLUSIVE then event A can happen OR event B can happen but they cannot both happen. (Venn diagram (e) ABOVE).

In terms of probability this means

$$P(A \cup B) = P(A) + P(B)$$

The Generalised Addition Law

This links the probability of the intersection with the probability of the union of two events A and B

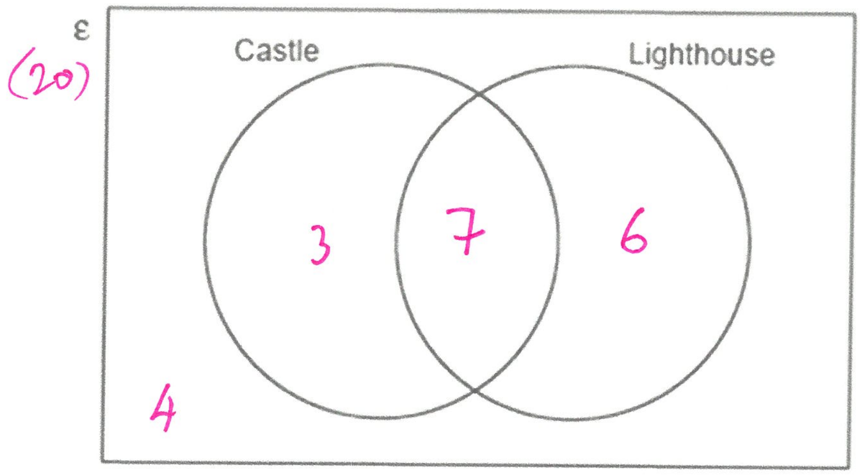
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Eg2 Consider the following GCSE PPQ

A group of 20 people visited Anglesey for a weekend break.

- 10 of the group visited Beaumaris Castle.
- 13 of the group visited South Stack Lighthouse.
- 4 of the group did not visit either of these places.

(a) Complete the Venn diagram below to show this information.
The universal set, ε, contains all of the 20 people in the group. [3]



$C = \{10\}$

$L = \{13\}$

$C \cup L = 16$

$$(C \cup L) = C + L - (C \cap L)$$
$$16 = 10 + 13 - (C \cap L)$$
$$C \cap L = 10 + 13 - 16$$
$$= 7$$

(b) One person is chosen at random from the group.
What is the probability that this person visited only one of the two places? [2]

$$\frac{9}{20}$$

Multiplication Law for Independent Events

When an event has no impact on another event, they are said to be independent events. For such events A and B, the following applies:

$$P(A \cap B) = P(A) \times P(B)$$

Eg3 Events A and B are such that $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cup B) = 0.44$

(a) Show that A and B are independent

(b) Calculate the probability of exactly one of the two events occurring.

(a) If indep then $P(A \cap B) = P(A) \times P(B)$

$= 0.3 \times 0.2$

$= 0.06$

Also $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

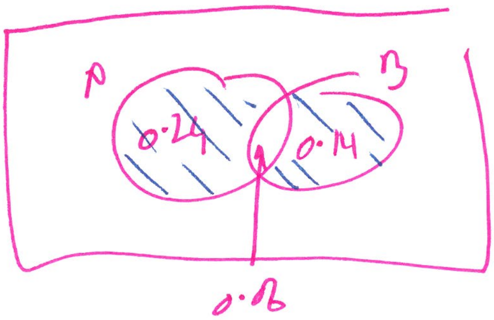
$0.44 = 0.3 + 0.2 - P(A \cap B)$

$P(A \cap B) = 0.5 - 0.44$

$= 0.06$

$P(A \cap B)$ is consistent.

(b)



$P(\text{exactly one}) = P(A \cup B) - P(A \cap B)$

$P(\text{exactly 1}) = 0.24 + 0.14$

$= 0.38.$

1 The events A and B are such that:

$P(A) = 0.45, P(B) = 0.30, P(A \cap B) = 0.25$

Calculate the probability of:

(a) $P(A \cup B)$

(b) $P(A' \cap B')$.

2 Amy and Bethany each throw a fair cubical die with faces numbered, 1, 2, 3, 4, 5, 6.

(a) Calculate the probability that the score on Amy's die is:

(i) equal to the score on Bethany's die

(ii) greater than the score on Bethany's die.

(b) Given that the sum of the scores on the two dice is 4, find the probability that the two scores are equal.

3 A and B are two independent events such that:

$P(A) = 0.3$ and $P(A \cup B) = 0.5$.

(a) Calculate $P(B)$.

(b) Calculate the probability that exactly one of the two events occurs.

4 The events A and B are such that:

$P(A) = 0.4, P(B) = 0.35$ and $P(A' \cap B') = 0.4$.

Determine whether:

(a) A and B are mutually exclusive

(b) A and B are independent.

5 Two independent events A and B are such that:

$P(A) = 0.4, P(B) = 0.3$

Find:

(a) $P(A \cap B)$ [1]

(b) $P(A \cup B)$ [3]

(c) the probability that neither A nor B occurs. [3]

6 Two events A and B are such that:

$P(A) = 0.4, P(B) = 0.2, P(A \cup B) = 0.5$

(a) Calculate $P(A \cap B)$. [2]

(b) Determine whether or not A and B are independent. [2]

7 Events A and B are such that

$P(A) = 0{\cdot}2, P(B) = 0{\cdot}4, P(A \cup B) = 0{\cdot}52$.

(a) Show that A and B are independent. [5]

(b) Calculate the probability of exactly one of the two events occurring. [2]

8 The events A and B are such that

$P(A) = 0{\cdot}3, P(B) = 0{\cdot}4$.

Evaluate $P(A \cup B)$ in each of the following cases.

(a) A and B are mutually exclusive.

(b) A and B are independent.

9 The events A, B are such that $P(A) = 0.2, P(B) = 0.3$. Determine the value of $P(A \cup B)$ when

(a) A, B are mutually exclusive, [2]

(b) A, B are independent, [3]

(c) $A \subset B$. [1]

Numerical Answers

(1a) 0.5 (b) 0.52 (2ai) 6/36 (ii) 15/36 (b) 1/3 (3a) 2/7 (b) 29/70 (5a) 0.12 (b) 0.58 (c) 0.42

(6a) 0.1 (7b) 0.44 (8a) 0.7 (b) 0.58 (9a) 0.5 (b) ~~0.54~~ 0.44 (c) 0.3

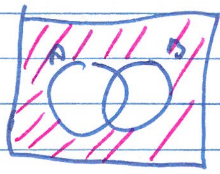
AS Probability (Textbook 'Test Yourself')

(1) $P(A) = 0.45 \qquad P(B) = 0.30 \qquad P(A \cap B) = 0.25$

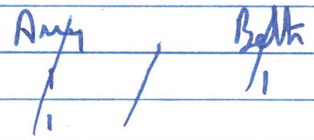
(a) $P(A \cup B) = 0.45 + 0.30 - 0.25$

$= 0.5$

(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.5$



(~~2~~)



(2)

		Amy 1	2	3	4	5	6
	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
B	3	3,1	3,2	3,3	3,4	3,5	3,6
E							
T	4	4,1	4,2	4,3	4,4	4,5	4,6
N	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

(a) (i) $P(\text{equal scores}) = \frac{6}{36}$

(ii) $P(A > B) = \frac{15}{36}$

(b) Possible outcomes are $(1,3), (2,2), (3,1)$

If we know it is one of these the $P(A=B) = \frac{1}{3}$

(3) (a) A and B are independent

$P(A) = 0.3 \qquad P(A \cup B) = 0.5$

Now $P(A \cap B) = P(A) \times P(B)$

$= 0.3\,P(B)$

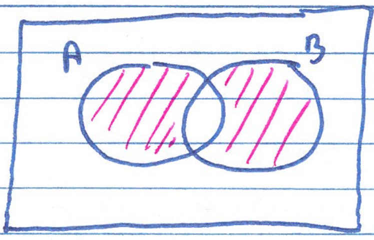
Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.5 = 0.3 + P(B) - 0.3P(B)$

$0.2 = 0.7\,P(B) \quad \therefore \; P(B) = \frac{2}{7}$

(3) contd

(b)



$P(\text{exactly } 1) = P(A \cup B) - P(A \cap B)$

$= 0.5 - \frac{3}{35}$

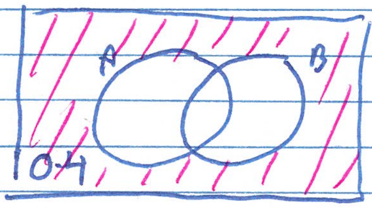
$= \frac{29}{70} \quad (0.4143)$

$P(A \cap B) = 0.3 \times \frac{2}{7} = \frac{3}{35}$

(4)(a) $P(A) = 0.4 \quad P(B) = 0.35 \quad P(A' \cap B') = 0.4$

If A and B are Mut exc then $P(A \cap B) = 0$

$P(A' \cap B') =$



So $P(A \cup B) = 0.6$

Now $0.6 = 0.4 + 0.35 - P(A \cap B)$

$P(A \cap B) = 0.75 - 0.6 = 0.15$

$P(A \cap B) \neq 0 \therefore$ events aren't mut exc.

(b) if independent then $P(A) \times P(B) = P(A \cap B) = 0.15$

$0.4 \times 0.35 = 0.14 \neq 0.15$

$\therefore$ events aren't independent either.

(5) $P(A) = 0.4$, $P(B) = 0.3$

We are given A & B are independent

(a) $\cancel{P(A)}$ $P(A \cap B) = 0.4 \times 0.3 = 0.12$

(b) $P(A \cup B) = 0.4 + 0.3 - 0.12$

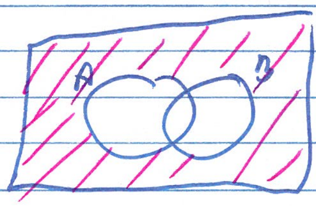
$= 0.58$

(c) P(neither A nor B) $= P(A' \cup B')$

$= 1 - P(A \cup B)$

$= 1 - 0.58$

$= 0.42$



(6) $P(A) = 0.4$ $P(B) = 0.2$ $P(A \cup B) = 0.52$

(a) $0.52 = 0.4 + 0.2 - P(A \cap B)$

$P(A \cap B) = 0.6 - 0.52 = 0.08$

(b) If A & B are independent then $P(A) \times P(B) = 0.08$

$0.4 \times 0.2 = 0.08$

So they are.

(7) $P(A) = 0.\cancel{3}2$ $P(B) = 0.4$ $P(A \cup B) = 0.52$

(a) $0.52 = 0.6 +$

(6) $P(A) = 0.4$ $P(B) = 0.2$ $P(A \cup B) = 0.5$

(a) $0.5 = 0.4 + 0.2 - P(A \cap B)$

$P(A \cap B) = 0.6 - 0.5 = 0.1$

(b) If A & B are independent then $P(A) \times P(B) = 0.1$

$0.4 \times 0.2 = 0.08$

So they aren't.

(7) $P(A) = 0.2$ $P(B) = 0.4$ $P(A \cup B) = 0.52$

(a) $0.52 = 0.2 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.6 - 0.52 = 0.08$

If A & B are independent then $P(A) \times P(B) = 0.08$

$0.2 \times 0.4 = 0.08$

So they are

(b) $P(\text{exactly one occur}) = P(A \cup B) - P(A \cap B)$

$= 0.52 - 0.08$

$= 0.44$

(8) $P(A) = 0.3$ $P(B) = 0.4$

(a) If Mut exc then $P(A \cup B) = 0.3 + 0.4 = 0.7$

(b) If independent then $P(A \cup B) = 0.3 + 0.4 - (0.3 \times 0.4)$

$= 0.7 - 0.12$

$= 0.58$

(9) $P(A) = 0.2 \quad P(B) = 0.3$

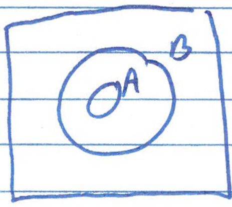
(a) If mut exc then $P(A \cup B) = 0.5$

(b) If independent then $P(A \cup B) = 0.2 + 0.3 - (0.2 \times 0.3)$

$= 0.5 - 0.06$

$= 0.44$

(c) $A \subset B$

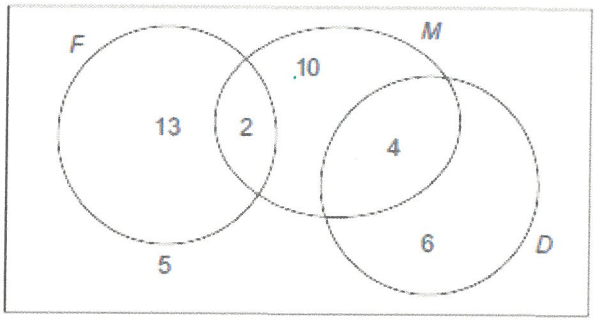


$P(A \cap B) = P(B) = 0.3$

PPQs

AS U2 June 2018

0 2 The Venn diagram shows the subjects studied by 40 sixth form students. F represents the set of students who study French, M represents the set of students who study Mathematics and D represents the set of students who study Drama.
The diagram shows the number of students in each set.



a) Explain what $M \cap D'$ means in this context. [1]

b) One of these students is chosen at random. Find the probability that this student studies

i) exactly two of these subjects,

ii) Mathematics or French or both. [3]

c) Determine whether studying Mathematics and studying Drama are statistically independent for these students. [3]

AS U2 June 2019

0 1 Three events A, B, C are such that $P(B) = \frac{1}{5}$, $P(C) = \frac{1}{6}$, $P(A \cup B) = \frac{3}{4}$, $P(A \cup C) = \frac{5}{6}$.

The events A and B are independent.

a) Find $P(A)$. [3]

b) Determine whether or not A and C are independent. [3]

c) What can be said about the events B and C if $B \cap C = \varnothing$? [1]

June 18 U2

Q2 (a) They study maths and don't study drama. E1

(b) (i) $P(\text{exactly } 2) = \frac{6}{40}$ B1

(ii) $P(M) = \frac{16}{40}$

$P(F) = \frac{15}{40}$

$P(M \cap F) = \frac{2}{40}$

$P(M \cup F) = P(M) + P(F) - P(M \cap F)$

$= \frac{16}{40} + \frac{15}{40} - \frac{2}{40}$ M1

$= \frac{29}{40}$ A1

(c) If independent $P(M \cap D) = P(M) \times P(D)$

$= \frac{16}{40} \times \frac{10}{40}$

$= \frac{160}{1600} = \frac{1}{10}$ B1

From Venn Diagram $= P(M \cap D) = \frac{4}{40} = \frac{1}{10}$ B1

These are consistent $\therefore$ the events are independent E1

$\frac{M}{7}$

AS Mathematics Unit 2: Applied Mathematics A

Solutions and Mark Scheme Summer 2018

SECTION A – Statistics

Qu. No.	Solution	Mark	Notes
1.	$P(X=7)=\binom{16}{7}\times 0.3^7\times 0.7^9$	M1	M0 if no calculation shown.
	$P(X=7)=0.1009(6\ldots)$	A1	Accept 0.101.
		[2]	
2(a)	(The set of) students who study Mathematics and not Drama.	E1	Do not accept reference to 'number of students' or 'probability'
(b) (i)	$\frac{6}{40}$ OR $\frac{3}{20}$ OR 0.15	B1	
(ii)	$P(M\cup F)=\frac{13+2+10+4}{40}$ oe	M1	
	$=\frac{29}{40}$ OR 0.725	A1	
(c)	$P(M)=\frac{16}{40}$ $P(D)=\frac{10}{40}$ $P(M\cap D)=\frac{4}{40}$	B1	All 3 correct (0.4, 0.25, 0.1)
	$P(M)\times P(D)=\frac{16}{40}\times\frac{10}{40}=\frac{1}{10}(=P(M\cap D))$	B1	Correctly evaluating 'their P(M)' x 'their P(D)' provided at least one correct. Accept alternative method
	Since $P(M)\times P(D)=P(M\cap D)$ they are statistically independent.	E1	FT candidate's probabilities provided B1 awarded. Convincing.
		[7]	

June 19 U2

Q1 (a) A & B are independent

$\therefore \; P(A \cap B) = P(A) \times P(B)$

$= P(A) \times \frac{1}{5}$

$= \frac{P(A)}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} = P(A) + \frac{1}{5} - \frac{P(A)}{5}$ M1

A1

$\frac{11}{20} = \frac{4}{5}P(A)$

$P(A) = \frac{11}{16}$ A1

(b) $P(A) = \frac{11}{16}$ $\quad P(C) = \frac{1}{6}$

If independent then $P(A \cap C) = \frac{11}{16} \times \frac{1}{6} = \frac{11}{96}$. B1

also $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$\frac{5}{6} = \frac{11}{16} + \frac{1}{6} - P(A \cap C)$

$P(A \cap C) = \frac{11}{16} + \frac{1}{6} - \frac{5}{6} = \frac{1}{48}$ B1

Inconsistent $\therefore$ A & C aren't independent. E1

(c) Mutually exclusive E1

M/7

GCE MATHEMATICS

AS UNIT 2 APPLIED MATHEMATICS A

SUMMER 2019 MARK SCHEME

SECTION A - STATISTICS

Qu. No.	Solution	Mark	Notes
1(a)	Correct use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	M1	Use of addition formula with at least $P(A \cup B)$ and $P(B)$ correct.
	$\frac{3}{4} = P(A) + \frac{1}{5} - \frac{1}{5}P(A)$	A1	
	$P(A) = \frac{11}{16}$ (0.6875)	A1	
(b)	$P(A \cap C) = \frac{11}{16} + \frac{1}{6} - \frac{5}{6}$ $= \frac{1}{48}$ $(0.0208333\ldots)$	B1	FT 'their P(A)' provided $0 \le P(A) \le 1$ and leads to $P(A \cap C)$ being between 0 and 1.
	$P(A) \times P(C) = \frac{11}{16} \times \frac{1}{6} = \frac{11}{96}$ $(0.11458333\ldots)$	B1	FT 'their $P(A)$'
	Since $\frac{1}{48} \ne \frac{11}{96}$, A and C are not independent.	E1	Award only from appropriate working, provided B1B1 awarded.
	OR		
	If A and C are independent, then $P(A \cap C) = P(A) \times P(C)$ $= \frac{11}{16} \times \frac{1}{6} = \frac{11}{96}$ $(0.11458333\ldots)$	(B1)	si FT 'their P(A)' provided $0 \le P(A) \le 1$ and leads to $P(A \cap C)$ being between 0 and 1.
	$P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{71}{96}$	(B1)	FT 'their $P(A)$'
	Since $\frac{71}{96} \ne \frac{5}{6}$, A and C are not independent.	(E1)	Award only from appropriate working, provided B1B1 awarded.