Conditional Probability & Two-way tables

Eg10 In a college there are 100 students taking A level French, German or Spanish. Of these students, 64 are female and the rest are male. There are 50 French students of whom 40 are female and 30 German students of whom 10 are female.

Find the probability that a randomly chosen student
(a) is taking Spanish
(b) is male, given that the student is taking Spanish

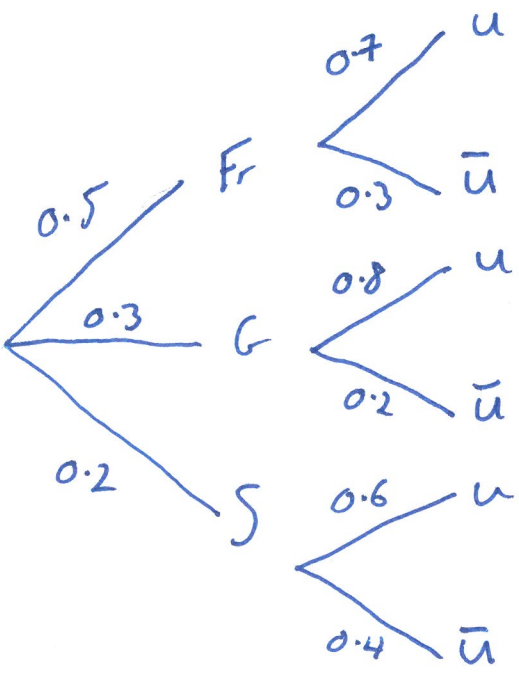
College records indicate that 70% of the French students, 80% of the German students and 60% of the Spanish students have applied for University.

A student is chosen at random.
(c) Find the probability that this student has applied for University.
(d) Given that the student had applied to University, find the probability that the student is studying French

	F	M	T
Fr	40	10	50
G	10	20	30
S	14	6	20
T	64	36	100

(a) $P(\text{Spanish}) = \frac{20}{100}$

(b) $P(M \mid Sp) = \frac{6}{20}$



(c) $P(U) = P(Fr \cap U) + P(G \cap U) + P(S \cap U)$

$= (0.5 \times 0.7) + (0.3 \times 0.8) + (0.2 \times 0.6)$

$= 0.35 + 0.24 + 0.12$

$= 0.71$

(d) $P(F \mid U) = \frac{P(F \cap U)}{P(U)}$

$= \frac{0.35}{0.71} = \frac{35}{71}$

$= 0.493$

If the information is provided in terms of percentages, care must be taken when considering the probabilities:

Eg10 In a town 40% of the people are male
20% of males are left-handed
26% of the people in the town are left-handed

A person is selected at random. Given that this person is a female, what is the probability that she is left-handed?

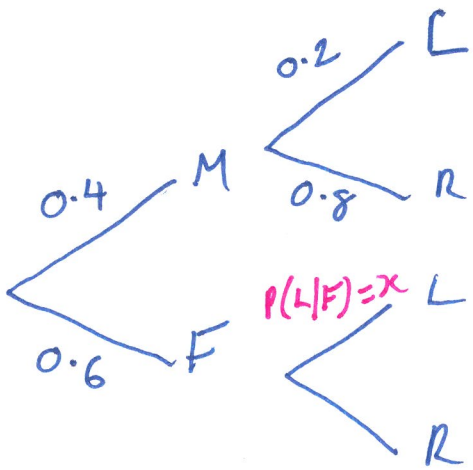
Consider a population of 100 people.

	L	R	T
M	8	32	40
F	18	42	60
T	26	74	100

20% of Males are L
20% of 40 = 8

$$P(L|F) = \frac{18}{60} = 0.3$$

As a tree diagram:



$P(L) = 0.26$

$P(L) = P(M \cap L) + P(F \cap L)$

$0.26 = (0.4 \times 0.2) + (0.6 \times x)$

$0.26 = 0.08 + 0.6x$

$x = \frac{0.26 - 0.08}{0.6}$

$x = 0.3$

Exercise 3

1. Debbie counts the videos and DVDs she has and places them into categories.

	Videos	DVDs	TOTAL
Film	43	16	59
Comedy	12	5	17
Sport	21	3	24
TOTAL	76	24	100

a) Complete the table above.

One of the items is chosen. Calculate the probability of choosing

b) a DVD. $\frac{24}{100}$
c) a comedy on video. $\frac{12}{100}$
d) it being sport, given that it is a dvd $\frac{3}{24}$
e) it is a video, given that it is not a film $\frac{33}{41}$

2. Jamie investigated hair and eye colour.

	Fair hair	Dark hair	TOTAL
Blue eyes	8	5	13
Other	7	10	17
TOTAL	15	15	30

a) Complete the table above.

One of the people is chosen. Calculate the probability of choosing

b) someone with blue eyes. $\frac{13}{30}$
c) someone with fair hair and blue eyes. $\frac{8}{30}$

Given that a person with dark hair is chosen. Calculate the probability of

d) them having blue eyes. $\frac{5}{15}$
e) them not having blue eyes. $\frac{10}{15}$

3. A travel agent recorded the bookings made on one Saturday.

	France	Spain	Germany	TOTAL
Car/Ferry	15	8	5	28
Plane	3	6	3	12
TOTAL	18	14	8	40

a) Complete the table above.

One of the bookings is chosen. Calculate the probability of choosing

b) a booking for Germany. 8/40
c) it being to France given that it is by plane 3/12
d) a ferry journey, given it is to Germany 5/8

4. Carol records some cars that pass by her house.

	Volvo	Renault	Ford	TOTAL
Grey	8	9	14	31
Red	4	11	8	23
Blue	0	3	3	6
TOTAL	12	23	25	60

a) Complete the table above.

One of the cars is chosen. Calculate the probability of choosing

b) a red car. 23/60
c) a Volvo given that it is a grey car 8/31
d) it being blue given that it is a Ford 3/25

5. An electrical store records the following information.

	Under 21	21 -45	Over 45	TOTAL
Satellite	48	23	19	90
Terrestrial	28	21	11	60
Cable	21	43	86	150
TOTAL	97	87	116	300

a) Complete the table above.

One of the records is chosen. Calculate the probability of choosing

b) someone under 21 with cable. 21/300
c) them not having cable given they are over 45 30/116

6. Put the following information into a two-way table.

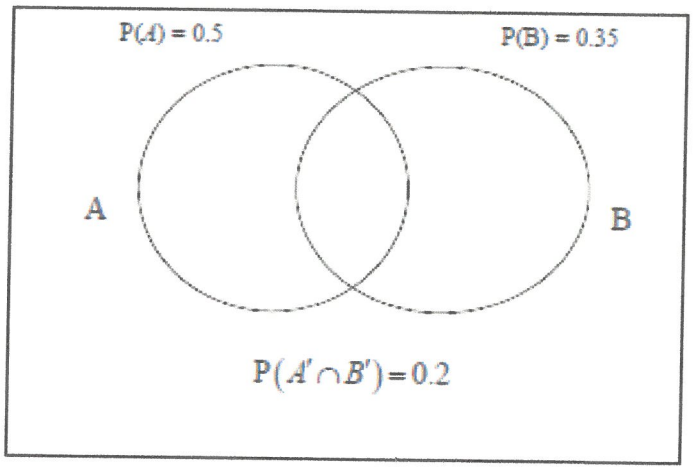
- Ford – Blue
- Vauxhall – Red
- Vauxhall – White
- Ford – Blue
- Ford – Red
- Ford – Red
- VW – Blue
- Ford – Green
- Nissan – White
- Ford – Green
- Rover – Silver
- Rover – White
- VW – Silver
- VW – Blue
- Ford – Silver

	F	V	Vw	N	R	T
B	2	0	2	0	0	4
R	2	1	0	0	0	3
W	0	1	0	1	1	3
G	2	0	0	0	0	2
S	1	0	1	0	1	3
T	7	2	3	1	2	15

A2 Unit 4 Assignment#1 – Probability

Due 30/09/19

1. Probabilities relating to the events A and B are given in the diagram below:



Find the probability that both A and B occur.

2. The number of students selecting English and History is as follows:

	History	Not History	Total
English	9	21	30
Not English	6	14	20
Total	15	35	50

Let E be the event that the student studies English.
Let H be the event that the student studies History.
Are E and H independent events?

3. A student regularly has to take two train journeys on a Sunday.
Over a long period of time she has worked out the probability that the first train is late is 0.4.
If the first train is late, the probability that the second train is late is 0.5.
If the first train is not late, the probability that the second train is late is 0.3.
   (i) Draw a tree diagram to show the possible outcomes for her next 2 journeys on a Sunday.
   (ii) Using the tree diagram calculate the probability
      (A) that exactly one journey is on time,
      (B) that both trains are on time, given that the second journey is on time.

4. Of a group of students studying at a particular college, 56% are male. The probability that a male student is studying maths is $\frac{1}{5}$ and the probability that a female student is studying maths is $\frac{1}{11}$.
   A student is chosen at random from the college. Find the probability that this student is
   (i) a female student who is studying maths

   (ii) a student who does not study maths
   (iii) a male student given that it is a student who is studying maths.

5. In the build up to the Olympics a high jumper measured her success at a particular height. She has a maximum of 3 attempts at this height; once she has jumped successfully she does not jump that height again.
   On 60% of occasions she clears it at the first attempt.
   When she attempts the height for the second time, she is successful in 75% of the attempts.
   When she attempts the height for the third time, she is successful in only 30% of the attempts.
   (i) Draw a tree diagram to show the possible outcomes.
   (ii) From the tree diagram calculate the probability
      (A) that she successfully clears the height in her three attempts,
      (B) that, given she clears the height, that she does so successfully at the first attempt.

FMSP Ex2

(1) $P(A \cup B) = 1 - 0.2 = 0.8$.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.35 - 0.8$

$= 0.05$

(2) $P(E) = \frac{30}{50}$, $P(H) = \frac{15}{50}$, P(E∩H)

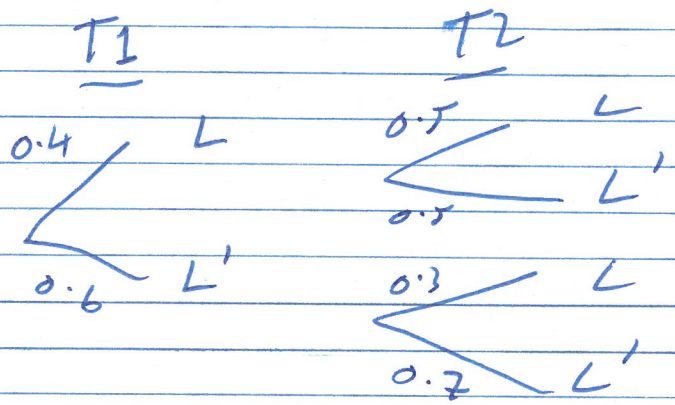
if independent then $P(E \cap H) = \frac{30}{50} \times \frac{15}{50} = \frac{9}{50}$

P(E∩A) = P

From table n° taking E AND H = $\frac{9}{50}$, so yes.

(3)

(I)



(II) (a) $P(\text{exact one on time}) = (0.4 \times 0.5) + (0.6 \times 0.3)$

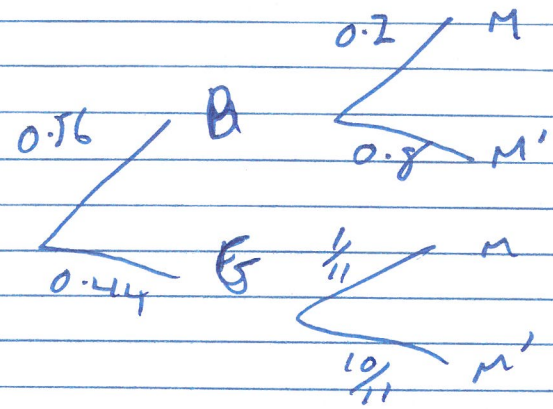
$= 0.38$

(b) $P(T1 \text{ on time}) = 0.6$

$P(T2 \text{ on time}) = (0.4 \times 0.5) + (0.6 \times 0.7) = 0.62$

$P(T1 \text{ on time} \mid T2 \text{ on time}) = \frac{P(T1 \cap T2)}{P(T2)} = \frac{0.6 \times 0.7}{0.62} = 0.677$

(4)



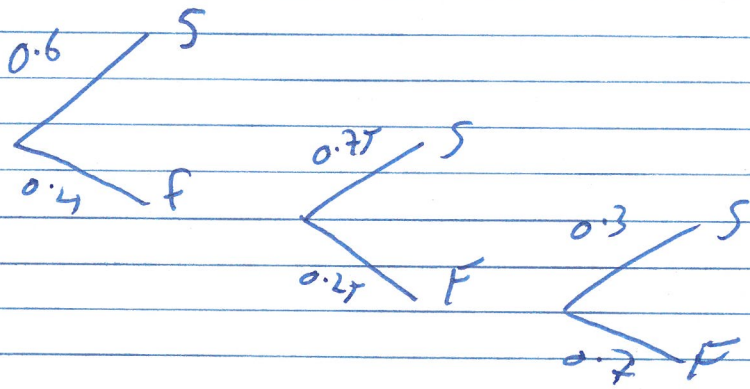
(i) $P(B' \cap M) = 0.44 \times \frac{1}{11} = \frac{1}{25}$

(ii) $P(M') = (0.76 \times 0.8) + (0.44 \times \frac{10}{11}) = 0.849$

(iii) $P(B|M) = \frac{P(B \cap M)}{P(M)} = \frac{0.76 \times 0.2}{1 - 0.849} = 0.737.$

(5)

(i)



(ii) (A) $P(\text{succon}) = P(S) + P(FS) + P(FFS)$

$= 0.6 + (0.4 \times 0.75) + P(0.4 \times 0.25 \times 0.3)$

$= 0.93$

(B) $P(S1|S) = \frac{P(S1 \cap S)}{P(S)} = \frac{0.6}{0.93} = 0.645.$