**5. Managing Variable Acceleration Using Vectors**

**From Unit 2:**

When acceleration is not uniform and the position, velocity and accelerations are given as functions of time then calculus is used to model particle motion:

DIFFERENTIATE with respect to *t*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Displacement*****s*** |  | **Velocity*****v*** |  | **Acceleration*****a*** |

INTEGRATE with respect to *t*

**Eg1** A particle P is moving along a straight line. At time t = 0, the particle is at point A and is moving with velocity 8ms-1 towards a point B on the line, where AB = 30m. At time t seconds (where t ≥ 0), the acceleration of P is (2 – 2t)ms-2 in the direction $\vec{AB}$.

1. Find an expression, in terms of t, for the displacement of P from A at t seconds
2. Show that P does not reach B.
3. Find the value of t when P returns to A, giving your answer to 3 significant figures.
4. Find the total distance travelled by P in the interval between the two instants when it passes through A.

**Eg2** A small metal ball moving in a magnetic field is modelled as a particle P of mass 0.2kg, moving in a straight line under the action of a single variable force F Newtons. At time t seconds, the displacement, x metres is given by $x=3\sin(2t)$.

 Find the magnitude of F when $t=\frac{π}{6}$

**Unit 4:**

When a particle is moving in a plane you can describe its position **r**, its velocity **v** and its acceleration **a** using vectors. The relationships between position (displacement), velocity and acceleration are the same in two dimensions as in one dimension.



**Eg3** A particle P is moving in a plane. At time t seconds, its velocity **V**ms-1 is given by

 $v=3ti+\frac{1}{2}t^{2}j$

When t = 0, the position vector of P is (2**i** - 3**j**)m. Find

1. the position vector of P at time t seconds
2. the acceleration of P when t = 3.

**Eg4** A particle P is moving in a plane so that, at time t seconds, its acceleration is (4**i** – 2t**j**) ms-2. When t = 3, the velocity of P is 6**i** ms-1 and the position vector of P is (20**i** + 3**j**)m with respect to the origin. Find

1. the angle between the direction of motion of P and **i** when t = 2
2. the distance of P from O when t = 0.

**Exercise 5.1 Qs 1 to 4**

**Eg5** A particle P of mass 0.5kg is moving under the action of a single force **F** newtons. At time t seconds, the position vector of P, **r** metres, is given by

 $r=\left(\frac{3t^{2}}{2}-\frac{t^{3}}{3}\right)i+\left(2t^{2}-8t\right)j$

 Find

1. the value of t when P is moving parallel to the vector **i**
2. the magnitude of **F** when t = 3.5

**Eg6** The velocity of a particle P at time t seconds is ((3t2 – 8)**i** + 5**j**) ms-1. When t = 0, the position vector of P with respect to a fixed origin is (2**i** – 4**j**)m.

1. Find the position vector of P after t seconds

A second particle Q moves with constant velocity (8**i** + 4**j**) ms-1. When t = 0 the position vector of Q with respect to the origin is 2**i** m.

1. Prove that P and Q collide.



**Ex 5.1 Numerical Answers**



**Exercise 5.1 Remaining Odds**

**Exercise 5.1**



