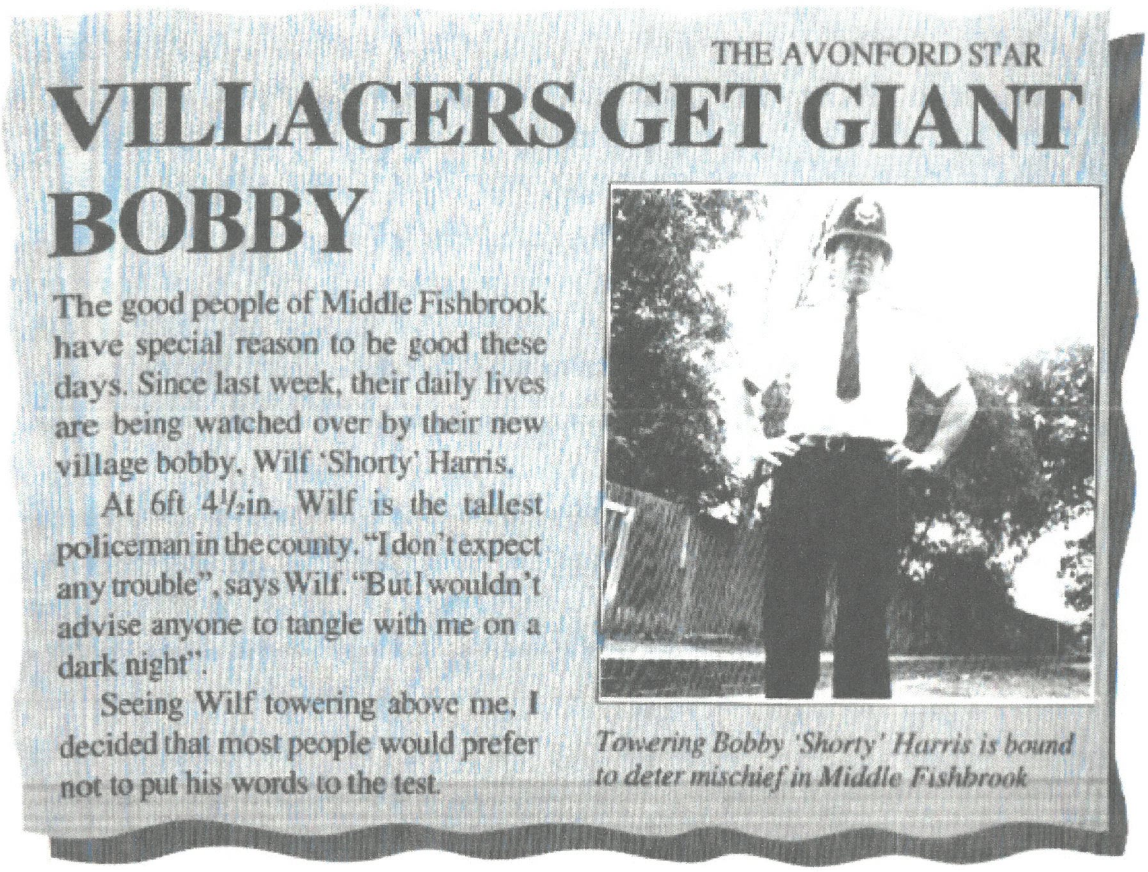
The Normal Distribution

Just as the binomial and Poisson distributions are important examples of the special distributions of the discrete kind, so the Normal distribution can be described as the single, most important continuous distribution in statistics. The form of the data approximates very well to data of the 'natural phenomenon' type, such as weights, heights and ages; data that occurs naturally in all types of situations.



THE AVONFORD STAR

VILLAGERS GET GIANT BOBBY

The good people of Middle Fishbrook have special reason to be good these days. Since last week, their daily lives are being watched over by their new village bobby, Wilf 'Shorty' Harris.

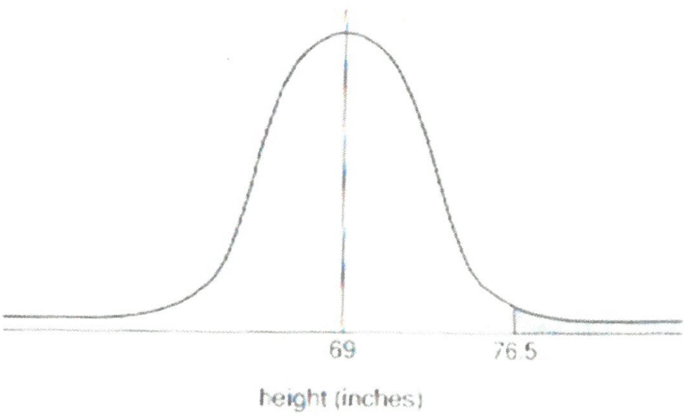
At 6ft $4\frac{1}{2}$in, Wilf is the tallest policeman in the county. "I don't expect any trouble", says Wilf. "But I wouldn't advise anyone to tangle with me on a dark night".

Seeing Wilf towering above me, I decided that most people would prefer not to put his words to the test.

Towering Bobby 'Shorty' Harris is bound to deter mischief in Middle Fishbrook

Wilf is clearly very tall, but how much so? Is he one in a hundred, or a thousand or even a million?

To answer this question we need to know the distribution of the heights of adult British men. This may be modelled by the Normal distribution which has the distinctive pdf shown below:



As always with probability density functions, the area beneath the curve represents probability, so the shaded area to the right of 76.5in represents the probability that a randomly selected adult male is over 6ft 4½ inches tall.

Before we are able to find this area we need to know the mean and standard deviation of the distribution. For adult British males these are 69 inches and 2.5 inches respectively.

We can summarise this as

for the continuous random variable H, where $H \sim N(69, 2.5^2)$, find $P(H > 76.5)$

we can use our calculators:

Normal CD
Lower = 76.5
Upper = 1×10^{99}
$\sigma = 2.5$
$\mu = 69$

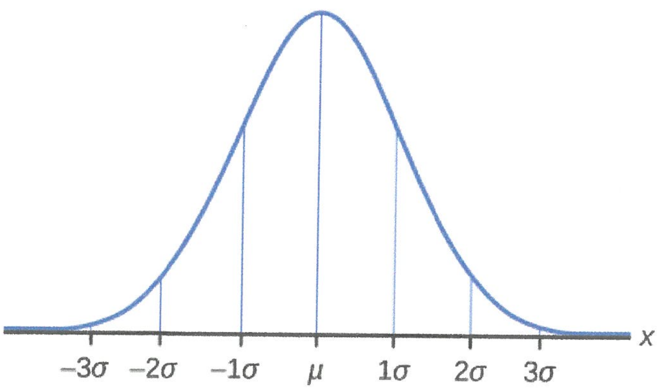
which will produce $P(H > 76.5) =$ 0.0013

So the probability of a randomly selected adult male being at least as tall is Wilf is 0.0013, ie just over 1 man in a thousand.

The key properties of a Normal distribution can be summarised as:

- The distribution is symmetrical about the mean, μ
- The mode, median and mean are all equal, due to the symmetry of the distribution
- The range of x is from $-\infty$ to ∞
- The horizontal axis is asymptotic to the curve
- The total area beneath the curve is unity
- 68% of the values in a Normal distribution lie within ± 1 standard deviation of the mean
- 95% of the values lie within ± 2 standard deviations of the mean
- 99.75% of the values lie within ± 3 standard deviations of the mean

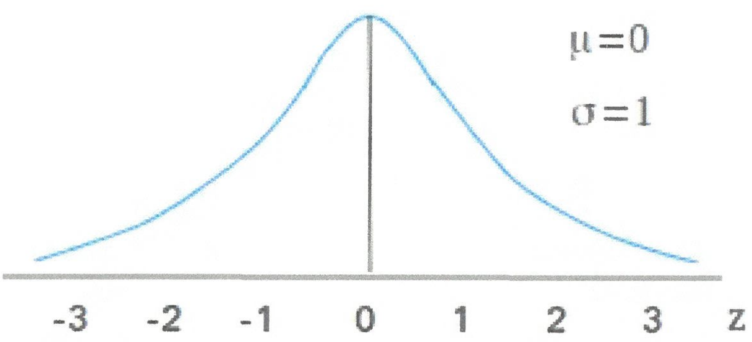
$X \sim N(\mu, \sigma^2)$



The Standard Normal Distribution, Z ~ N(0, 1^2)

This is a Normal distribution centred at 0 with a variance and hence a standard deviation of 1. All Normal distributions with mean μ and variance σ^2 can be adjusted to fit this curve as we will see later.

The standard normal distribution curve

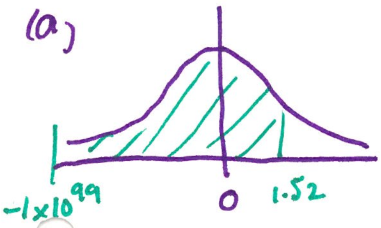


$Z \sim N(0, 1^2)$

Even though the use of calculators make working with Normal distributions fairly straightforward, it is always a good idea to sketch a diagram to represent the probability you are trying to find.

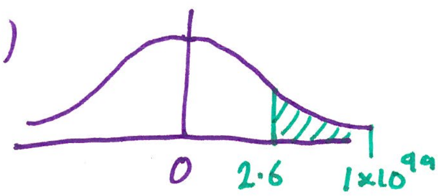
Eg10 Find (a) P(Z < 1.52) (b) P(Z > 2.60) (c) P(Z < -0.75) (d) P(-1.18 < Z < 1.43)

(a)



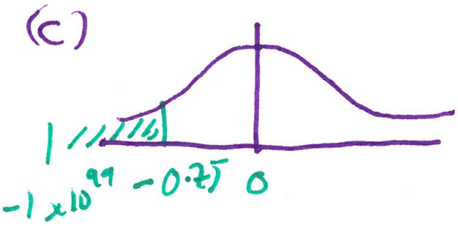
$P(Z<1.52) = 0.9357$

(b)



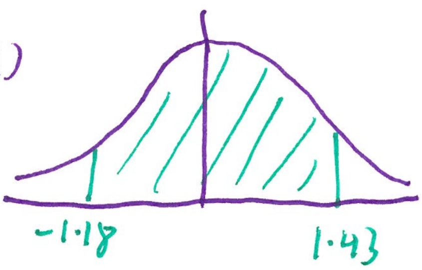
$P(Z>2.60) = 0.0047$

(c)



$P(Z<-0.75) = 0.2266$

(d)

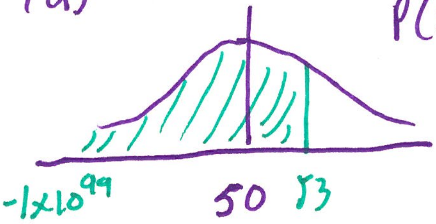


$P(-1.18<Z<1.43) = 0.8046$

Eg11 The random variable $X \sim N(50, 4^2)$.

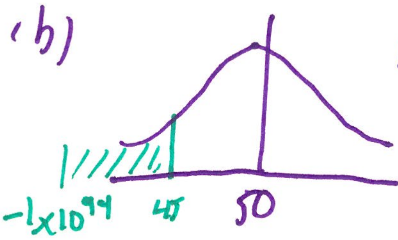
Find (a) $P(X < 53)$, (b) $P(X \leq 45)$

1a)



$P(x<53) = 0.7734$

(b)



$P(x\leq 45)$

$= 0.1056$

Eg12 When a butcher takes an order for a Christmas Turkey, he asks the customer what weight in kg the bird should be. He then sends his order to a turkey farmer who supplies birds of about the requested weight. For any particular , their error in kg may be taken to be normally distributed with mean 0 and standard deviation 0.75.

Mrs Jones orders a 10kg turkey from the butcher. Find the probability that the one she receives is

(i) over 12kg

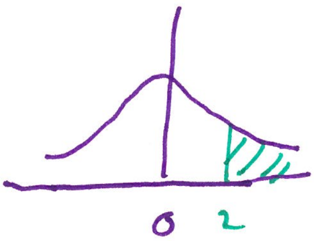
(ii) under 10kg

(iii) within 0.5kg of the weight she actually ordered.

$E \sim N(0, 0.75^2)$

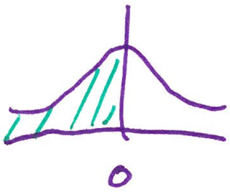
(i) $P(E > 2)$

$= 0.0038$



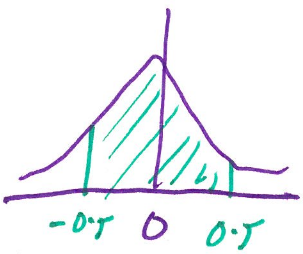
(ii) $P(E < 0)$

$= 0.5$



(iii) $P(-0.5 < E < 0.5)$

$= 0.495$



Exercise 2.3

Find the following

1 **a** $P(Z < 2.12)$ **b** $P(Z < 1.36)$

c $P(Z > 0.84)$ **d** $P(Z < -0.38)$

2 **a** $P(Z > 1.25)$ **b** $P(Z > -1.68)$

c $P(Z < -1.52)$ **d** $P(Z < 3.15)$

3 **a** $P(Z > -2.24)$ **b** $P(0 < Z < 1.42)$

c $P(-2.30 < Z < 0)$ **d** $P(Z < -1.63)$

4 **a** $P(1.25 < Z < 2.16)$ **b** $P(-1.67 < Z < 2.38)$

c $P(-2.16 < Z < -0.85)$ **d** $P(-1.57 < Z < 1.57)$

Answers

1	**a** 0.9830	**b** 0.9131	**c** 0.2005	**d** 0.3520
2	**a** 0.1056	**b** 0.9535	**c** 0.0643	**d** 0.9992
3	**a** 0.9875	**b** 0.4222	**c** 0.4893	**d** 0.0516
4	**a** 0.0902	**b** 0.9438	**c** 0.1823	**d** 0.8836

Exercise 2.4

1. The random variable $X \sim N(30, 2^2)$
   Find (a) $P(X < 33)$ (b) $P(X > 26)$

2. The random variable $X \sim N(40, 9)$.
   Find **a** $P(X > 45)$. **b** $P(X < 38)$.

3. The random variable $Y \sim N(25, 25)$.
   Find **a** $P(Y < 20)$. **b** $P(18 < Y < 26)$.

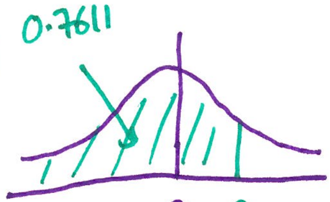
4. The random variable $X \sim N(18, 10)$.
   Find **a** $P(X > 20)$. **b** $P(X < 15)$.

1	**a**	0.9332	**b**	0.9772
2	**a**	0.0475	**b**	0.2514
3	**a**	0.1587	**b**	0.4985
4	**a**	0.264	**b**	0.171

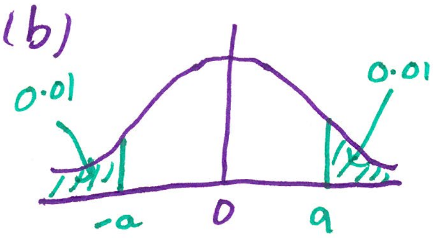
Using a given probability to find the corresponding boundary parameter

Once again, the calculator has made this a pretty straightforward process using the 'Inverse Normal' mode.

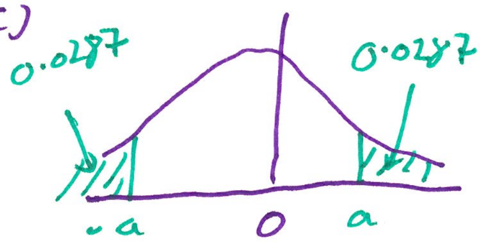
Eg13 Find the value of the constant *a* such that
(a) $P(Z < a) = 0.7611$ (b) $P(Z > a) = 0.01$ (c) $P(Z > a) = 0.0287$ (d) $P(Z < a) = 0.0170$

(a)


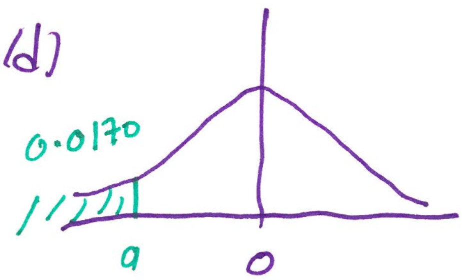
$a = 0.71$

(b)


-2.33, $a = 2.33$

(c)


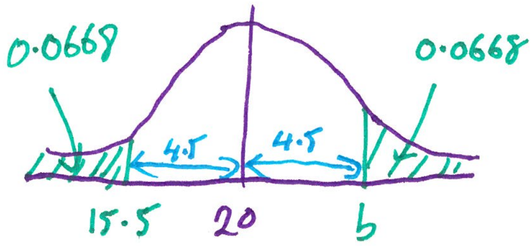
$= -1.9$, $a = 1.9$

(d)


$a = -2.12$

3^2

Eg14 The random variable $Y \sim N(20, 9)$. Find the value of b such that $P(Y > b) = 0.0668$



$= 24.5$

Eg15 In a particular experiment, the length of a metal bar is measured many times. The measured values are distributed approximately Normally with mean 1.340m and standard deviation 0.021m. Find the probabilities that any one measured value

(i) exceeds 1.370m

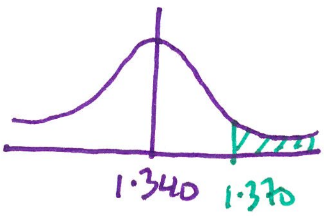
(ii) lies between 1.310m and 1.370m

(iii) lies between 1.330m and 1.390m

Find the length *l* for which the probability that any one measured value is less than *l* is 0.1.

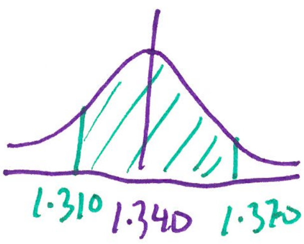
$l \sim N(1.340, 0.021^2)$

(i)



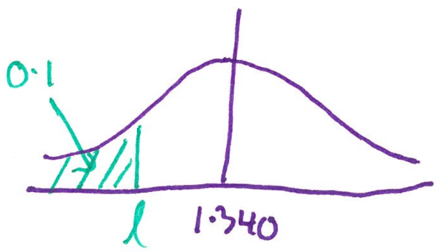
$P(l > 1.370)$
$= 0.077$

(ii) $P(1.310 < l < 1.370)$



$= 0.847$

(iii) $P(1.330 < l < 1.390) = 0.674$



$P(L < l) = 0.1$

$l = 1.313$

Exercise 2.5

5 The random variable $X \sim N(20, 8)$.

Find **a** $P(X > 15)$, **b** the value of a such that $P(X < a) = 0.8051$.

6 The random variable $Y \sim N(30, 5^2)$.

Find the value of a such that $P(Y > a) = 0.30$.

7 The random variable $X \sim N(15, 3^2)$.

Find the value of a such that $P(X > a) = 0.15$.

8 The random variable $X \sim N(20, 12)$.

Find the value of a and the value of b such that

a $P(X < a) = 0.40$, **b** $P(X > b) = 0.6915$.

c Write down $P(b < X < a)$.

9 The random variable $Y \sim N(100, 15^2)$.

Find the value of a and the value of b such that

a $P(Y > a) = 0.975$, **b** $P(Y < b) = 0.10$.

c Write down $P(a < Y < b)$.

10 The random variable $X \sim N(80, 16)$.

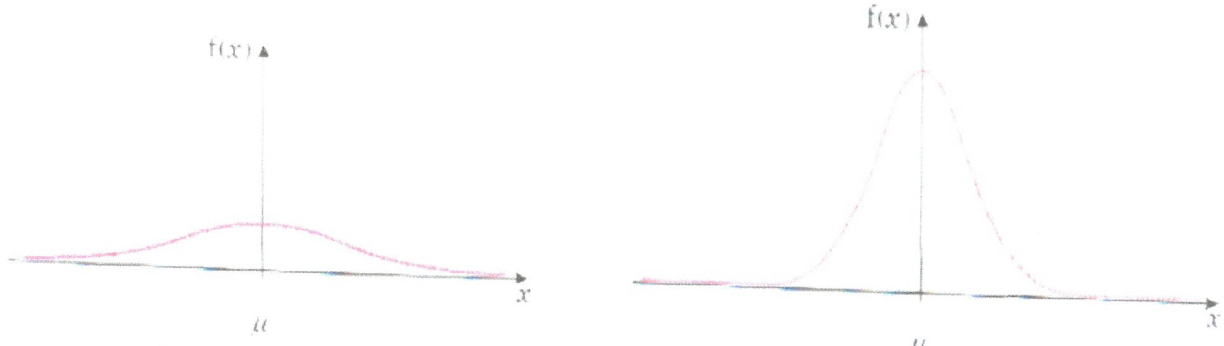
Find the value of a and the value of b such that

a $P(X > a) = 0.40$, **b** $P(X < b) = 0.5636$.

c Write down $P(b < X < a)$.

5	**a** 0.961 or 0.962		**b** 22.4
6	32.6		
7	18.1		
8	**a** 19.1	**b** 18.3	**c** 0.0915
9	**a** 70.6	**b** 80.8	**c** 0.025
10	**a** 81.0	**b** 80.6	**c** 0.0364

Using the mappability of the Standard Normal Distribution, Z, to find the mean, standard deviation, or both for a Normal distribution, X



large value of σ small value of σ

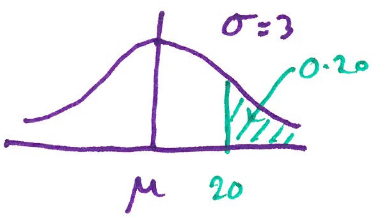
The distribution will have the same familiar 'bell' shape but the mean μ and the standard deviation σ will be different.

$X \sim N(\mu, \sigma^2)$ can be transformed into $Z \sim N(0, 1^2)$ and vice-versa using the formula

$$Z = \frac{X - \mu}{\sigma}$$

when tables rather than calculators were used to find probabilities of Normal distributions this had to be used to transform any given distribution into the Z distribution in order to read from the tables. This is no longer necessary. However, this process still needs to be used where situations call for you to determine an unknown mean or standard deviation value for a given distribution.

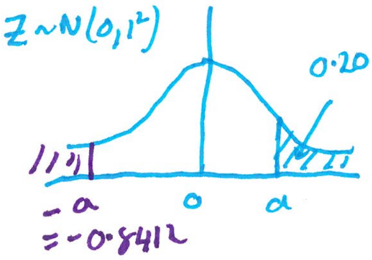
Eg16 The random variable $X \sim N(\mu, 3^2)$. Given that $P(X > 20) = 0.20$, find the value of μ.



from $Z \sim N(0,1^2)$

$P(Z < a) = 0.20$

$a = 0.8412$



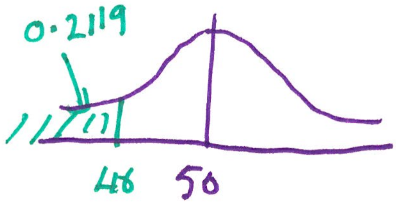
Using $a = \frac{X-\mu}{\sigma}$

$0.8412 = \frac{20-\mu}{3}$

$\mu = 20 - 3(0.8412)$

$= 17.5$

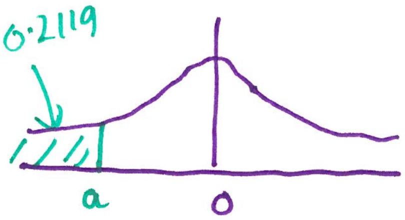
Eg17 The random variable $X \sim N(50, \sigma^2)$. Given that $P(X < 46) = 0.2119$, find the value of σ.



From $Z \sim N(0, 1^2)$

$P(Z < a) = 0.2119$

$a = -0.7998$



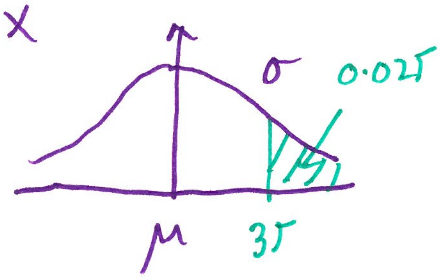
Using $Z = \dfrac{X - \mu}{\sigma}$

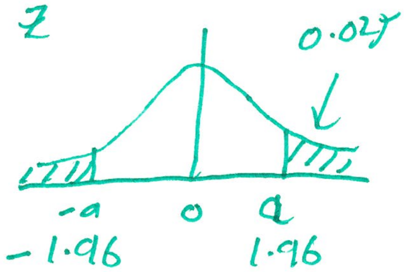
$-0.7998 = \dfrac{46 - 50}{\sigma}$

$-0.7998\sigma = -4$

$\sigma = \dfrac{-4}{-0.7998} = 5.0$

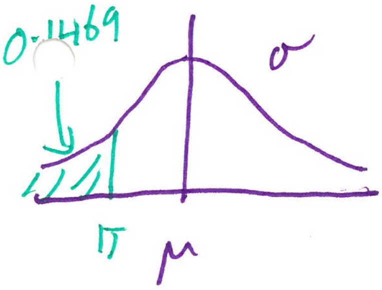
Eg18 The random variable $X \sim N(\mu, \sigma^2)$. Given that $P(X > 35) = 0.025$ and $P(X < 15) = 0.1469$, find the mean and standard deviation of the distribution.

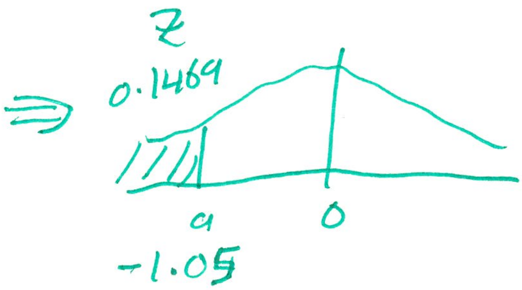




$1.96 = \dfrac{35 - \mu}{\sigma}$

$\boxed{1.96\sigma + \mu = 35 \quad \text{— (A)}}$





$-1.05 = \dfrac{15 - \mu}{\sigma}$

$\boxed{-1.05\sigma + \mu = 15 \quad \text{— (B)}}$

(A) − (B) $\quad 3\sigma = 20$

$\sigma = \dfrac{20}{3} = 6.64$

in (A) $\quad 1.96(6.64) + \mu = 35$

$\mu = 21.98$

Exercise 2.6

1 The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$.
Find the value of μ.

2 The random variable $X \sim N(11, \sigma^2)$ and $P(X > 20) = 0.01$.
Find the value of σ.

3 The random variable $Y \sim N(\mu, 40)$ and $P(Y < 25) = 0.15$.
Find the value of μ.

4 The random variable $Y \sim N(50, \sigma^2)$ and $P(Y > 40) = 0.6554$.
Find the value of σ.

5 The random variable $X \sim N(\mu, \sigma^2)$.
Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and the value of σ.

6 The random variable $Y \sim N(\mu, \sigma^2)$.
Given that $P(Y < 25) = 0.10$ and $P(Y > 35) = 0.005$, find the value of μ and the value of σ.

7 The random variable $X \sim N(\mu, \sigma^2)$.
Given that $P(X > 15) = 0.20$ and $P(X < 9) = 0.20$, find the value of μ and the value of σ.

Hint for Question 7:
Draw a diagram and use symmetry to find μ.

8 The random variable $X \sim N(\mu, \sigma^2)$.
The lower quartile of X is 25 and the upper quartile of X is 45.
Find the value of μ and the value of σ.

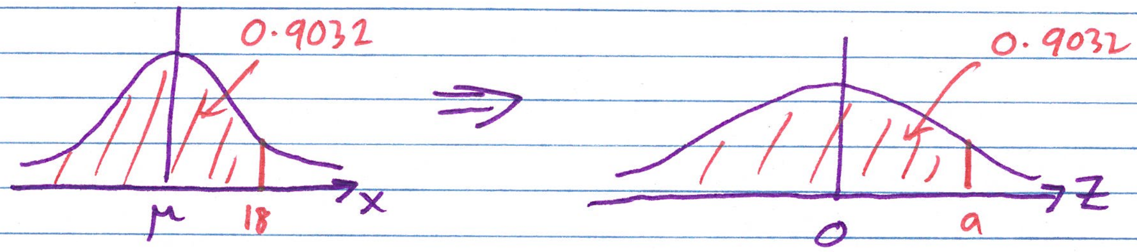
9 The random variable $X \sim N(0, \sigma^2)$.
Given that $P(-4 < X < 4) = 0.6$, find the value of σ.

10 The random variable $X \sim N(2.68, \sigma^2)$.
Given that $P(X > 2a) = 0.2$ and $P(X < a) = 0.4$, find the value of σ and the value of a

1	11.5
2	3.87
3	31.6
4	25
5	$\mu = 13.1$, $\sigma = 4.32$.
6	$\mu = 28.3$, $\sigma = 2.59$.
7	$\mu = 12$, $\sigma = 3.56$.
8	$\mu = 35$, $\sigma = 14.8$ or $\sigma = 14.9$.
9	4.75
10	$\sigma = 1.99$, $a = 2.18$.

Ex 2.6

① $X \sim N(\mu, 5^2)$ $\quad P(X < 18) = 0.9032$

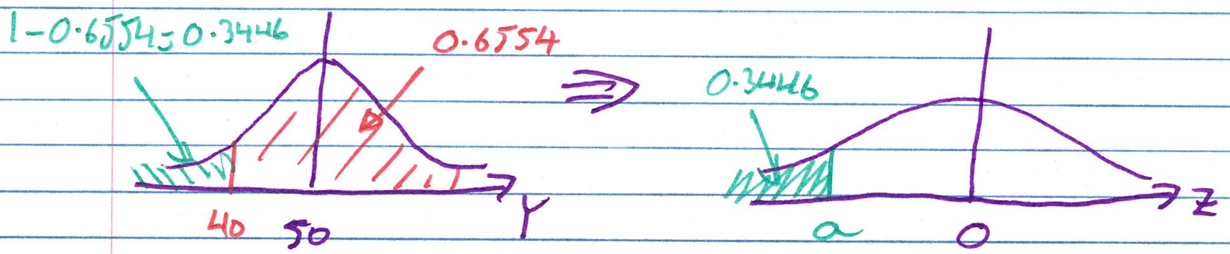


$a = 1.3$

Now $1.3 = \dfrac{18 - \mu}{5}$

$\mu = 18 - 6.5 = 11.5$

④. $Y \sim N(50, \sigma^2)$ $\quad P(Y > 40) = 0.6554$



$a = -0.4$

Now $-0.4 = \dfrac{40 - 50}{\sigma}$

$-0.4\sigma = -10$

$\sigma = \dfrac{-10}{-0.4} = 25$

Eg19 A characteristic of the shape of a human skull is measured by a number n. People are classed into three groups: A (for which $n \le 75$), B ($75 < n \le 80$) and C ($n > 80$). In a certain population the percentages of people within these groups are 58, 38 and 4 respectively. Assuming that n is distributed Normally within this population, determine its mean and standard deviation.

Three people are chosen at random from this population. Determine the probabilities that

(i) each of the three has a value of n greater than 70;

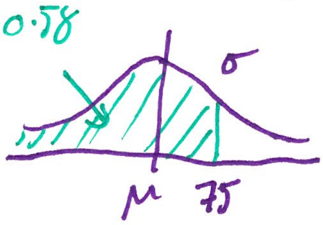
(ii) at least one of the three has a value of n less than 70.

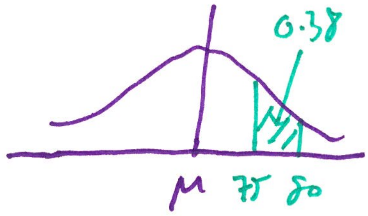
$n \sim N(\mu, \sigma^2)$

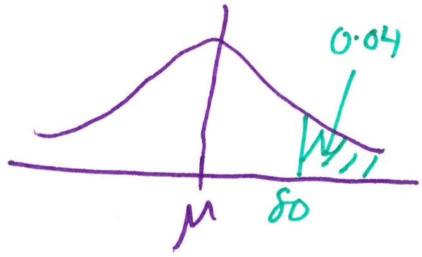
A: $P(n \le 75) = 0.58$

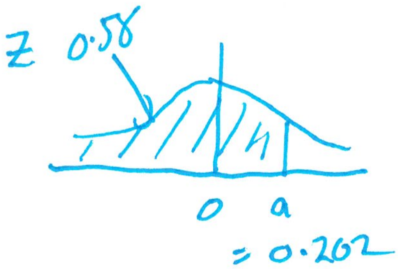
B: $P(75 < n \le 80) = 0.38$

C: $P(n > 80) = 0.04$







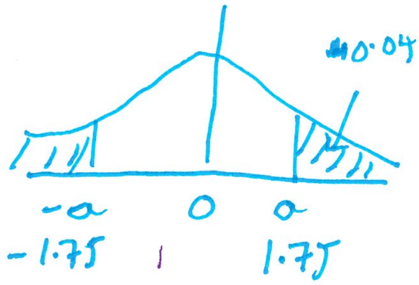


Now $0.202 = \dfrac{75-\mu}{\sigma}$

$\mu + 0.202\sigma = 75$ — (A)

And $1.75 = \dfrac{80-\mu}{\sigma}$

$\mu + 1.75\sigma = 80$ — (B)

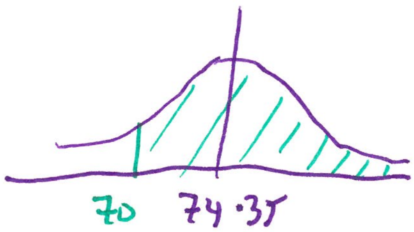


(B) − (A) $\quad 1.549\sigma = 5$

$\sigma = 3.23$

in (A) $\quad \mu = 75 - 0.202(3.23) = 74.35$

$\therefore \quad n \sim N(74.35, 3.23^2)$



(i) $P(n > 70) = 0.911$

Now $P(n>70) \times P(n>70) \times P(n>70) = 0.911^3 = 0.76$

(ii) $1 - 0.76 = 0.244$

Problem Solving – Exercise 2.7

1 The heights of a large group of men are normally distributed with a mean of 178 cm and a standard deviation of 4 cm.

A man is selected at random from this group.

a Find the probability that he is taller than 185 cm.

A manufacturer of door frames wants to ensure that fewer than 0.005 men have to stoop to pass through the frame.

b On the basis of this group, find the minimum height of a door frame.

2 The weights of steel sheets produced by a factory are known to be normally distributed with mean 32.5 kg and standard deviation 2.2 kg.

a Find the percentage of sheets that weigh less than 30 kg.

Bob requires sheets that weigh between 31.6 kg and 34.8 kg.

b Find the percentage of sheets produced that satisfy Bob's requirements.

3 The time a mobile phone battery lasts before needing to be recharged is assumed to be normally distributed with a mean of 48 hours and a standard deviation of 8 hours.

a Find the probability that a battery will last for more than 60 hours.

b Find the probability that the battery lasts less than 35 hours.

4 The random variable $X \sim \mathrm{N}(24, \sigma^2)$.

Given that $\mathrm{P}(X > 30) = 0.05$, find

a the value of σ,

b $\mathrm{P}(X < 20)$,

c the value of d so that $\mathrm{P}(X > d) = 0.01$.

5 A machine dispenses liquid into plastic cups in such a way that the volume of liquid dispensed is normally distributed with a mean of 120 ml. The cups have a capacity of 140 ml and the probability that the machine dispenses too much liquid so that the cup overflows is 0.01.

a Find the standard deviation of the volume of liquid dispensed.

b Find the probability that the machine dispenses less than 110 ml.

Ten percent of customers complain that the machine has not dispensed enough liquid.

c Find the largest volume of liquid that will lead to a complaint.

6 The random variable $X \sim \mathrm{N}(\mu, \sigma^2)$. The lower quartile of X is 20 and the upper quartile is 40.

Find μ and σ.

7 The heights of seedlings are normally distributed. Given that 10% of the seedlings are taller than 15 cm and 5% are shorter than 4 cm, find the mean and standard deviation of the heights.

8 A psychologist gives a student two different tests. The first test has a mean of 80 and a standard deviation of 10 and the student scored 85.

a Find the probability of scoring 85 or more on the first test.

The second test has a mean of 100 and a standard deviation of 15. The student scored 105 on the second test.

b Find the probability of a score of 105 or more on the second test.

c State, giving a reason, which of the student's two test scores was better.

9 Jam is sold in jars and the mean weight of the contents is 108 grams. Only 3% of jars have contents weighing less than 100 grams. Assuming that the weight of jam in a jar is normally distributed find

a the standard deviation of the weight of jam in a jar,

b the proportion of jars where the contents weigh more than 115 grams.

10 The waiting time at a doctor's surgery is assumed to be normally distributed with standard deviation of 3.8 minutes. Given that the probability of waiting more than 15 minutes is 0.0446, find

a the mean waiting time,

b the probability of waiting fewer than 5 minutes.

11 The thickness of some plastic shelving produced by a factory is normally distributed. As part of the production process the shelving is tested with two gauges. The first gauge is 7 mm thick and 98.61% of the shelving passes through this gauge. The second gauge is 5.2 mm thick and only 1.02% of the shelves pass through this gauge.

Find the mean and standard deviation of the thickness of the shelving.

1 **a** 0.0401 **b** 188 cm
2 **a** 12.7% or 12.8% **b** 51.1% or 51.2%
3 **a** 0.0668 **b** 0.052
4 **a** 3.65 **b** 0.1357 **c** 32.5
5 **a** 8.60 ml **b** 0.123 **c** 109 ml
6 $\mu = 30$, $\sigma = 14.8$ or $\sigma = 14.9$
7 mean 3.76 cm, standard deviation 10.2 cm

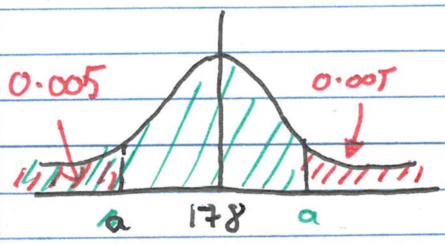
8 **a** 0.3085 **b** 0.370 or 0.371
c The first score was better, since fewer of the students got this score or more.
9 **a** 4.25 or 4.26 **b** 0.050
10 **a** 8.54 minutes **b** 0.176
11 mean 6.12 mm, standard deviation 0.398 mm

Ex2.7 (Edexcel S1 Ex9E)

(1) $H \sim N(178, 4^2)$

(a) $P(H > 185) = 0.04$

(b)



$P(H < a) = 1 - 0.005$

$a = 188.3\text{cm}$

(2) $W \sim N(32.5, 2.2^2)$

(a) $P(W < 30) = 0.128$ so 12.8%

(b) $P(31.6 < W < 34.8) = 0.511$ so 51.1%

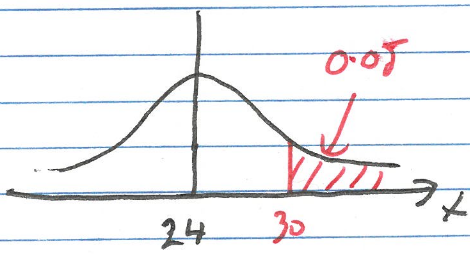
(3) $T \sim N(48, 8^2)$

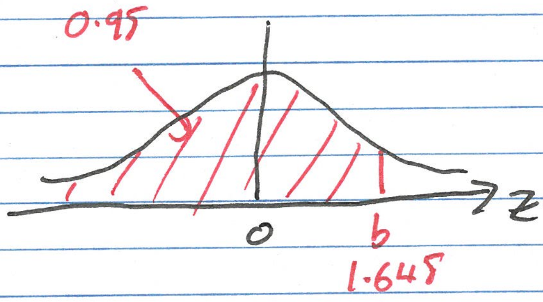
(a) $P(T > 60) = 0.0668$

(b) $P(T < 35) = 0.052$

(4) $X \sim N(24, \sigma^2)$

(a)





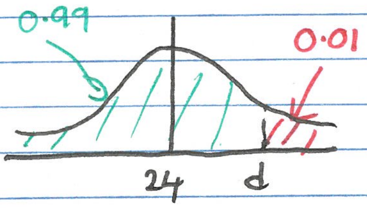
$$1.645 = \frac{30 - 24}{\sigma}$$

$$1.645\sigma = 6$$

$$\sigma = 3.65$$

④ (b) $P(X<20) = 0.136$

(c) $P(X>d) = 0.01$

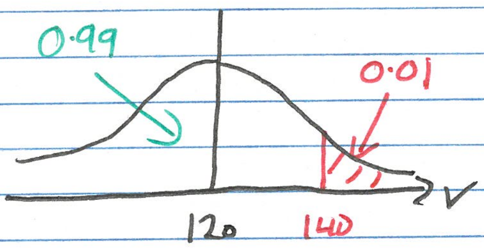


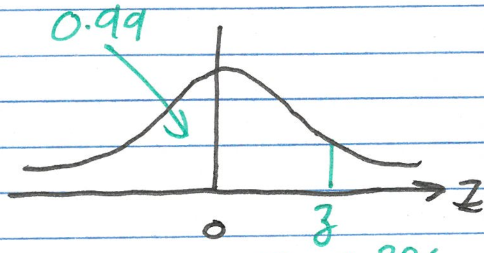
$d = 32.5$

⑤ $V \sim N(120, \sigma^2)$

$P(V>140) = 0.01$

(a)





$z = 2.326$

$$2.326 = \frac{140-120}{\sigma}$$

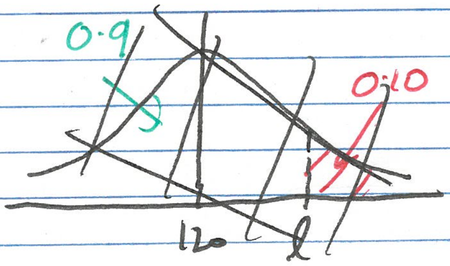
$$2.326\sigma = 20$$

$$\sigma = 8.60 \text{ ml}$$

(b) $V \sim N(120, 8.60^2)$

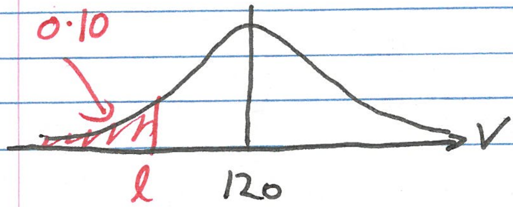
$P(V<110) = 0.122$

(c)



$\cancel{P(L<l) = 0.9}$

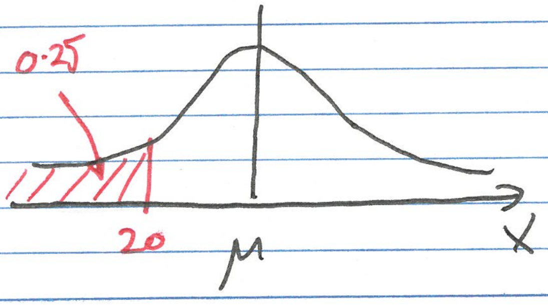
$\cancel{l =}$

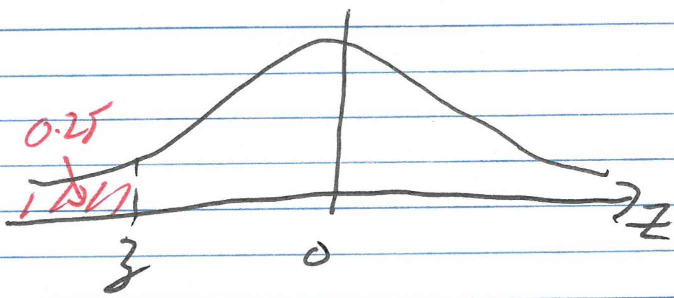


$P(L<l) = 0.1$

$l = 109$ ml

(6) $X \sim N(\mu, \sigma^2)$

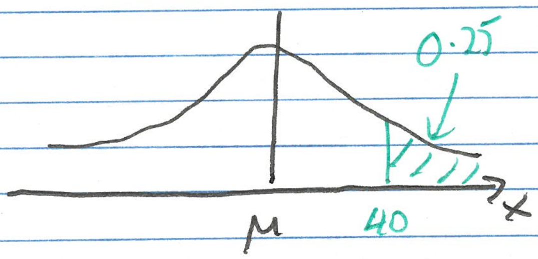


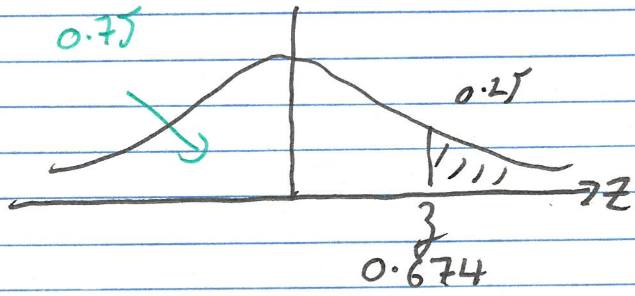


$z = -0.674$

Now $-0.674 = \dfrac{20-\mu}{\sigma}$

$$\mu - 0.674\sigma = 20 \quad — (1)$$





$$0.674 = \frac{40-\mu}{\sigma}$$

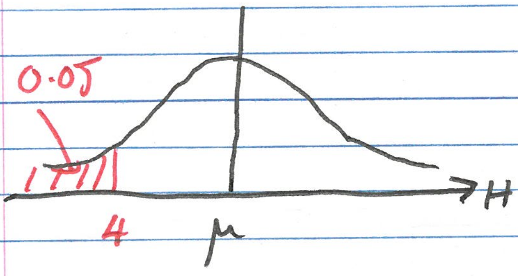
$$\mu + 0.674\sigma = 40 \quad — (2)$$

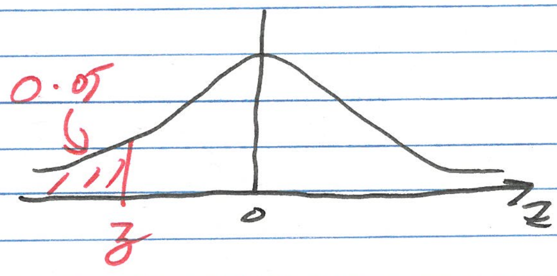
From calculator, $\mu = 30$, $\sigma = 14.8$

7 $H \sim N(\mu, \sigma^2)$

$P(H > 15) = 0.10$

$P(H < 4) = 0.05$

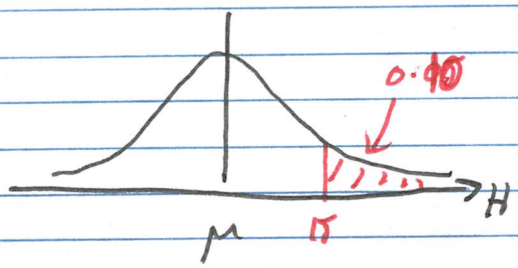


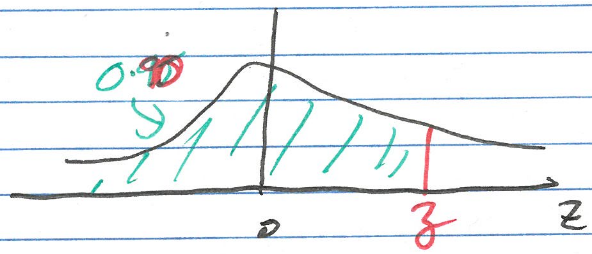


$z = -1.645$

$$-1.645 = \frac{4-\mu}{\sigma}$$

$$\mu - 1.645\sigma = 4 \quad -(1)$$





$z = 1.282$

$$1.282 = \frac{15-\mu}{\sigma}$$

$$\mu + 1.282\sigma = 15$$

from calculator, $\mu = 10.18$, $\sigma = 3.76$ (answers in book other way around)

(8) $T_1 \sim N(80, 10^2)$

(a) $P(T_1 > 85) = 0.3085$

$T_2 \sim N(100, 15^2)$

(b) $P(T_2 > 105) = 0.369$

(c) T_1 30.85% scored higher

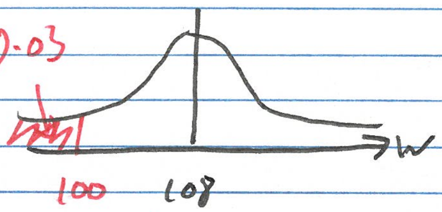
T_2 36.9% scored higher

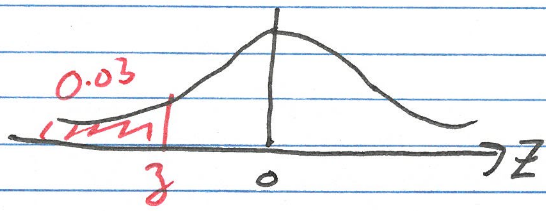
So he did best on first test.

(9) $W \sim N(108, \sigma^2)$

$P(W < 100) = 0.03$

(a)





$z = -1.881$

$$-1.881 = \frac{100 - 108 \; \cancel{108 - 100}}{\sigma}$$

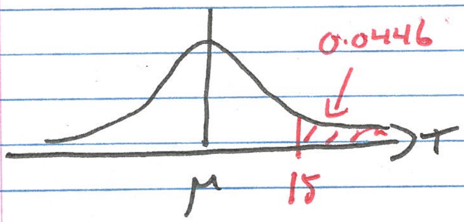
$$\sigma = \frac{-8}{-1.881} = 4.25$$

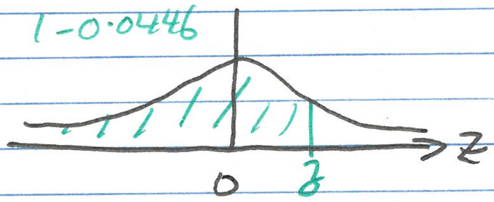
(b) $P(W > 115) = 0.050$

(10) $T \sim N(\mu, 3.8^2)$

$P(T > 15) = 0.0446$

(a)





$z = 1.6996$

$$1.6996 = \frac{15-\mu}{3.8}$$

$$\mu = 15 - 1.6996(3.8)$$

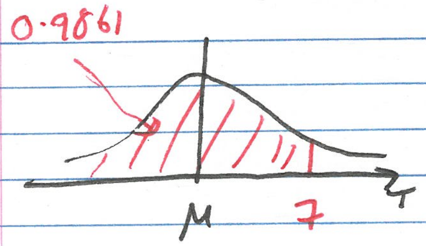
$$= 8.54 \text{ min}$$

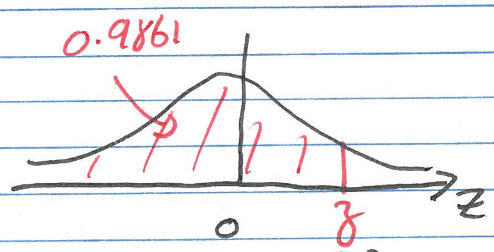
(b) $P(X < 5) = 0.176$

(11) $T \sim N(\mu, \sigma^2)$

$P(T < 7) = 0.9861$

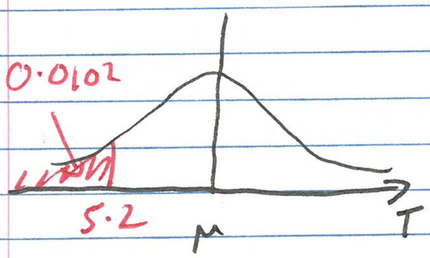
$P(T < 5.2) = 0.0102$

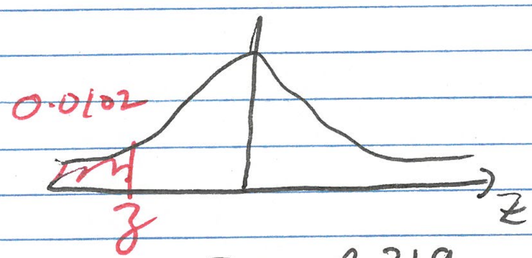




$z = 2.2$

$$2.2 = \frac{7-\mu}{\sigma} \Rightarrow \mu + 2.2\sigma = 7 \quad \text{— (1)}$$





$z = -2.319$

(11) cont'd $\quad -2.319 = \frac{5.2-\mu}{\sigma} \Rightarrow \mu - 2.319\sigma = 5.2$ — (2)

From calc, $\mu = 6.12$, $\sigma = 0.398$

(12) $X \sim N(14, 3^2)$

(a) $P(X \geq 11) = 0.8413$

(b) $P(9 < X < 11) = 0.111$

(13) $X \sim N(20, 5^2)$

(a) $P(X \leq 16) = 0.2119$

(b) $P(X < d) = 0.95$

$d = 28.2$