MARKING SCHEME

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS

SUMMER 2016

INTRODUCTION

This marking scheme was used by WJEC for the 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

	Additional Mathematics Summer 2016		Final
1	(a)(i)(7x +2)(3x - 2)	B2	B1 (7x - 2)(3x + 2) or 7x(3x -2) + 2(3x − 2) or (x - ⅔)(21x +6) or ‘(7x + 2) and/or (3x − 2)’ or ‘(7x + 2) + (3x − 2)’ or equivalent
	(ii) -2/7 with 2/3 or -0.2857… or -0.286 with 0.666..	B2	If a restart in (ii) to factorise, do not alter marking in (i), unless the candidate is clearly replacing their answer (i) Ignore sight of "=0" Must be from factorising. STRICT FT for their factors. B1 for each answer Do not accept from the use of the quadratic formula
	(b)(i) (x+ 6)² ± … …….. + 13	B1 B1	Sight of (x+6)² or (x + ¹²/₂)² Ignore sight of ‘=0’ Accept 49 - 36 if not evaluated, otherwise mark final value. Do not accept ‘= -13’ or ‘=13’ *(x + 6)² + 13, B1, B1 ISW.*
	Least value (+)13	B1	Must follow completing the square FT their value but not 49 or - 36
	(ii) (x =) -6	B1	FT from ‘their (x + 6)²’ Do not accept (-6, 13)
		8	
2	(a) 36x³ + 8x (+0)	B3	B1 for 36x³ (not 9×4x³), B1 for +8x (not 4×2x), and B1 for +0 (or blank) provided at least 1 other mark awarded. Mark final answer
	(b) -8x⁻⁹	B1	Mark final answer
	(c) ¾ x⁻¹ᐟ⁴	B1	Index needs to be simplified. Mark final answer
		5	
3		M1	Attempt to use common denominator, may be implied by sight of 35(3x) -14(x-6) + 10(2x+3) without sight of /70 May be seen in stages
	{ 35(3x) -14(x-6) + 10(2x+3)} (/70)	B1	Or equivalent. May be seen in stages, as intention of method
	{105x-14x+84+20x+30} (/70)	B1	B1 for 1 slip (e.g. -84). Must be as a sum of 5 terms.
	(111x + 114)/70 or showing LHS ≡ RHS	A1	Convincing **must** follow from fully correct working at each stage *If no denominator then possible M1 (see note above), B1 B1 A0, however if denominator replaced later all marks are allowable*
		4	

4	(a) (y+δy =) (x+δx)² + 3(x+δx)
	 Intention to subtract (y=) x² + 3x to find δy
	  (δy =) 2xδx + (δx)² +3δx
	  Dividing by δx **and** (lim) δx→0
	          dy/dx = lim δy/δx = 2x + 3
	                    δx→0
		B1
		M1
		A1
		M1
		A1
			Or alternative notation. Allow if final bracket omitted
			
			Accept δx² as meaning (δx)²
			FT equivalent level of difficulty
			CAO. Must follow from correct working and notation
			All notation throughout the working must be correct in order to award the final A1
			Do not accept dy/dx = lim 2x + 3 as a final answer
			                          x→0
			
			Use of dy/dx throughout max 4 marks only, final A0

		5	

			Final
5	(Area of circle(s) =) $(2\times)$ $\pi\times4^2$ or $(2\times)16\pi$	M1	
	(Curved surface =) $2\times\pi\times4 \times 18$ or 144π	M1	
	(Area of card =) $(18 +8 + 8)\times 2\times\pi\times4$	M2	M1 for sight of $(\ldots+ 8 + 8) \times 2\times\pi\times4$ or
	$\quad\quad\quad\quad\quad\quad (= 34\times 8\pi)$		$(18 + \ldots + \ldots) \times 2\times\pi\times4$
	(Area wasted =)	m1	Intention to subtract areas of 2 circles and a rectangle from
	$((18+8+8)\times 2\pi\times4)$ - $(2\times\pi\times4\times18 +2\times\pi\times4^2)$		the card, depends on at least M3 previously awarded
			CAO. Accept an answer in the range provided supported
	$(=)$ 96π (cm²) or answer in the range	A1	by correct working
	$\quad\quad\quad$ 300 (cm²) to 302(cm²)		
			Alternative:
			(Width of card =) $2\times \pi \times 4$ (= 25·1327 cm) M1
			(M0 if stated as 'area of the circle' unless clearly used as
			width of the card)
			(Area of the small rectangle =) $8 \times 2 \times \pi \times 4$ M1
			(Area of a circle(s) =) $(2 \times) \pi \times 4^2$ M1
			(Area wasted =) $2 \times 8 \times 2 \times \pi \times 4 - 2 \times \pi \times 4^2$ M2
			(or M1 for $8 \times 2 \times \pi \times 4 - \pi \times 4^2$)
			(Area wasted =) 96π (cm²) or 300 to 302 (cm²) CAO A1
	QWC2: Candidates will be expected to	QWC 2	QWC2 Presents relevant material in a coherent and logical
	• present work clearly, with words explaining process or steps		manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar.
	AND		
	• make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer		QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar
			OR
	QWC1: Candidates will be expected to		evident weaknesses in organisation of material but using
	• present work clearly, with words explaining process or steps		acceptable mathematical form, with few if any errors in spelling, punctuation and grammar.
	OR		
	• make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer		QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
		8	
6	(a) Multiplier $(5-\sqrt2) / (5-\sqrt2)$	M1	Allow if the multiplier is stated as $(5-\sqrt2)$ provided it is used as $(5-\sqrt2)/ (5-\sqrt2)$
	Denominator		
	$25 + 5\sqrt2 - 5\sqrt2 - 2$ OR $25 - 2$ OR 23	A1	
	$3 (5 -\sqrt2) /23$ or $(15 - 3\sqrt2)/23$	A1	CAO. Mark final answer
			Unsupported answer is awarded no marks.
	(b)(i) $x^{15/5}/x^{1/2}$ or $x^3/ x^{1/2}$	B1	Or equivalent first stage of working with indices
	$\quad\quad\quad\quad\quad\quad = x^{5/2}$	B1	CAO. Accept $x^{2.5}$ or $x^{2\frac12}$
	(ii) Correctly extracting a factor of $x^{1/9}$	M1	
	(numerator) or $\dfrac{8x^{1/9} + 1(x^{2/9})}{x^{2/9}\quad\quad (x^{2/9})}$		
	$\dfrac{8+x^{1/9}}{x^{1/9}}$ or $8x^{-1/9} + 1$ or $8/x^{1/9} + 1$	A1	CAO. Mark final answer
		7	

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	Additional Mathematics Summer 2016		Final
7	(a) FG^2 = (-2 -4)^2 + (14-6)^2 (= 6^2 +8^2) FG = √100 (= 10)	M1 A1	Or equivalent. Allow 1 slip or error CAO
	(b) Gradient FG (14-6)/(-2-4) = 8/-6 (= - 4/3)	M1 A1	Do not ignore incorrect cancelling, mark final answer
	(c) (-2 + 4)/2 or (14 + 6)/2	M1	Sight of (1, …) or (… , 10) implies M1 provided no incorrect working is seen
	Mid point (1, 10)	A1	
	Perpendicular gradient ¾ (or 6/8)	B1	FT -1/ 'their answer in (b)'
	$\frac{y-10}{x-1} = \frac{3}{4}$ or 10 = ¾ × 1 + c	M1	OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' or 'their answer in (b)' AND 'their mid point' or for 'points F or G' used
	y – 10 = ¾ (x - 1) or 4(y – 10) = 3(x – 1) or 4y = 3x + 37 or c = 9¼ or c = 37/4	m1	**Do not allow** for use gradient from their answer in (b), and/or points F or G. Only FT for 'their perpendicular gradient' (with B1 previously awarded) AND 'their mid point' CAO. Must be in this form with '=0'
	4y – 3x – 37 = 0 or 3x – 4y + 37 = 0	A1 10	

8	(dy/dx=) 3x^2 – 6x
	dy/dx = 0 or 3x^2 – 6x= 0 or 3x^2 = 6x
	                       x = 0 and y = 11
	                       x = 2 and y = 7
	
	d^{2}y/dx^2 = 6x - 6
	
	(0, (11)): d^{2}y/dx^2 <0, point is a maximum
	(2, (7)): d^{2}y/dx^2 >0, point is a minimum
		B1
		M1
		A1
		A1
		
		M1
		
		A1
		A1
		
		7
			
			FT their dy/dx form ax^2 ± bx
			
			Answer only, no working shown M0 A0 A0
			
			Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation)
			FT for their x value
			FT for their other x value provided this does not have the same interpretation as the first x value
			
			Answer only, no working shown M0 A0 A0
			If d^{2}y/dx^2 =cx + d where c≠0 and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded))

9	(a) $\frac{1/\sqrt2}{1/\sqrt2}$ = 1	B1	**Working must be shown throughout** Must be sight of 1/√2 / 1/√2 or √2/2 / √2/2 or cos45°/sin45° = 1/tan45°
	(b) (½ =) $\frac{1}{\sqrt3/1}$ $\frac{(\times\sqrt3)}{2\sqrt3 (\times\sqrt3)}$ or $\frac{½ \sqrt3}{3}$	M1	Must be sight of $\frac{½ \times \sqrt3}{\sqrt3 \sqrt3}$ leading to $\frac{½\sqrt3}{3}$ Do not accept ½ ÷ √3
	= √3 / 6	A1	
	(c) (√3/2)^2 + 1$^{(2)}$	M1	This stage must be seen, do not accept starting with ¾ + 1, this is M0, A0
	7/4 or 1¾ or 1.75	A1 5	

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	Additional Mathematics Summer 2016		Final

10	(a) $(4)^3 + 6(4)^2 - (4) - 30$ (= 64 + 96 – 4 - 30)
	
	$= 126$
	
	(b)(i) Substitute $x = 2$
	   Showing $f(2) = 0$
	
	(ii) $(x-2)(x^2 + bx + c)$
	 or intention to divide by $(x-2)$ with x^2 shown
	   $((x - 2))$ $(x^2 + 8x + 15)$
	
	   $((x - 2))(x + 3)(x + 5)$
		M1
		A1
		
		M1
		A1
		
		M1
		
		A2
		
		A1
		8
			Or division method giving $x^2 + 10x$ …
			
			Or division method giving $x^2 + 8x$ …
			Convincing, working shown
			Allow $2^3 + 6(2)^2 - (2) – 30 = 0$
			
			A1 for $+8x$ or $+15$.
			Or use of factor theorem A1 $(x+3)$, A1 $(x+5)$
			CAO. Mark final answer, *but ignore attempts to 'solve'*

11	(a) Correct shaped graph with $(0°,)$ 180° & 360° labelled on the x-axis AND 2 & 8 labelled on the y-axis
	
	(b) Maximum value 8 AND
	          Minimum value 2
		B3
		
		B2
		
		5
			Intention for approximately (90°, 5) and (270°, 5)
			B2 awarded a for correct shape graph with conditions:
			  • $\cos x$ reflected
			  • with one complete period, labelled 0° to 360°
			  • with difference in y values between maximum and minimum of 6, for their labels
			OR
			B1 for a correct shape graph with any two of the 3 bullet points above met
			
			If no marks, award SC1 for a curve through at least 5 correct points across the full range with all other conditions met. Do not accept a parabola or straight lines
			
			Accept Maximum (180(°), 8) and Minimum (360(°), 2)
			Allow unsupported correct responses
			FT provided at least B2 previously awarded in (a)
			
			B1 for either value correct, or for a difference between their max and min of 6, or if their answers are reversed (including Maximum (180(°), 2) and Minimum (360(°), 8))

12	(a) (dy/dx=) $21x^6 + 4$
	   (d²y/dx²=) $126x^5$
	
	(b) (4/4) x^4 + (2/2)x^2 + (4/-1) x^{-1}
	                      + c
	(constant)
	
	(c) $8x^2/2 + 2x$
	$[8x^2/2 + 2x]^3_2$ and with intention to substitute and subtract
	
	$=(8×3^2/2 + 2×3) – (8×2^2/2 + 2×2)$ (= 42 – 20)
	
	$=$ 22
		B1
		B1
		
		B3
		B1
		
		B2
		M1
		
		A1
		
		A1
		11
			Accept sight of $21x^6 + 4$
			FT to 2nd B1 from dy/dx = kx^n (+ m)
			
			B1 for each term. Accept unsimplified. ISW
			Award if at least B1 given for integration
			
			B1 for $8x^2/2$ or $2x$
			Intention to use 3, 2 (in either order) **and** subtract
			FT their integration, not the same terms as given or differentiated, this includes if there is only1 term seen.
			
			FT for correct use of limits provided working with 2 terms from 'their integration'
			CAO, not FT.
			Answer only, no working shown, M0 A0 A0

			Final
13	(When x = 3) y = 33	B1	
	(Gradient when x = 3, dy/dx =) 3×2x	M1	For differentiation, before substitution of x = 3
	$\quad$ 18	A1	
	Equation $\dfrac{y - 33}{x - 3}$ = 18 or 33 = 18×3 + c	M1	FT values for 'their 33' and 'their 18' provided at least one of these is correct.
	y − 33 = 18(x − 3) or c = -21	m1	Implies previous M1
	y = 18x − 21 or equivalent	A1	CAO. Mark final answer
		6	
14	Method to solve simultaneously, e.g. use of y = 10 − x or x = 10 − y into the first equation	M1	$10 - x = x^2 - 6x + 14$ or $y = (10 - y)^2 - 6(10 - y) + 14$
	$x^2 - 5x + 4 = 0$ or $y^2 - 15y + 54 = 0$	A1	Must '=0' or implied in further working
	(x − 4)(x − 1) (=0) or (y -9)(y − 6) (=0)	m1	OR x = (5 ± √9)/2 or y = (15 ± √9)/2 FT from their quadratic
	(4, 6) and (1, 9)	A1	CAO Need not be in this form, accept x=4, y=6 with x=1, y=9 x & y values must be given Do not accept unsupported responses
		4	Do not accept trial & improvement
15	(a) Intention to substitute x=2 and x = 4 into $y = -x^2 + 6x - 8$	M1	OR substituting either value and showing y = 0 OR attempt to factorise as a pair of brackets ((-)x ..2)(x ..4) Do not accept $(-2)^2 + 6×2 - 8$ and $(-4)^2 + 6×4 - 8$ Accept $-2^2 + 6×2 - 8$ and $-4^2 + 6×4 - 8$
	Showing y = 0 for both values	A1	OR factorised as (-)(x − 2)(x − 4) or equivalent
	(b) Intention to integrate	M1	Intention to integrate (manipulated given, hence not using given or differentiated)
	$-x^3/3 + 6x^2/2 - 8x$	A2	A1 one term correct.
	Use of correct limits 4 & 2 in the correct order **and** intention to subtract	m1	The limits must be used in the correct order
	$\quad$ 4/3 or 1.33(3…)	A1	CAO. Only allow 1.3 from correct working and sight of 4/3 *Answer only gets no marks* *No marks for use of the trapezium rule*
		7	

Level 2 Certificate in Additional Mathematics MS Summer 2016