Area and Perimeter

You need to know how to calculate the area and perimeter of the following standard shapes:

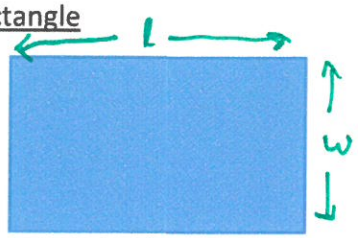
Rectangle, Triangle, Trapezium, Circle

as well as shapes that are made up of these – ***called composite shapes***.

Perimeter measures the total length of the boundary of the shape. Its units are typically mm, cm, m, km.

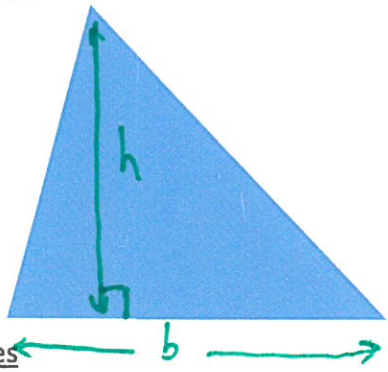
Area measures how much surface is covered by a shape. Its units are typically mm^2, cm^2, m^2, km^2.

The Rectangle



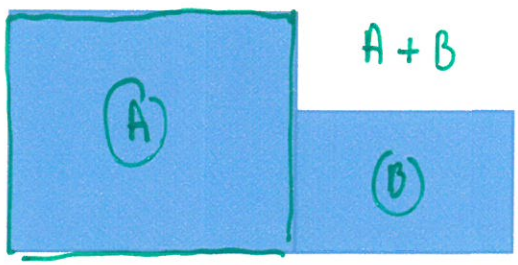
Area = $l \times w$

The Triangle

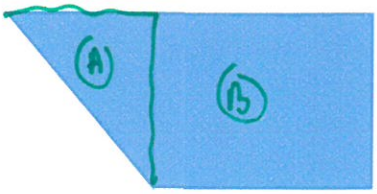


Area = $\frac{1}{2} \times b \times h$

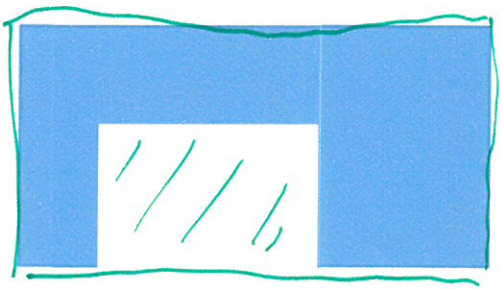
Composites



A + B

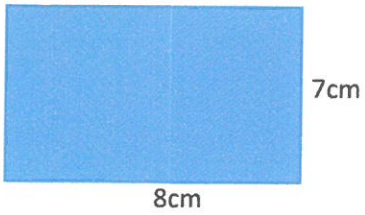


Area of △(A) + Area of (B)

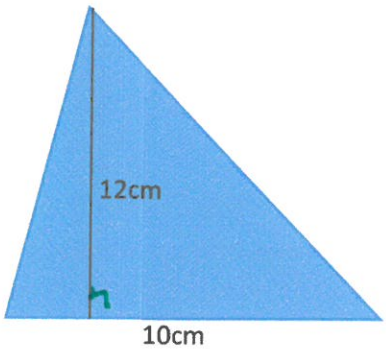


Area whole rectangle – Area of small rectangle

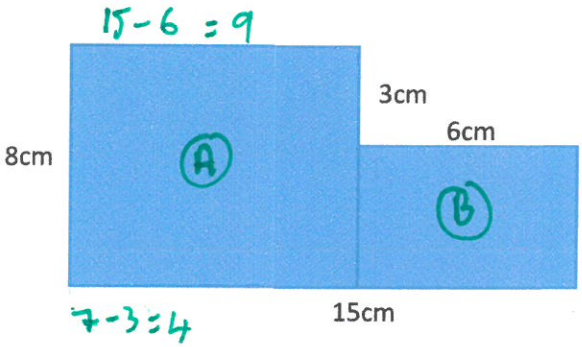
Examples



Area: $7 \times 8 = 56\ cm^2$



Area: $\frac{1}{2} \times 10 \times 12 = 60\ cm^2$

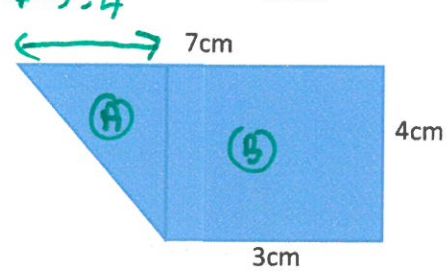


Area of (A) = $8 \times 9 = 72$

Area of (B) = $6 \times 5 = 30$ +

8-3 = 5

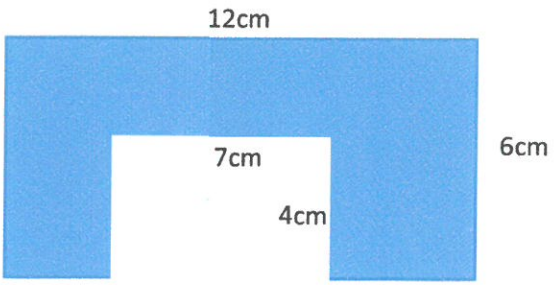
Total Area 102 cm^2



Area of $\triangle$(A) = $\frac{1}{2} \times 4 \times 4 = 8$

Area of (B) = $3 \times 4 = 12$ +

Total Area 20 cm^2



Area = $7 \times 6 - 7 \times 4$

$= 42$

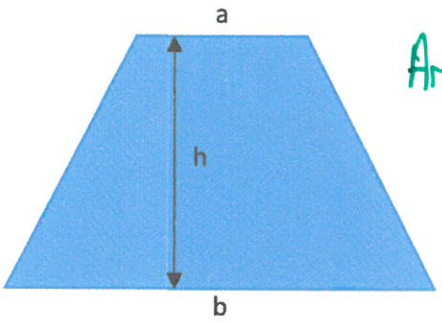
$- 28$

$14\ cm^2$

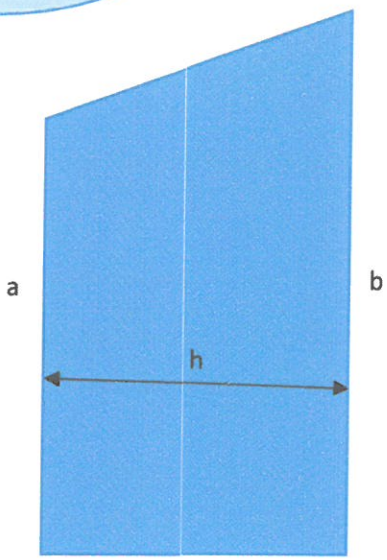
Area and Perimeter (2)

The Trapezium

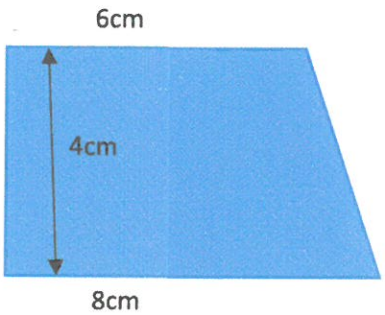
This is a quadrilateral (4 sided shape) with ONE pair of parallel sides



Area $= \frac{1}{2} \times (a+b) \times h$



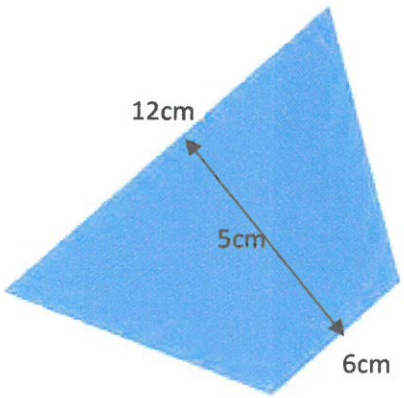
Examples



Area $= \frac{1}{2} \times (6+8) \times 4$

$= \frac{1}{2} \times 14 \times 4$

$= 7 \times 4 = 28\ cm^2$



Area $= \frac{1}{2} \times (12+6) \times 5$

$= \frac{1}{2} \times 18 \times 5$

$= 9 \times 5 = 45\ cm^2$

Area and Perimeter (3)

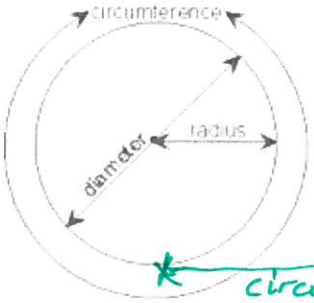
The Circle

In order to calculate the perimeter and area of a circle, we need to know the value of π (pi).

π = 3.14159265359....

Pi is an example of an irrational number, it cannot be written as a fraction and has an infinite run of digits following the decimal point.

It is the value you get should you divide the exact perimeter of a circle by its exact diameter (distance across the widest part of the circle).



circumf = dist travelled in one complete turn

As you can see, the perimeter of a circle has its own special name, the circumference.

You need to memorize and learn how to use the following two formulas:

For a circle with a radius of r cm,

Area = π x r^2

Circumference = π x 2r

On a calculator allowed paper, use the π button shift x10^x

On a non-calculator paper, use 3.14 as the value of π.

Examples

1. A circle has a radius of 4.3cm. Find its area and circumference. r = 4.3

$A = \pi \times 4.3^2 = 58.1\ cm^2$

$C = \pi \times 2 \times 4.3 = 27.0\ cm$

2. A wheel has a diameter of 60cm. How many times will it turn as it travels 1km?

diameter = 0.6 m distance = 1000 m

radius = 0.3 m

$C = \pi \times 2 \times 0.3 = 1.88\ m$

So number of turns = $1000 \div 1.88 = 530.5$

So 530 complete turns in 1 km

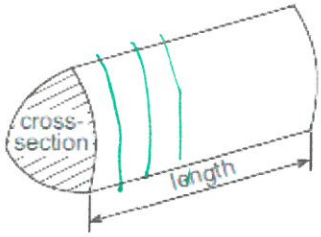
Volume

- Volume is a measure of how much space is occupied by a 3-dimensional object.
- Units of volume are typically cm^3, m^3
- However, if the volume is a liquid, it tends to be called capacity and the units are ml and litres, where 1 litre = 1000 cm^3

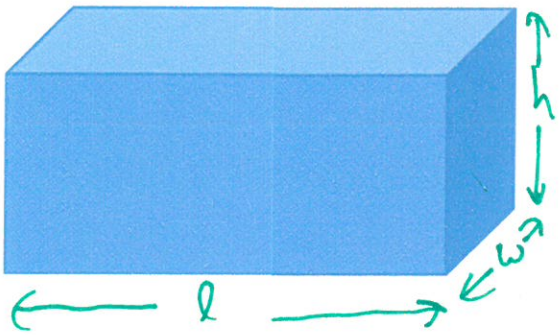
Prisms and Cuboids

A prism is an object with a uniform cross-section, on a GCSE paper you are told:

Volume of prism = area of cross-section × length



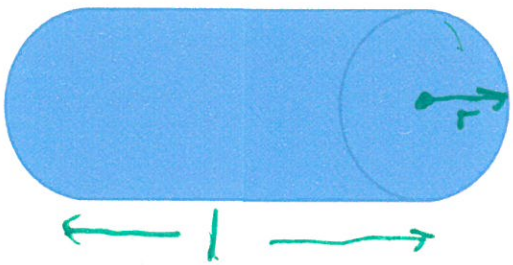
A cuboid is a prism whose cross section is a rectangle:



Volume = $l \times w \times h$

The cylinder

A cylinder is a prism with a circular cross-section.

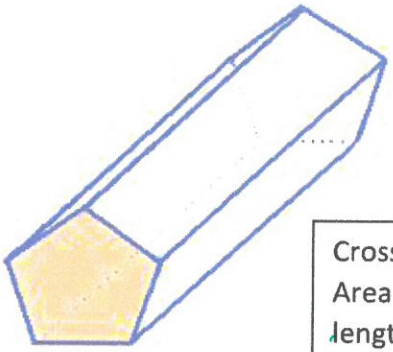


Volume = $\pi \times r^2 \times$ length

Examples

Calculate the volume of the following:

1.



Pentagonal prism

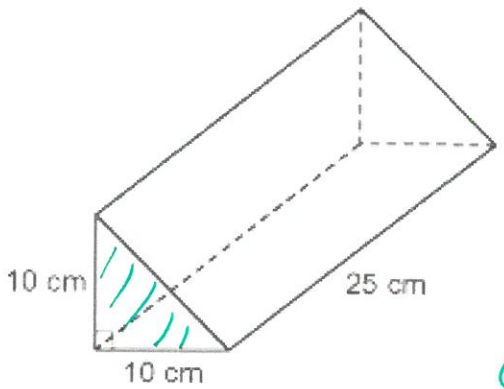
Cross-sectional Area = 50cm², length 1.5m

length = 150 cm

Volume = 50 × 150 = 7500 cm³

Capacity = 7500 ÷ 1000
= 7.5 litres

2.

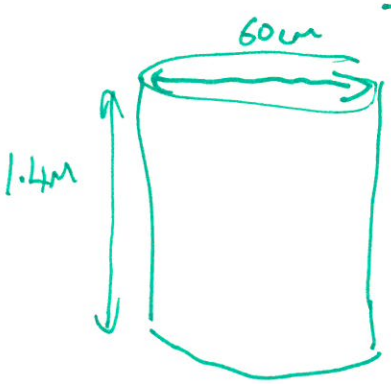


Area of cross-section = $\frac{1}{2} \times 10 \times 10$
= 50 cm²

Volume = 50 × 25
= 1250 cm³

Capacity = 1250 ÷ 1000
= 1.25 litres

3. A cylindrical drum has a diameter of 60cm and a height of 1.4 metres. Find the capacity of the drum in litres.



Need volume in cm³
So height = 140 cm

diameter = 60 cm
So radius = 30 cm

Volume = $\pi \times 30^2 \times 140$
= 395840.6744 cm³

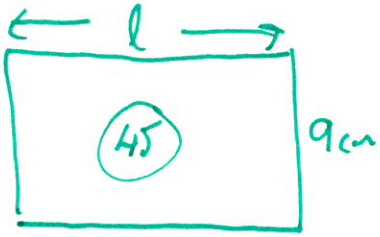
Capacity = 395.8 litres
= 400 litres to 1 sig fig

Perimeter, Area & Volume: Reverse Problems

Sometimes you may be given a question where you are given the perimeter, area or volume and asked to find the length(s) of a side(s):

Examples

1. A rectangle has an area of 45cm^2. If one of the sides is 9cm, find the length of the other side and the perimeter of the shape.

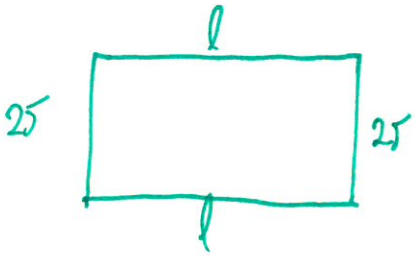


Area = $l \times 9$

$45 = l \times 9$

$l = 45 \div 9 = 5$cm

2. A rectangle has a perimeter of 140 cm. If the length of the shorter side is 25cm, find the dimensions of the rectangle and its area.



$140 - 25 - 25 = 90$

$l = 90 \div 2 = 45$ cm

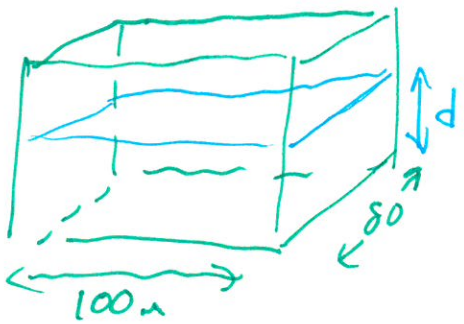
3. A square has an area of 64cm^2. What is the length of its sides?

$l \times l = 64$

$l^2 = 64$

$l = \sqrt{64} = 8$cm

4. A fish tank, in the shape of a cuboid, contains 400 litres of water. If the base of the tank measures 1metre by 80cm, how deep will the water be?



$V = l \times w \times h$

$V = 400\ l$

$V = 400 \times 1000 = 400000$ cm^3

$400000 = 100 \times 80 \times d$

$400000 = 8000 \times d$

So $d = 400000 \div 8000 = 50$cm