Geometric Sequences and Series

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1. A geometric series is $a + ar + ar^2 + ...$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + ...$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

Leave blank

2. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(b) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(d) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)

Leave blank

3. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,
(2)

(b) the first term,
(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.
(2)

Leave blank

4. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.
(4)

(b) Hence show that $k = 12$.
(2)

(c) Find the common ratio of this series.
(2)

(d) Find the sum to infinity of this series.
(2)

Leave blank

5. A car was purchased for £18 000 on 1st January.
On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.
(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(b) Find the value of n.
(3)

An insurance company has a scheme to cover the maintenance of the car.
The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.
(2)

(d) Find the total cost of the insurance scheme for the first 15 years.
(3)

Leave blank

6. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(a) the common ratio of the series,

(3)

(b) the first term of the series,

(2)

(c) the sum to infinity of the series.

(2)

Leave blank

7. A geometric series has first term a and common ratio r.
The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2 - 25r + 4 = 0$.

(4)

(b) Find the two possible values of r.

(2)

(c) Find the corresponding two possible values of a.

(2)

(d) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24.

(2)

Leave blank

8. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

Leave blank

9. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(a) the 20th term of the series, to 3 decimal places,
(2)

(b) the sum to infinity of the series.
(2)

Given that the sum to k terms of the series is greater than 24.95,

(c) show that $k > \frac{\log 0.002}{\log 0.8}$,
(4)

(d) find the smallest possible value of k.
(1)

Leave blank

10. The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$
(2)

(b) Find the first term of the sequence.
(2)

(c) Find the sum of the first 15 terms of the sequence.
(3)

(d) Find the sum to infinity of the sequence.
(2)