C3

Chapter 8

Differentiation

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7. $$f(x) = x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.

(3)

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(c) Given that $f(x) = (x-2)(x^3 + ax^2 + bx + c)$, find the values of the constants, a, b and c.

(3)

(d) In the space provided on page 21, sketch the graph of $y = f(x)$.

(3)

(e) Hence sketch the graph of $y = |f(x)|$.

(1)

Leave blank

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

Leave blank

8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

Leave blank

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

Leave blank

5. The functions f and g are defined by

$$\mathrm{f}: x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$\mathrm{fg}: x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{fg}(x)\right] = x(x\mathrm{e}^{x^2} + 2)$.

(6)

Leave blank

4. (i) Given that $y = \dfrac{\ln(x^2+1)}{x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(ii) Given that $x = \tan y$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{1+x^2}$.

(5)

N 3 5 3 8 1 A 0 8 2 8

5.

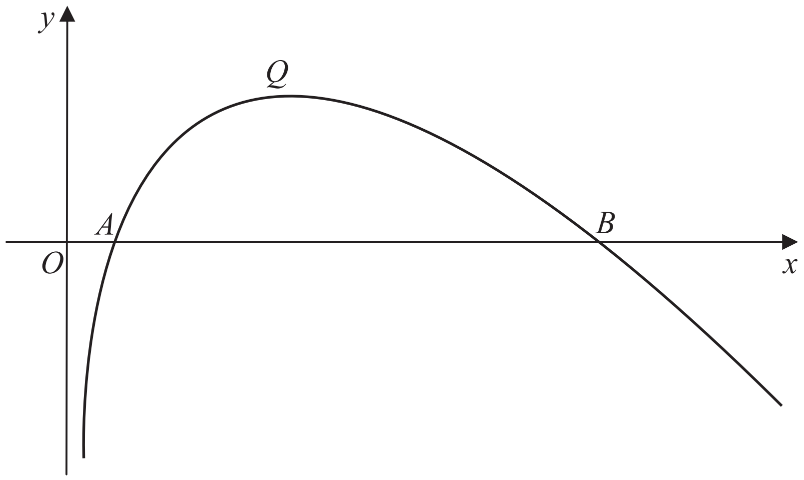

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x)\ln x, \quad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B.

(2)

(b) Find $f'(x)$.

(3)

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(d) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

Leave blank

Leave blank

7.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

Leave blank

8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x+\alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x}\cos 3x$$

(b) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x}\cos(3x+\alpha)$$

where R and α are the constants found in part (a).

(5)

(c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)

Leave blank

5.

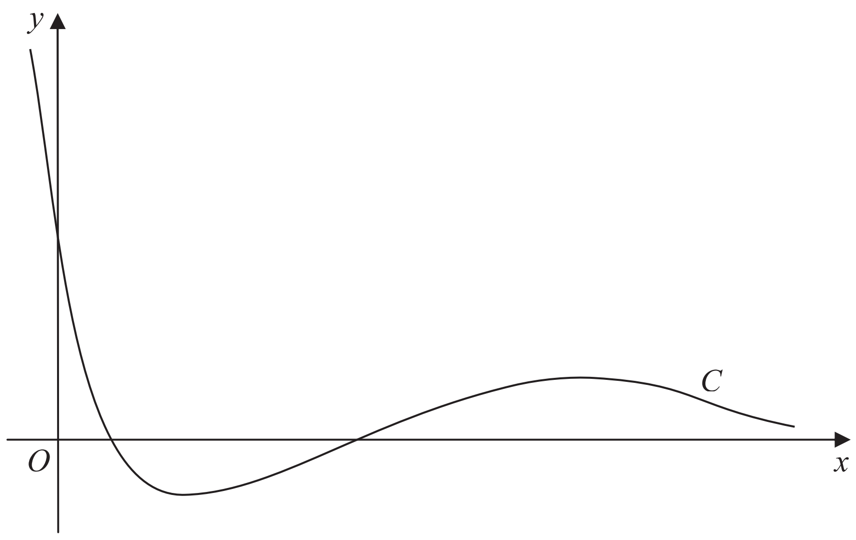

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\frac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

Leave blank

2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

Leave blank

1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

Leave blank

4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

P 4 0 0 8 4 A 0 8 2 4

Leave blank

1. Differentiate with respect to x

(a) $\ln\left(x^2 + 3x + 5\right)$

(2)

(b) $\dfrac{\cos x}{x^2}$

(3)

Leave blank

2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$\mathrm{f}(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$\mathrm{f}(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate $\mathrm{f}(x)$ and find $\mathrm{f}'(2)$.

(3)

Leave blank

1. (a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

Leave blank

7. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

Leave blank

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

Leave blank

8. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

Leave blank

1. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\dfrac{\sin 4x}{x^3}$

(5)

Leave blank

2.
$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \qquad x > \frac{1}{2}.$$

(a) Show that $f(x) = \dfrac{4x-6}{2x-1}$.

(7)

(b) Hence, or otherwise, find $f'(x)$ in its simplest form.

(3)

Leave blank

3. A curve C has equation

$$y = x^2e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.
(3)

(b) Hence find the coordinates of the turning points of C.
(3)

(c) Find $\frac{d^2y}{dx^2}$.
(2)

(d) Determine the nature of each turning point of the curve C.
(2)

Leave blank

6. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x+2)$.

(3)

Given that $y = \dfrac{3x^2+6x-7}{(x+1)^2}$, $\quad x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

Leave blank

4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \; y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \frac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \frac{e^x}{(e^x - 2)^2}$

(3)

(c) Find the exact values of x for which $g'(x) = 1$

(4)

Leave blank

3. The curve C has equation

$$x = 2\sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

Leave blank

4. (i) The curve C has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = (1 + e^{2x})^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)

N 2 3 5 8 3 A 0 8 2 4

Leave blank

2.

$$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$

(3)