Differentiation

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1. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

Leave blank

2. **Figure 4**

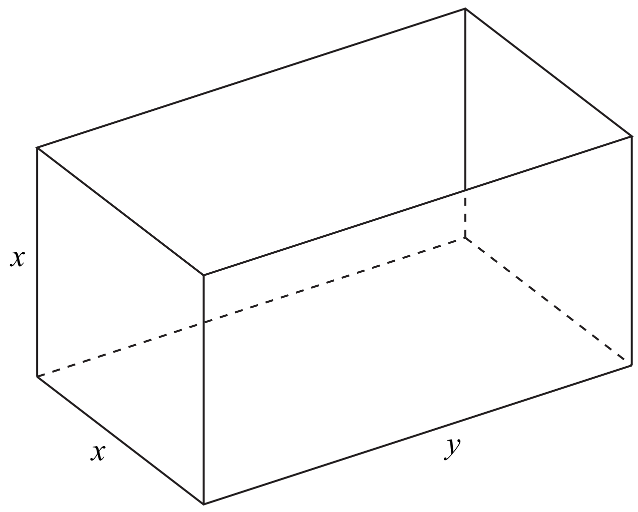

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is $100\,\text{m}^3$.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

Leave blank

3. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)

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4. The curve C has equation $y = 12\sqrt{x} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\dfrac{d^2y}{dx^2}$.

(2)

(c) State the nature of the turning point.

(1)

5. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

6.

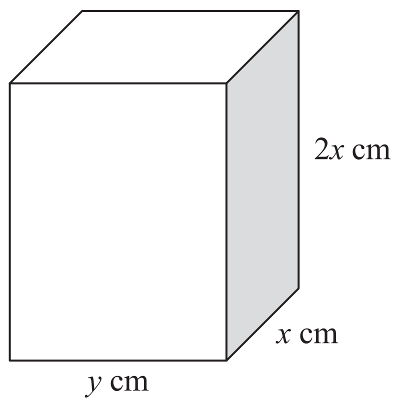

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.
(5)

(c) Justify that the value of V you have found is a maximum.
(2)

7.

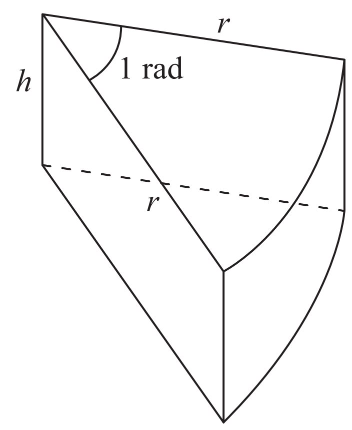

Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}$$

(5)

(b) Use calculus to find the value of r for which S is stationary.

(4)

(c) Prove that this value of r gives a minimum value of S.

(2)

(d) Find, to the nearest cm^2, this minimum value of S.

(2)

Leave blank

8.

$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(a) Find $\frac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)