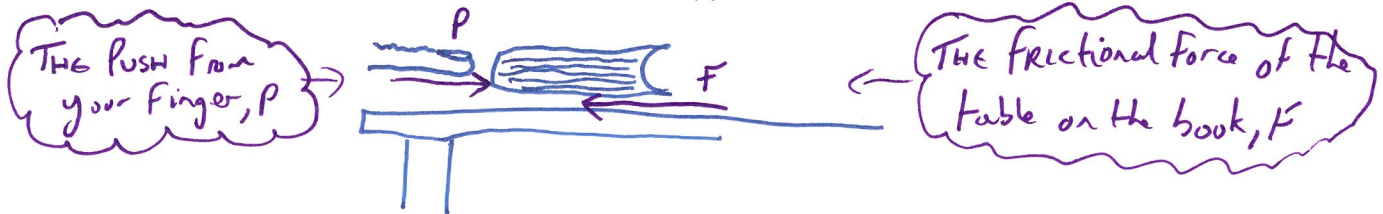


## Friction

Place a heavy book on a table and push it lightly with your finger. Nothing happens. The force from your finger is balanced by an equal frictional force in the opposite direction.



Now increase the force  $P$  with which your finger is pushing the book. As  $P$  increases, so does the frictional force  $F$  opposing it. They balance each other, so

$$P = F$$

Until ... the book moves. At that point the frictional force  $F$  has reached the greatest value it can take, and it is no longer able to balance  $P$ .

So the frictional force  $F$  between an object and surface is not constant, but increases as the applied force  $P$  increases until the force  $F$  reaches a value  $F_{max}$  beyond which it cannot increase. The book is then on the point of moving and is said to be in a state of **limiting equilibrium**.

In our situation with the book, whilst  $P < F_{max}$ , the book will not move. When  $P = F_{max}$ , the book is in limiting equilibrium (on the point of moving). When  $P > F_{max}$ , the book moves.

A frictional force will always act in the direction opposed to motion. If an object is moving, the frictional force will take its greatest possible value.

## Coefficient of friction

The magnitude of the maximum frictional force is a fraction of the normal reaction,  $R$ . This fraction is called the coefficient of friction ( $\mu$ ) for the two surfaces in contact.

$$F_{max} = \mu R$$

From Wikipedia...

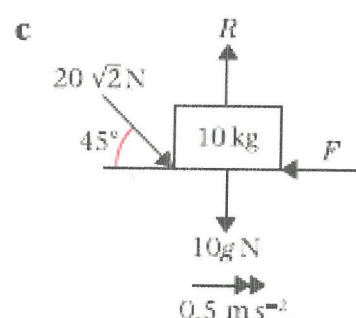
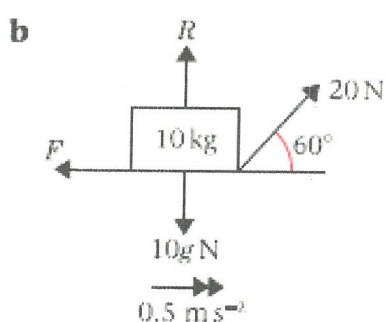
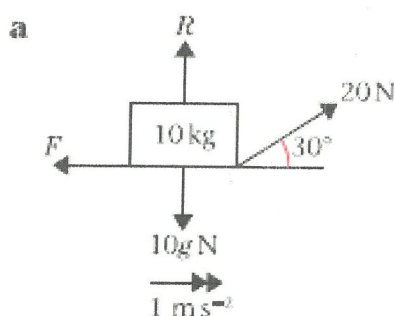
Most dry materials in combination have friction coefficient values between 0.3 and 0.6. Values outside this range are rarer, but [teflon](#), for example, can have a coefficient as low as 0.04. A value of zero would mean no friction at all, an elusive property – even [magnetic levitation vehicles](#) have [drag](#). Rubber in contact with other surfaces can yield friction coefficients from 1 to 2. Occasionally it is maintained that  $\mu$  is always  $< 1$ , but this is not true. While in most relevant applications  $\mu < 1$ , a value above 1 merely implies that the force required to slide an object along the surface is greater than the normal force of the surface on the object. For example, [silicone rubber](#) or [acrylic rubber](#)-coated surfaces have a coefficient of friction that can be substantially larger than 1.

**Eg7** A block of mass 5kg rests on a rough horizontal plane, the coefficient of friction between the block and the plane being 0.6. Calculate the frictional force acting on the block when a horizontal force  $P$  is applied to the block and the magnitude of  $P$  is (a) 12N, (b) 28N, (c) 36N. Also calculate the magnitude of any acceleration that may occur.

**Eg8** A 10kg trunk lies on a rough horizontal floor. The coefficient of friction between the trunk and the floor is  $\frac{\sqrt{3}}{4}$ . Calculate the magnitude of the force  $P$  which is necessary to pull the trunk horizontally if  $P$  is applied (a) horizontally, (b) at  $30^\circ$  above the horizontal.

### Exercise 6.3

In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane.  $R$  is the normal reaction and  $F$  is the friction force. Find the coefficient of friction in each case.



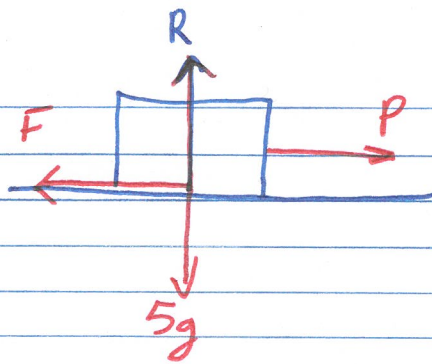
### Answers

(3 s.f.)  $R = 88 \text{ N}$ ,  $\mu = 0.083$   
 (2 s.f.)  $R = 80.7 \text{ N}$ ,  $\mu = 0.062$   
 (2 s.f.)  $R = 118 \text{ N}$ ,  $\mu = 0.13$

**Eg9** A mass of 6kg rests in limiting equilibrium on a rough plane inclined at  $30^\circ$  to the horizontal. Find the coefficient of friction between the mass and the plane.

**Eg10** A mass of 0.5kg is resting on a rough plane. The coefficient of friction between the mass and the plane is  $\frac{1}{\sqrt{2}}$ , and the plane is inclined at angle  $\theta$  to the horizontal such that  $\sin \theta = \frac{1}{3}$ . A mass then experiences a force of 6N applied up the plane along a line of greatest slope. Calculate the magnitude of the acceleration of the mass up the slope.

Ex 7



$$\mu = 0.6$$

$$\Sigma F_y: R - 5g = 0$$

$$R = 5g$$

$$\text{Now } F_{\max} = \mu R$$

$$= 0.6 \times 5g$$

$$= 29.4 \text{ N}$$

∴ when (a)  $P = 12$  no movement  $P < F_{\max}$

(b)  $P = 28$  no movement  $P < F_{\max}$

(c)  $P = 36$ , block will accelerate as  $P > F_{\max}$

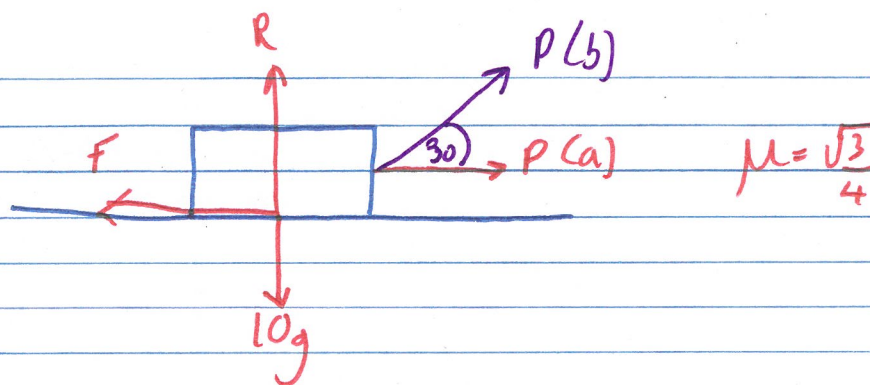
$$\text{NZL } P - F = Ma$$

$$36 - 29.4 = 5a$$

$$a = 1.3 \text{ ms}^{-2}$$



Eg 8



$$\Sigma F_y: R - 10g = 0$$

$$R = 10g$$

$$F_{\max} = \frac{\sqrt{3}}{4} \times 10g$$

(a) At limiting equilibrium,  $P - F_{\max} = 0$

$$\therefore P = \frac{\sqrt{3}}{4} \times 10g = 42.4 \text{ N}$$

(b) At limiting equilibrium

$$\text{Now } \Sigma F_y \text{ changes } R + P \sin 30 - 10g = 0 \quad \text{--- (1)}$$

$$F_{\max} = \frac{\sqrt{3}}{4} \times 10g \quad \text{--- (2)}$$

$$\Sigma F_x \text{ @ limiting Equilib, } P \cos 30 - F_{\max} = 0 \quad \text{--- (3)}$$

$$\text{From (1) } R = 10g - P \sin 30$$

$$\text{in (2) } F_{\max} = \frac{\sqrt{3}}{4} [10g - P \sin 30]$$

$$\text{in (3) } P \cos 30 - \frac{\sqrt{3}}{4} [10g - P \sin 30] = 0$$

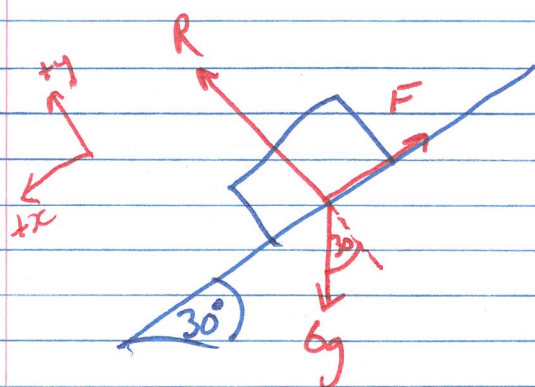
$$P \cos 30 - \frac{\sqrt{3}}{4} 10g + \frac{\sqrt{3}}{4} P \sin 30 = 0$$

$$P \left[ \cos 30 + \frac{\sqrt{3}}{4} \sin 30 \right] = \frac{\sqrt{3}}{4} 10g$$

$$P = 39.2 \text{ N}$$



-g9



About to slide down plane  
 ~~$\mu = \frac{1}{\sqrt{3}}$~~

$$\Sigma F_x: Gg \sin 30^\circ - F = 0 \quad \text{--- (1)}$$

$$\Sigma F_y: R - Gg \cos 30^\circ = 0 \quad \text{--- (2)}$$

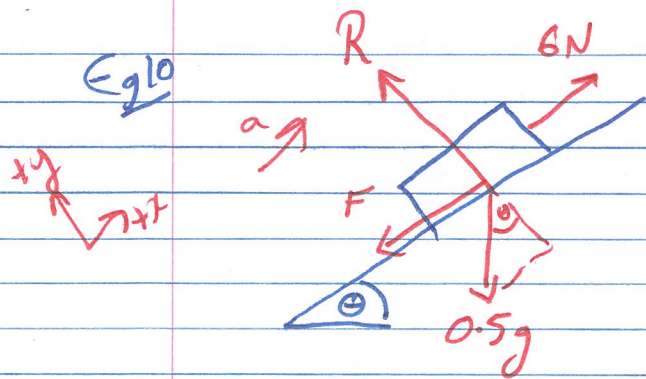
$$F_{\text{max}} = \mu R \quad \text{--- (3)}$$

From (1)  $F = Gg \sin 30^\circ$

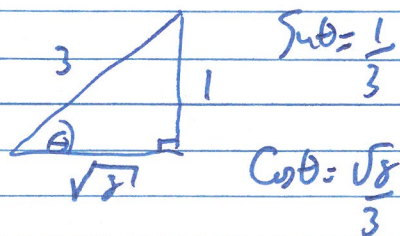
From (2)  $R = Gg \cos 30^\circ$

$$\text{in (3)} \quad \mu = \frac{F}{R} = \frac{Gg \sin 30^\circ}{Gg \cos 30^\circ} = \tan 30^\circ$$

$$\mu = \frac{\sqrt{3}}{3}$$



$$\mu = \frac{1}{\sqrt{2}}$$



$$\Sigma F_x: 6 - 0.5g \sin \theta - F = 0.5a \quad \text{--- (1)}$$

$$\Sigma F_y: R - 0.5g \cos \theta = 0 \quad \text{--- (2)}$$

$$F_{\max} = \frac{1}{\sqrt{2}} R \quad \text{--- (3)}$$

$$\text{From (2)} \quad R = 0.5g \frac{\sqrt{3}}{2}$$

$$\text{u (3)} \quad F_{\max} = \frac{1}{\sqrt{2}} \times 0.5g \frac{\sqrt{3}}{2} = \frac{49}{15}$$

$$\text{u (1)} \quad 6 - 0.5g \frac{1}{2} - \frac{49}{15} = 0.5a$$

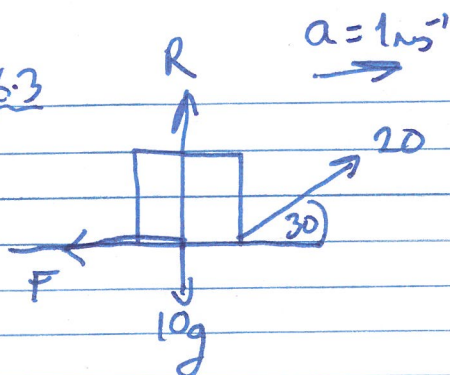
$$1.1 = 0.5a$$

$$a = 2.2 \text{ ms}^{-2}$$



Ex 6.3

(a)



$$\Sigma F_x: 20 \cos 30 - F = 10 \times 1 \quad \text{--- (1)}$$

$$\Sigma F_y: R + 20 \sin 30 - 10g = 0 \quad \text{--- (2)}$$

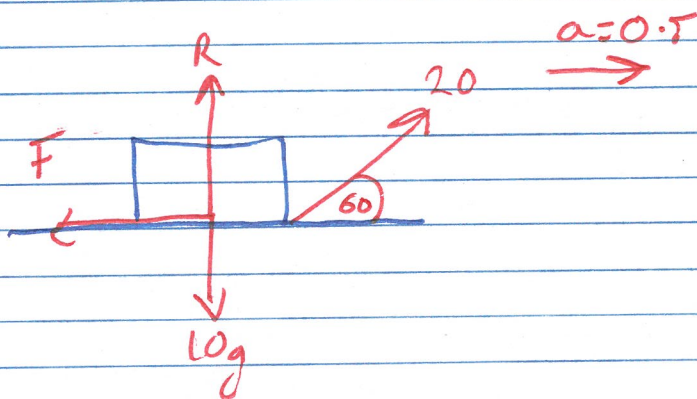
$$F_{\max} = \mu R \quad \text{--- (3)}$$

From (1)  $F = 20 \cos 30 - 10$

From (2)  $R = 10g - 20 \sin 30$

u(3)  $\mu = \frac{F}{R} = \frac{20 \cos 30 - 10}{10g - 20 \sin 30} = \underline{0.083}$

(b)



$$\Sigma F_x: 20 \cos 60 - F = 10 \times 0.5 \quad \text{--- (1)}$$

$$\Sigma F_y: R + 20 \sin 60 - 10g = 0 \quad \text{--- (2)}$$

$$F_n = \mu R \quad \text{--- (3)}$$

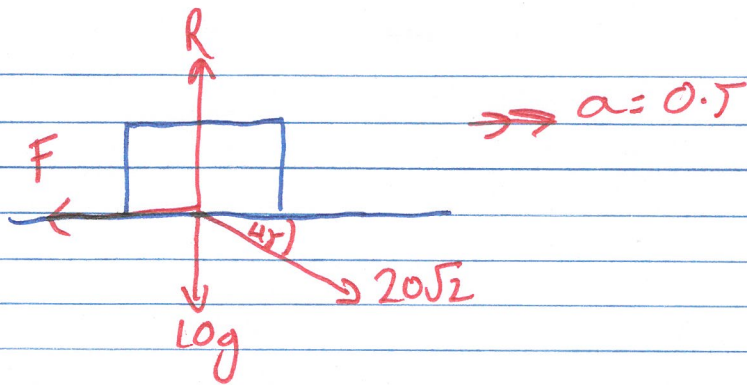
From (1)  $F = 20 \cos 60 - 5$

From (2)  $R = 10g - 20 \sin 60$

u(3)  $\mu = \frac{F}{R} = \frac{20 \cos 60 - 5}{10g - 20 \sin 60} = \underline{0.062}$



(C)



$$\sum F_x : 20\sqrt{2} \cos 45^\circ - F = 10 \times 0.5 \quad \text{--- (1)}$$

$$\sum F_y : R - 20\sqrt{2} \sin 45^\circ - 10g = 0 \quad \text{--- (2)}$$

$$F_{\max} = \mu R \quad \text{--- (3)}$$

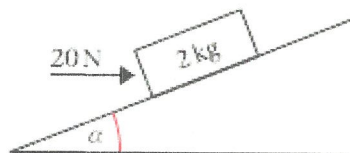
$$\text{From (1)} \quad F = 20\sqrt{2} \cos 45^\circ - 5$$

$$\text{From (2)} \quad R = 10g + 20\sqrt{2} \sin 45^\circ$$

$$\text{in (3)} \quad \mu = \frac{20\sqrt{2} \cos 45^\circ - 5}{10g + 20\sqrt{2} \sin 45^\circ} = 0.127$$

### Exercise 6.4

- 1 A particle of mass  $0.5 \text{ kg}$  is placed on a smooth inclined plane. Given that the plane makes an angle of  $20^\circ$  with the horizontal, find the acceleration of the particle.
- 2 The diagram shows a box of mass  $2 \text{ kg}$  being pushed up a smooth plane by a horizontal force of magnitude  $20 \text{ N}$ . The plane is inclined to the horizontal at an angle  $\alpha$ , where  $\tan \alpha = \frac{3}{4}$ .



Find

- a the normal reaction between the box and the plane,
- b the acceleration of the box up the plane.

- 3 A boy of mass  $40 \text{ kg}$  slides from rest down a straight slide of length  $5 \text{ m}$ . The slide is inclined to the horizontal at an angle of  $20^\circ$ . The coefficient of friction between the boy and the slide is  $0.1$ . By modelling the boy as a particle, find

- a the acceleration of the boy,
- b the speed of the boy at the bottom of the slide.

- 4 A block of mass  $20 \text{ kg}$  is released from rest at the top of a rough slope. The slope is inclined to the horizontal at an angle of  $30^\circ$ . After  $6 \text{ s}$  the speed of the block is  $21 \text{ m s}^{-1}$ . As the block slides down the slope it is subject to a constant resistance of magnitude  $R \text{ N}$ . Find the value of  $R$ .

- 5 A book of mass  $2 \text{ kg}$  slides down a rough plane inclined at  $20^\circ$  to the horizontal. The acceleration of the book is  $1.5 \text{ m s}^{-2}$ . Find the coefficient of friction between the book and the plane.

- 6 A block of mass  $4 \text{ kg}$  is pulled up a rough slope, inclined at  $25^\circ$  to the horizontal, by means of a rope. The rope lies along the line of the slope. The tension in the rope is  $30 \text{ N}$ . Given that the acceleration of the block is  $2 \text{ m s}^{-2}$  find the coefficient of friction between the block and the plane.

- 7 A parcel of mass  $10 \text{ kg}$  is released from rest on a rough plane which is inclined at  $25^\circ$  to the horizontal.

- a Find the normal reaction between the parcel and the plane.

Two seconds after being released the parcel has moved  $4 \text{ m}$  down the plane.

- b Find the coefficient of friction between the parcel and the plane.

- 8 A particle  $P$  is projected up a rough plane which is inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ . The coefficient of friction between the particle and the plane is  $\frac{1}{3}$ . The particle is projected from the point  $A$  with speed  $20 \text{ m s}^{-1}$  and comes to instantaneous rest at the point  $B$ .

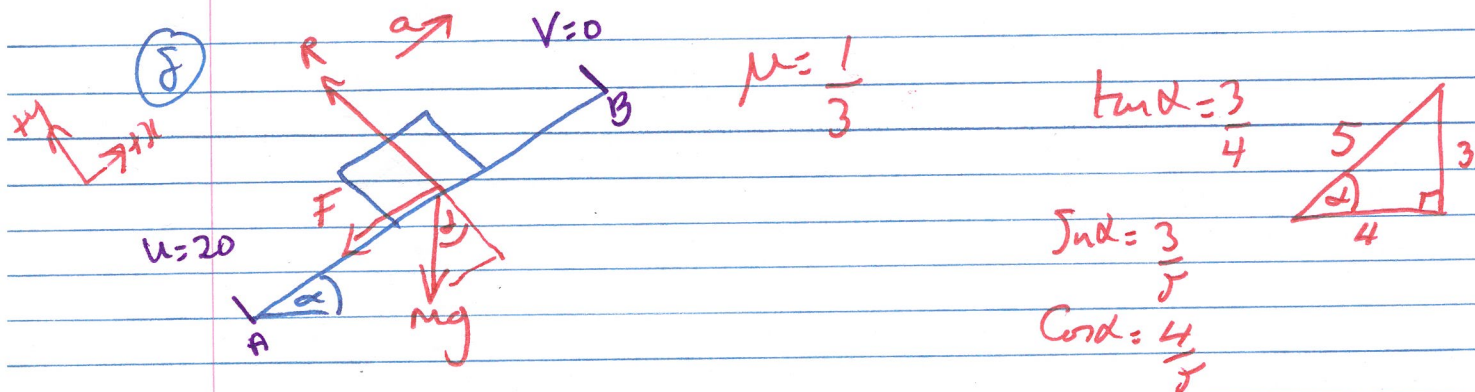
- a Show that while  $P$  is moving up the plane its deceleration is  $\frac{13g}{15}$ .
- b Find, to three significant figures, the distance  $AB$ .
- c Find, to three significant figures, the time taken for  $P$  to move from  $A$  to  $B$ .
- d Find the speed of  $P$  when it returns to  $A$ .

Answers

- 1  $3.35 \text{ m s}^{-2}$  (3 s.f.)
- 2 a  $27.7 \text{ N}$  (3 s.f.) b  $2.12 \text{ m s}^{-2}$
- 3 a  $2.43 \text{ m s}^{-2}$  (3 s.f.) b  $4.93 \text{ m s}^{-1}$  (3 s.f.)
- 4  $28 \text{ N}$
- 5  $0.20$  (2 s.f.)
- 6  $0.15$  (2 s.f.)
- 7 a  $88.8 \text{ N}$  (3 s.f.) b  $0.24$  (2 s.f.)
- 8 a  $\frac{13}{2}$  b  $23.5 \text{ m}$  (3 s.f.) c  $2.35 \text{ s}$  (3 s.f.)
- d  $12.4 \text{ m s}^{-1}$  (3 s.f.)



Ex 6.4



$$(a) \quad \Sigma F_x: -F - Mg \sin \alpha = Ma \quad \text{--- (1)}$$

$$\Sigma F_y: R - Mg \cos \alpha = 0 \quad \text{--- (2)}$$

$$F = \frac{1}{3} R \quad \text{--- (3)}$$

From (2)  $R = Mg \cos \alpha$

in (3)  $F = \frac{1}{3} Mg \cos \alpha$

$$u(1) \quad -\frac{1}{3} Mg \cos \alpha - Mg \sin \alpha = ma$$

$$a = -\frac{1}{3} g \cdot \frac{4}{5} - g \cdot \frac{3}{5}$$

$$a = -\frac{4}{15} g - \frac{3}{5} g$$

$$a = -\frac{13g}{15} \text{ or required}$$

b)  $u = 20, v = 0, a = -\frac{13g}{15}, s = ?$

$$0^2 = 20^2 - 2 \times \frac{13g}{15} \times s$$

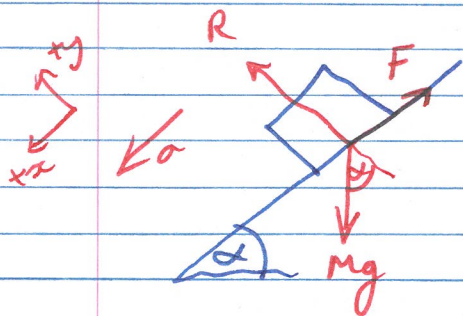
$$s = \frac{400}{\left(\frac{26g}{15}\right)} = 23.5 \text{ metres}$$

⑧(c)  $V = u + at$

$$0 = 20 - \frac{13g}{15}t$$

$$t = \frac{20}{\left(\frac{13g}{15}\right)} = 2.35 \text{ sec}$$

(d) Need to calculate a new accel.



$$\Sigma F_x: Mg \sin \alpha - F = ma \quad \text{--- (1)}$$

$$\Sigma F_y: R - Mg \cos \alpha = 0 \quad \text{--- (2)}$$

$$F = \mu R \quad \text{--- (3)}$$

$$\text{From (2)} \quad R = Mg \cos \alpha$$

$$\text{in (3)} \quad F = \frac{1}{3} Mg \cos \alpha$$

$$\text{in (1)} \quad \cancel{Mg \sin \alpha} - \frac{1}{3} \cancel{Mg \cos \alpha} = ma$$

$$a = \frac{3g}{5} - \frac{1g}{3} \times \frac{4}{5}$$

$$a = \frac{g}{3}$$

$$\text{Now } u = 0, v = ?, a = \frac{g}{3}, s = 23.5$$

$$v^2 = 0^2 + 2 \times \frac{g}{3} \times 23.5$$

$$v = \sqrt{153.53} = 12.4 \text{ ms}^{-1}$$



1	3.35 m s <sup>-2</sup> (3 s.f.)
2	a 27.7 N (3 s.f.) b 2.12 m s <sup>-2</sup>
3	a 2.43 m s <sup>-2</sup> (3 s.f.) b 4.93 m s <sup>-1</sup> (3 s.f.)
4	28 N
5	0.20 (2 s.f.)
6	0.15 (2 s.f.)
7	a 88.8 N (3 s.f.) b 0.24 (2 s.f.)
8	a $\frac{13}{12}$ b 23.5 m (3 s.f.) c 2.35 s (3 s.f.)
	d 12.4 m s <sup>-1</sup> (3 s.f.)

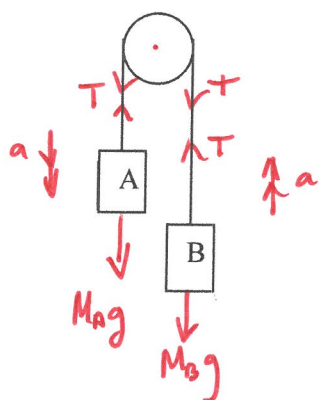
Answers

### The Effect of N2L on Pulleys

When connected particles pass over a pulley and released, the resulting motion will produce the same acceleration in each body. However, it is not possible to consider the system as a whole as the particles will be travelling in different directions.

A smooth pulley means that the tensions in the string are equal on both sides of the pulley.

If  $M_A > M_B$



**Eg 11** Two particles of mass 7kg and 3 kg are connected by a light, inextensible string passing over a smooth fixed pulley. Find the acceleration of the particles, the tension in the string and the force exerted on the pulley.

**Eg 12** Bodies of mass 3Mkg and Mkg are connected by a light inextensible string which passes over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find the acceleration of the system and the distance moved by the 3Mkg mass in the first 2 seconds of motion. After the 3M kg mass hits the floor 10metres below the point of release, how much farther will the Mkg body travel before beginning to fall again?

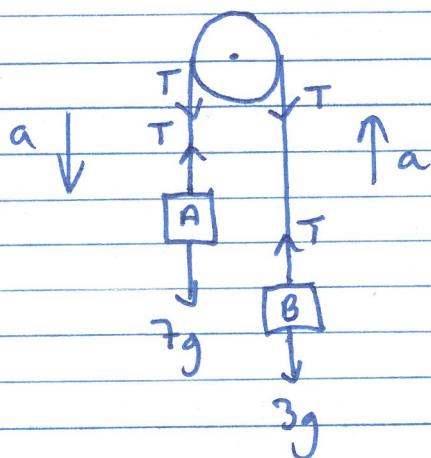
### Exercise 6.5 Q4

**Eg 13** A particle of mass 3kg rests on a rough horizontal table ( $\mu = 0.2$ ). It is connected by a light, inextensible string passing over a smooth pulley fixed at the edge of the table to a particle of mass 2kg which hangs freely. Find the acceleration of the system when it is released from rest. Find also the force exerted on the pulley.

### Ex 6.5 Q5



Ex 11



A: N2L  $7g - T = 7a$  — (1)

B: N2L  $T - 3g = 3a$  — (2)

(1) + (2)  $4g = 10a$

$a = \frac{4g}{10} = 3.9 \text{ m/s}^2$

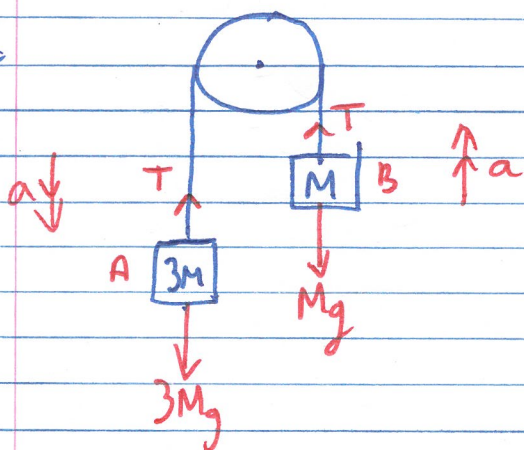
In (2)  $T = 3g + 3(3.9) = \frac{417}{41} \text{ N}$



Force exerted on pulley  $R - 2T = 0$

$R = 2T = 82 \text{ N}$

Ex 12



$$A: 3Mg - T = 3Ma \quad \text{--- (1)}$$

$$B: T - Mg = Ma \quad \text{--- (2)}$$

$$(1) + (2) \quad 2Mg = 4Ma$$

$$a = \frac{g}{2} = 4.9 \text{ ms}^{-2}$$

Now using SUVAT:  $u=0$ ,  $a=4.9$ ,  $t=2$ ,  $s=?$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times 4.9 \times 2^2$$

$$s = 9.8 \text{ metres.}$$

Speed of A when hits Floor = initial speed of B when string goes slack then moving under  $g$ .

$$u=0, s=10, a=4.9, v=?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 4.9 \times 10$$

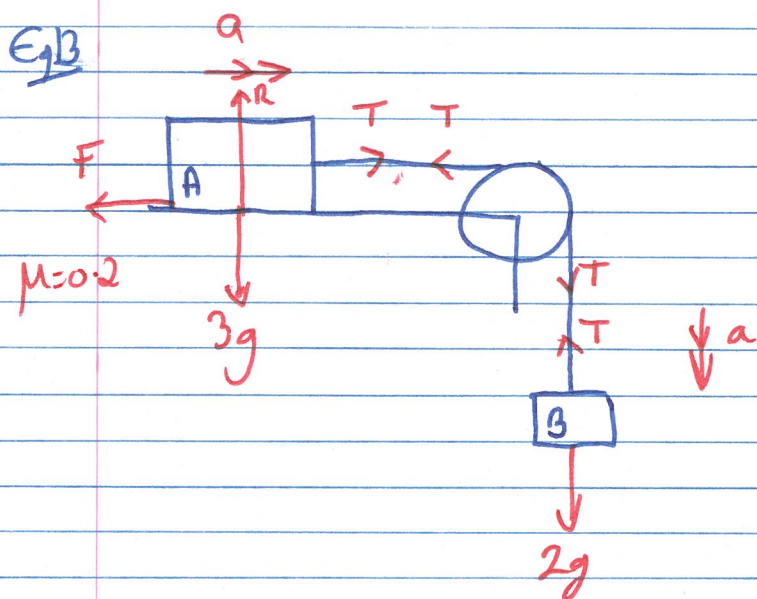
$$v = \sqrt{98}$$

$$\text{So for B: } u = \sqrt{98}, a = -9.8, v = 0, s = ?$$



Eg13 cont  $0^2 = 98 + 2 \times -9.8 \times S$

$$S = \frac{98}{19.6} = 5 \text{ metres.}$$



On A:  $\Sigma f_x = ma$   $T - F = 3a$  — (1)

$\Sigma f_y = 0$   $R - 3g = 0$  — (2)

$F = 0.2R$  — (3)

On B:  $\Sigma f_y = ma$   $2g - T = 2a$  — (4)

From (2)  $R = 3g$

∴ (3)  $F = 0.2(3g) = 0.6g$

∴ (1)  $T - 0.6g = 3a$

$T = 0.6g + 3a$  — (5)

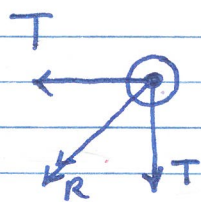
∴ (4)  $2g - 0.6g - 3a = 2a$

$1.4g = 5a$

$a = \frac{1.4g}{5} = 2.744, \quad 2.7 \text{ ms}^{-2} \quad 2 \text{ s.f.}$

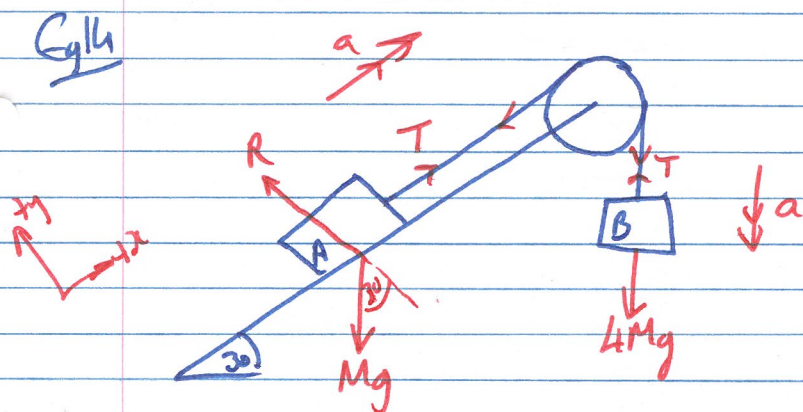


Q14 contd. in (5)  $T = 0.6g + (2.744) = 8.824 \text{ N}$   
 $= 14.112, 14 \text{ N (2sf)}$



$$R = \sqrt{T^2 + T^2} = 14.95...$$

$$= 20 \text{ N to 2sf.}$$



On A:  $T - Mg \sin 30 = Ma$  — (1)

$R - Mg \cos 30 = 0$  — (2)

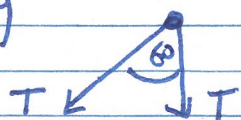
On B:  $4Mg - T = 4Ma$  — (3)

(1) + (3)  $4Mg - \frac{Mg}{2} = 5Ma$   
 $\frac{7g}{2} = 5a$

$a = 0.7g = 6.86, 6.9 \text{ m/s}^2 (2sf)$

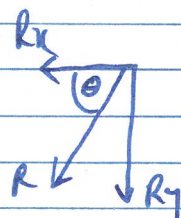
in (1)  $T = M(6.86) + \frac{Mg}{2} = 11.76 M$

Force on pulley



$R_x = T \sin 60$

$R_y = T + T \cos 60$



$\theta = \tan^{-1} \left( \frac{T + T \cos 60}{T \sin 60} \right) = 60^\circ$   
 below horizontal

$R = \sqrt{(T \sin 60)^2 + (T + T \cos 60)^2} = 20.36 M$   
 $= 20 M \text{ N}$

**Eg 14** A particle of mass  $M$  kg rests on a smooth plane inclined at an angle of  $30^\circ$  to the horizontal. It is connected by a light, inextensible string passing over a smooth pulley fixed at the top of the plane to a particle of mass  $4M$  kg which hangs freely. Find the acceleration of the system when it is released from rest, the tension in the string and also the force exerted by the string on the pulley.

**Ex 6.5** Q's 6, 7, 9

**Exercise 6.5**

- 1** Two particles  $P$  and  $Q$  of mass 8 kg and 2 kg respectively, are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of magnitude  $F$  is applied to  $P$  in a direction away from  $Q$  and when the string is taut the particles move with acceleration  $0.4 \text{ m s}^{-2}$ .
  - a** Find the value of  $F$ .
  - b** Find the tension in the string.
- 2** Two bricks  $P$  and  $Q$ , each of mass 5 kg, are connected by a light inextensible string. Brick  $P$  is held at rest and  $Q$  hangs freely, vertically below  $P$ . A force of 200 N is then applied vertically upwards to  $P$  causing it to accelerate at  $1.2 \text{ m s}^{-2}$ . Assuming there is a resistance to the motion of each of the bricks of magnitude  $R$  N, find
  - a** the value of  $R$ ,
  - b** the tension in the string connecting the bricks.
- 3** A car of mass 1500 kg is towing a trailer of mass 500 kg along a straight horizontal road. The car and the trailer are connected by a light inextensible tow-bar. The engine of the car exerts a driving force of magnitude 10 000 N and the car and the trailer experience resistances of magnitudes 3000 N and 1000 N respectively.
  - a** Find the acceleration of the car.
  - b** Find the tension in the tow-bar.
- 4** Two particles  $A$  and  $B$  of mass 4 kg and 3 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.
  - a** Find the tension in the string.When  $A$  has travelled a distance of 2 m it strikes the ground and immediately comes to rest.
  - b** Assuming that  $B$  does not hit the pulley find the greatest height that  $B$  reaches above its initial position.
- 5** Two particles  $A$  and  $B$  of mass 5 kg and 3 kg respectively are connected by a light inextensible string. Particle  $A$  lies on a rough horizontal table and the string passes over a small smooth pulley which is fixed at the edge of the table. Particle  $B$  hangs freely. The coefficient of friction between  $A$  and the table is 0.5. The system is released from rest. Find
  - a** the acceleration of the system,
  - b** the tension in the string,
  - c** the magnitude of the force exerted on the pulley by the string.



- 6** Two particles  $P$  and  $Q$  of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a smooth inclined plane. The plane is inclined to the horizontal at an angle  $\alpha$  where  $\tan \alpha = 0.75$ . Particle  $P$  is held at rest on the inclined plane at a distance of 2 m from the pulley and  $Q$  hangs freely on the edge of the plane at a distance of 3 m above the ground with the string vertical and taut. Particle  $P$  is released. Find the speed with which it hits the pulley.
- 7** Two particles  $P$  and  $Q$  of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed wedge. One face of the wedge is smooth and inclined to the horizontal at an angle of  $30^\circ$  and the other face of the wedge is rough and inclined to the horizontal at an angle of  $60^\circ$ . Particle  $P$  lies on the rough face and particle  $Q$  lies on the smooth face with the string connecting them taut. The coefficient of friction between  $P$  and the rough face is 0.5.
- a** Find the acceleration of the system.
  - b** Find the tension in the string.
- 8** A van of mass 900 kg is towing a trailer of mass 500 kg up a straight road which is inclined to the horizontal at an angle  $\alpha$  where  $\tan \alpha = 0.75$ . The van and the trailer are connected by a light inextensible tow-bar. The engine of the van exerts a driving force of magnitude 12 kN and the van and the trailer experience resistances to motion of magnitudes 1600 N and 600 N respectively.
- a** Find the acceleration of the van.
  - b** Find the tension in the tow-bar.
- 9** Two particles  $P$  and  $Q$  of mass 2 kg and 3 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of  $30^\circ$ . Particle  $P$  is held at rest on the inclined plane and  $Q$  hangs freely on the edge of the plane with the string vertical and taut. Particle  $P$  is released and it accelerates up the plane at  $2.5 \text{ m s}^{-2}$ . Find
- a** the tension in the string,
  - b** the coefficient of friction between  $P$  and the plane,
  - c** the force exerted by the string on the pulley.
- 10** A car of mass 900 kg is towing a trailer of mass 300 kg along a straight horizontal road. The car and the trailer are connected by a light inextensible tow-bar and when the speed of the car is  $20 \text{ m s}^{-1}$  the brakes are applied. This produces a braking force of 2400 N. Find
- a** the deceleration of the car,
  - b** the magnitude of the force in the tow-bar,
  - c** the distance travelled by the car before it stops.

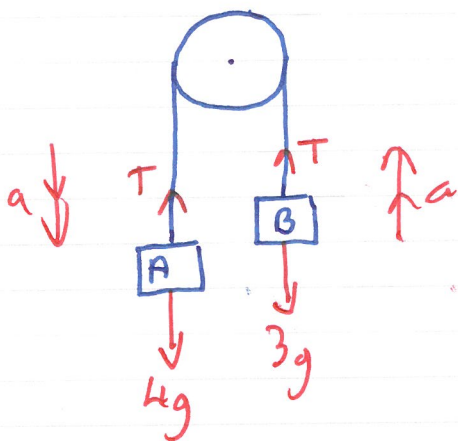


### Answers

- 1 a 4 N      b 0.8 N
- 2 a  $R = 45$       b 100 N
- 3 a  $3 \text{ m s}^{-2}$       b 2500 N
- 4 a 33.6 N (3 s.f.)      b  $2\frac{2}{3} \text{ m}$
- 5 a  $0.613 \text{ m s}^{-2}$  (3 s.f.)      b 27.6 N (3 s.f.)  
c 39.0 N (3 s.f.)
- 6  $2.8 \text{ m s}^{-1}$
- 7 a  $0.569 \text{ m s}^{-2}$  (3 s.f.)      b 0.56 mg
- 8 a  $1.12 \text{ m s}^{-2}$       b 4100 N
- 9 a 21.9 N      b 0.418 (3 s.f.)      c 38 N (2 s.f.)
- 10 a  $2 \text{ m s}^{-2}$       b 600 N      c 100 m

Ex 6.5

(4)



(a) N2L A:  $4g - T = 4a$  — (1)

B:  $T - 3g = 3a$  — (2)

(1) + (2)  $g = 7a$

$$a = \frac{g}{7}$$

u(2)  $T = \frac{3g}{7} + 3g = \frac{24g}{7} = 33.6 \text{ N}$

(b) Speed of A on impact:  $u=0$ ,  $a=\frac{g}{7}$ ,  $s=2$ ,  $V=?$

$$V^2 = u^2 + 2as$$

$$V^2 = 0 + 2 \times \frac{g}{7} \times 2$$

$$V = \sqrt{\frac{4g}{7}}$$

Now for motion of B:  $u = \sqrt{\frac{4g}{7}}$ ,  $a = -g$ ,  $V=0$ ,  $s=?$

$$V^2 = u^2 + 2as$$

$$0 = \frac{4g}{7} - 2gs$$

$s = \frac{2}{7} \text{ m}$  after A hit floor, so already travelled 2m  
ie  $2\frac{2}{7} \text{ m}$  above initial position