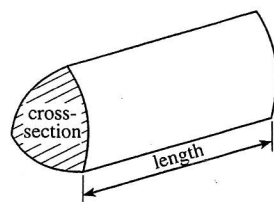JUNE 05 PAPER 2 Solutions

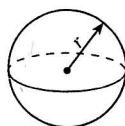## Formula List

Volume of prism = area of cross-section × length

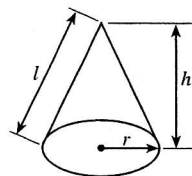**Volume of sphere** = $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone = $\frac{1}{3} \pi r^2 h$
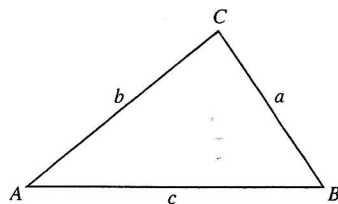
Curved surface area of cone = $\pi r l$

In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{ \frac{\sum x}{n} \right\}^2}$$
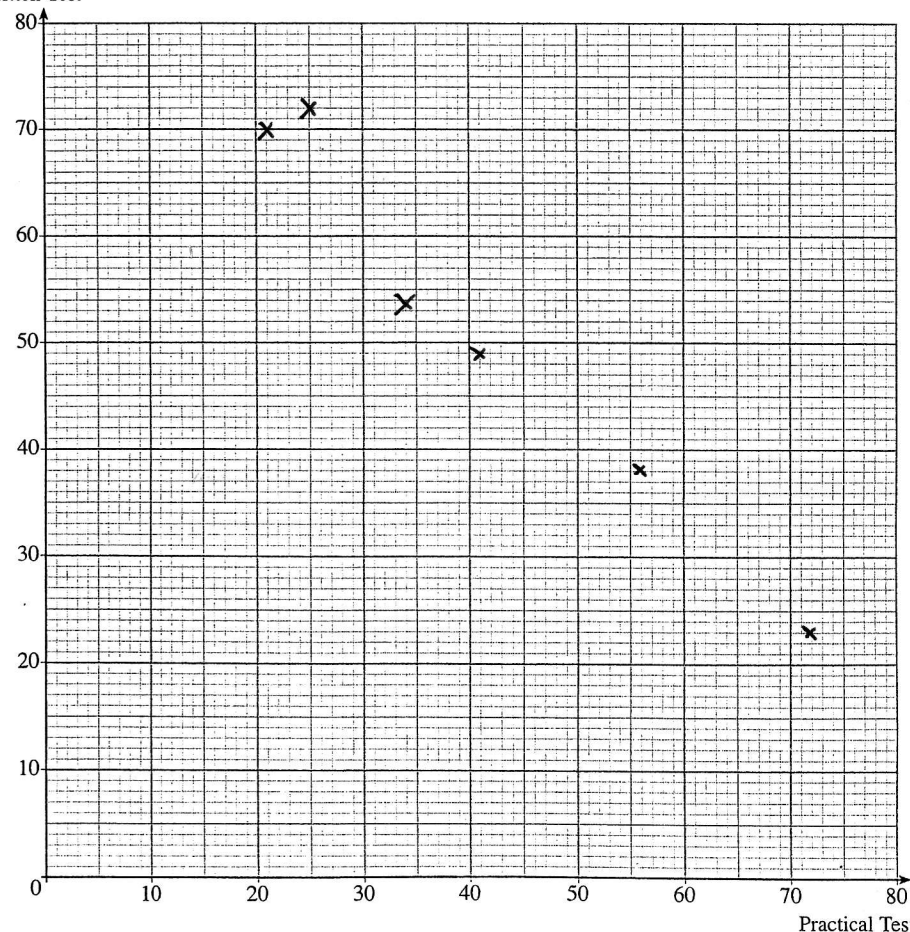
(T184/4)

1. In an examination, pupils have to undertake a practical test and a written test. The table below shows the results obtained by 6 pupils who took the tests.

Practical Test	21	34	72	56	25	41
Written Test	70	54	23	38	72	49

(a) On the graph paper provided draw a scatter diagram of these results.

[2]

Written Test / Practical Test

(b) Describe the correlation between the two sets of tests scores.

Negative

[1]

(T184/4)

Turn over.

John and Lynda share £248 in the ratio of 1 : 7. How much does each receive?

$1 + 7 = 8$ $248 \div 8 = 31$

John gets £31

Linda gets £31 × 7 = £217

John receives .... Lynda receives ....

[2]

A designer makes a piece of jewellery for £48 and sells it at a profit of 45%.
What is the selling price?

$1.45 \times 48 = £69.60$

[3]

4. Mrs. Hughes received an electricity bill. The details were as follows.

Present meter reading	69285
Previous meter reading	68672
Charge per unit	7.2 pence per unit
Service charge	£8.50

(a) Showing all your working, find the cost, in £, of the electricity including the service charge.

N° of Units used = 69285 - 68672 = 613

Cost of units = 613 × 7.2 = 4413.6 pence = £44.14

Total = 44.14 + 8.50 = £52.64

[3]

(b) Find the total cost when VAT is added at the rate of 5%.

$1.05 \times 52.64 = £55.27$

[3]

Turn over.

5. Alice, Bethan and Catrin are sisters.
Let Alice be x years old.

(a) Bethan is 7 years older than Alice. Write down in terms of x the age of Bethan.

$x+7$

[1]

(b) Catrin is twice as old as Alice. Write down in terms of x the age of Catrin.

$2x$

[1]

(c) The sum of the ages of Alice, Bethan and Catrin is 59 years. Form an equation in x and solve it to find Alice's age.
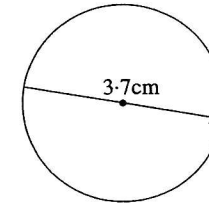
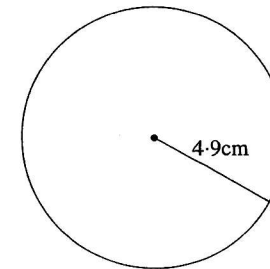$x + 2x + x + 7 = 59$

$4x = 52$

$x = \dfrac{52}{4} = 13$

[2]

6. (a) Calculate the circumference of a circle with a diameter of 3·7 cm.

3·7cm

$\pi \times 3.7 = 11.6 cm$

[2]

(b) Calculate the area of a circle with a radius of 4·9 cm.

4·9cm

$\pi \times 4.9^2 = 75.4 cm^2$

[2]

Turn over.

7. The table below shows the probability of selecting beads of different colours from a bag.

Colour	Red	Green	Yellow	Black
Probability	0·2	0·3	0·4	0·1

(a) Are there any other colours of bead in the bag? You must give a reason for your answer.

$$0.2 + 0.3 + 0.4 + 0.1 = 1$$

No other colour because total probability =1

[1]

(b) One bead is selected from the bag at random. What is the probability that the selected bead is not green?

$$1 - 0.3 = 0.7$$

[2]

8. Calculate the average speed, in m.p.h., of a car that travels 63 miles in one and a half hours.

$$\text{Speed} = \frac{dist}{time} = \frac{63}{1.5}$$

[2]

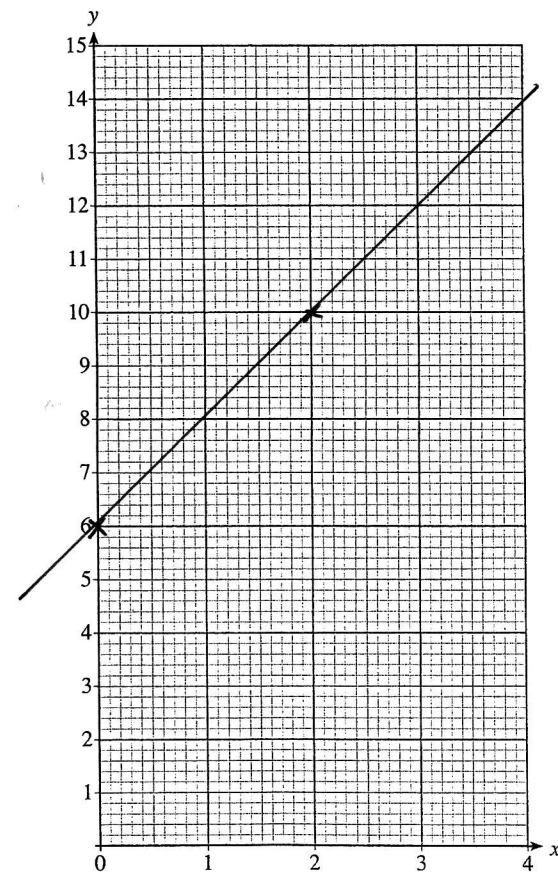9. Use the graph paper below to draw the graph of the straight line $y = 2x + 6$.

When $x=0$ $y = 2 \times 0 + 6 = 6$ (0,6)

when $x=2$ $y = 2 \times 2 + 6 = 4 + 6 = 10$ (2,10)

[3]

Turn over.

10. *(a)* Solve

$$5x - 2 = 7.$$

$5x = 9$

$x = \dfrac{9}{5}$

[2]

(b) Expand and simplify

$$2(5x + 6) + 3(x - 2).$$

$10x + 12 + 3x - 6$

$13x + 6$

[2]

(c) Simplify

$$m^3 \times m^7.$$

M^{10}

[1]

(d) Solve

$$\frac{25 + x}{2} = 20.$$

$25 + x = 20$

$x = 20 - 25$

$x = -5$

[3]

11. A prism has a uniform cross-section in the shape of a triangle FGH, in which $FG = 5 \cdot 1$ cm, $GH = 6 \cdot 8$ cm and $F\widehat{G}H = 90°$. The length of the prism is $3 \cdot 7$ cm.
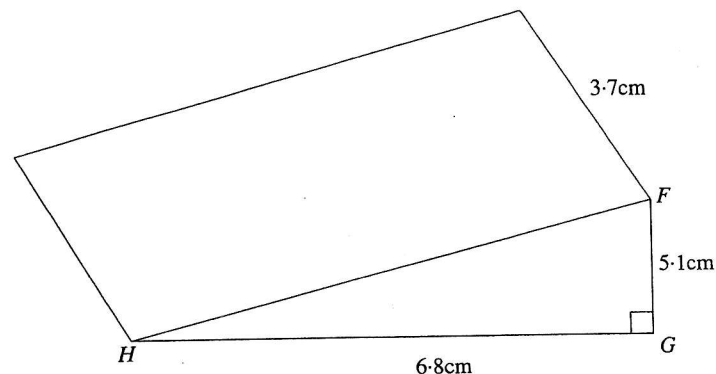

Diagram not drawn to scale.

(a) Calculate the volume of the prism.

Area of cross section $= \dfrac{1}{2} \times 6.8 \times 5.1 = 17.34 \text{cm}^2$

Volume of prism $= 17.34 \times 3.7 = 64.158 \text{cm}^3$

[3]

(b) The material from which the prism is made has a density of $3 \cdot 9$ g/cm³. Find the mass of the prism in kilograms.

$D = \dfrac{M}{V}$

$M = D \times V$

$= 3.9 \times 64.158$

$= 250.2162 \text{ g}$

$= 0.25 \text{ kg}$

[3]

Turn over.

12. A solution to the equation

$$x^3 + x = 45$$

lies between 3·4 and 3·5.
Use the method of trial and improvement to find this solution correct to 2 decimal places.

$x = 3.45 \qquad 44.5$ too small

$x = 3.47 \qquad 45.25$ too big

$x = 3.46 \qquad 44.88$ too small

So x lies between 3.46 & 3.47

Test using $x = 3.465 \qquad 45.07$ too big

$\therefore x = 3.46$ to 2dp

[4]

13. The heights of 50 pupils were measured to the nearest centimetre. The table shows a grouped frequency distribution of the heights.

Height of pupil (h centimetres)	Number of pupils
$140 \leqslant h \leqslant 149$	10
$150 \leqslant h \leqslant 159$	25
$160 \leqslant h \leqslant 169$	15

Find an estimate for the mean height of these pupils.

Midpt $144.5 \times 10 = 1445$

$154.5 \times 25 = 3862.5$ +

$164.5 \times 15 = 2467.5$ +

$\overline{7775}$

Mean $= \dfrac{7775}{50} = 155.5$ cm

[4]

14. Name the following shapes from the descriptions given.

(a) A quadrilateral with only 1 pair of parallel sides.

Trapezium

[1]

(b) A quadrilateral with 4 lines of symmetry.
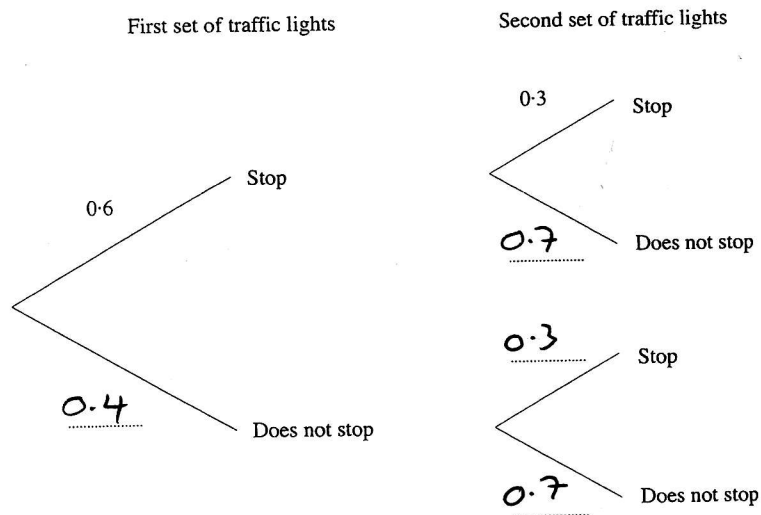
Square

[1]

(c) A regular polygon with rotational symmetry of order 5.

pentagon

[1]

15. Rodney goes through two sets of traffic lights on his way to work. The probability that he has to stop at the first set of traffic lights is 0·6. The probability that he has to stop at the second set of traffic lights is 0·3.

(a) Complete the following tree diagram.

First set of traffic lights   Second set of traffic lights

0·3 — Stop

0·6 — Stop

0·7 — Does not stop

0·3 — Stop

0·4 — Does not stop

0·7 — Does not stop

[2]

(b) Calculate the probability that Rodney does not have to stop at either set of lights.

$$0·4 \times 0·7 = 0·28$$

[2]

16. *(a)* Write **each** of the following numbers in standard form.

(i) 0·00073

$$7·3 \times 10^{-4}$$

(ii) 82 000 000

$$8·2 \times 10^{7}$$

[2]

(b) Find, in standard form, the value of:

$$\frac{4·7 \times 10^{18}}{8·9 \times 10^{3}}$$

$$5·28 \times 10^{14}$$

[2]

Turn over.

17. (a) The triangle *PQR* is a right-angled triangle with $\widehat{PQR} = 90°$. The length *QR* = 6·8 cm and the length *PR* = 14·2 cm.
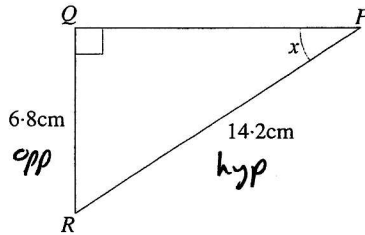
Q ⌐ P
x

6·8cm
opp

14·2cm
hyp

R

O
S H

Diagram not drawn to scale.

Calculate the size of the angle $\widehat{QPR}$ which is denoted by *x*.

$$Sin x = \frac{6.8}{14.2}$$

$$x = Sin^{-1}\left(\frac{6.8}{14.2}\right) = 28.6°$$

[3]

(b) The triangle *KLM* is a right-angled triangle with $\widehat{KLM} = 90°$. The length *KM* = 5·2 cm and $\widehat{LKM} = 42°$.
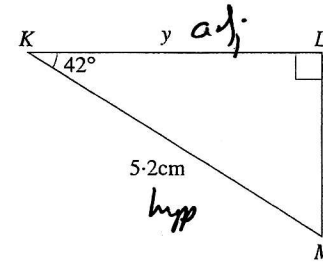
K y adj L
42°

5·2cm
hyp

M

C A H

Diagram not drawn to scale.

Calculate the length of *KL* which is denoted by *y*.

$$adj = Cos 42 \times hyp$$

$$y = Cos 42 \times 5.2 = 3.9 cm$$

[3]

18. Solve the inequality

$$6x - 5 > 2x + 11.$$

$$6x - 2x > 11 + 5$$

$$4x > 16$$

$$x > \frac{16}{4}$$

$$x > 4$$

[2]

19. (a) Factorise $4x^2 - 8xy$.

$$4x(x - 2y)$$

[2]

(b) Simplify $\sqrt{x^8}$.

$$(x^8)^{\frac{1}{2}} = x^4$$

[1]

(c) Simplify $(x^6)^{\frac{1}{3}}$.

$$x^2$$

[1]

20. Factorise the expression $12x^2 + 17x + 6$. Hence solve the equation $12x^2 + 17x + 6 = 0$.

$$\boxed{72} \quad 9,8$$

$$12x^2 + 9x + 8x + 6$$

$$3x(4x + 3) + 2(4x + 3)$$

$$(3x + 2)(4x + 3) = 0$$

$$\text{either} \quad 3x + 2 = 0 \quad \text{or} \quad 4x + 3 = 0$$

$$x = \frac{-2}{3} \qquad x = \frac{-3}{4}$$

[3]

21. Use the formula method to solve the equation $2x^2 + 32x + 13 = 0$, giving the solutions correct to two decimal places.

$$a = 2 \quad b = 32 \quad c = 13$$

$$x = \frac{-32 \pm \sqrt{32^2 - 4 \times 2 \times 13}}{2 \times 2}$$

$$x = \frac{-32 \pm \sqrt{1024 - 104}}{4}$$

$$x = \frac{-32 \pm \sqrt{920}}{4}$$

$$\text{either} \quad x = \frac{-32 + \sqrt{920}}{4} = -0.42$$

$$\text{or} \quad x = \frac{-32 - \sqrt{920}}{4} = -15.57$$

[3]

22. Make *g* the subject of the following formula.

$$4d(g + 3) = 3g + 5$$

$4dg + 12d = 3g + 5$

$4dg - 3g = 5 - 12d$

$g(4d - 3) = 5 - 12d$

$g = \dfrac{5 - 12d}{4d - 3}$

[4]

23. On the graph paper provided, draw the region which satisfies all of the following inequalities.

$$x + y \leqslant 7$$
$$y \geqslant 5x + 2$$
$$x \geqslant -2$$

Make sure that you clearly indicate the region that represents your answer.

$x + y = 7$ whan $x = 0$ $y = 7$ $(0, 7)$

$y = 0$ $x = 7$ $(7, 0)$

$y = 5x + 2$ whan $x = 0$ $y = 2$ $(0, 2)$

$x = 2$ $y = 5 \times 2 + 2 = 12$ $(2, 12)$

[3]

Turn over.
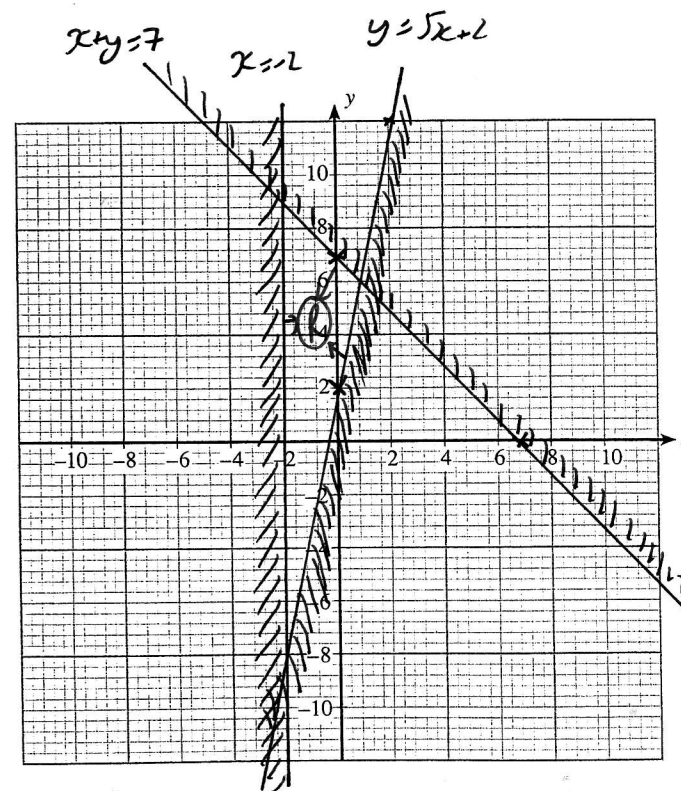
24. Using the axes below, **sketch** the graph of $y = \cos x$ for values of x from $-180°$ to $180°$.
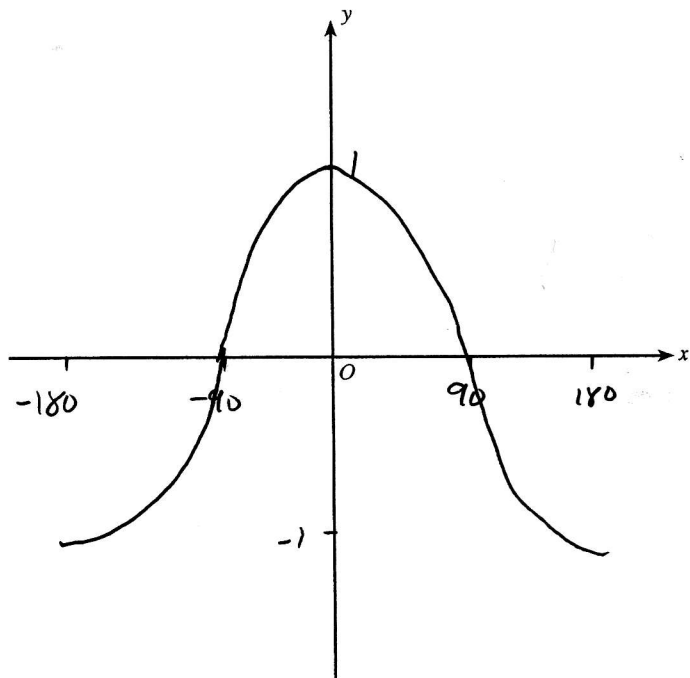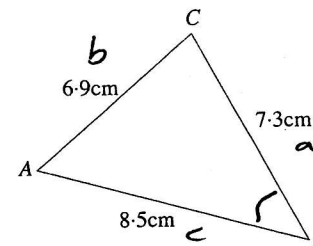
[2]

25. The diagram shows triangle ABC.

Diagram not drawn to scale.

Given that $AB = 8\cdot5$ cm, $AC = 6\cdot9$ cm and $BC = 7\cdot3$ cm, calculate $C\hat{B}A$.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{7\cdot3^2 + 8\cdot5^2 - 6\cdot9^2}{2 \times 7\cdot3 \times 8\cdot5}$$

$$\cos B = 0\cdot630$$

$$B = \cos^{-1}(0\cdot630)$$

$$= 51\cdot1°$$

[3]

Turn over.

26. Vectors **OP**, **OQ** and **OR** are shown in the diagram below.

R

a + 20b

Q

O   P
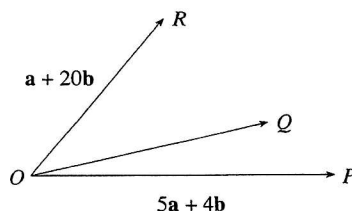
5a + 4b

Diagram not drawn to scale.

You are given that **OP** = 5**a** + 4**b** and **OR** = **a** + 20**b**.

(a) Show that **PR** = −4**a** + 16**b**.

$$\vec{PR} = -\vec{OP} + \vec{OR} = -5a - 4b + a + 20b$$
$$= -4a + 16b$$

[1]

(b) Express **RP** in terms of **a** and **b** in its simplest form.

$$\vec{RP} = -\vec{OR} + \vec{OP} = -a - 20b + 5a + 4b$$
$$= 4a - 16b$$

[1]

(c) Given that **PQ** = −**a** + 4**b**, what can you say about the lines *PQ* and *PR*?

$$\vec{RP} = 4(a - 4b)$$

They are parallel, RP = 4 times as long

and in opposite direction.

[1]

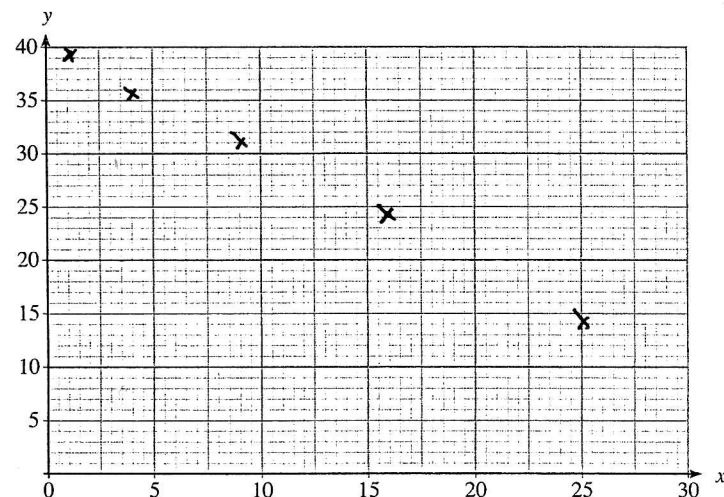**27.** The data in the table was recorded during an experiment. Results were recorded for the two variables *x* and *y*.

x^2	1	4	9	16	25
x	1	2	3	4	5
y	39·1	35·9	31·0	24·2	14·8

(a) On the graph paper, plot the value of *y* against the value of x^2.

[2]

Before starting the experiment it was already known that *y* is approximately equal to $ax^2 + b$. From the intersection of the graph with the *y* axis it is found that $b \approx 40$.

(b) Use your graph to estimate the gradient *a*.

$$y = ax^2 + 40$$
when x=3 y=31
31 = 9a + 40   9a = −9   a = −1

[1]

(c) Hence write down the approximate equation for *y* in terms of x^2.

$$y = -x^2 + 40$$

[1]