

JUNE 2006 PAPER 2 Solutions

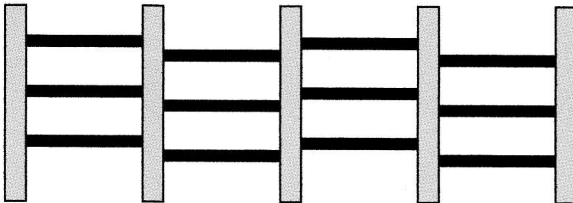
3

1. Gordon uses $\frac{3}{4}$ of a jar of peanut butter on a single slice of toast. How many jars of peanut butter would he use for 20 slices of toast?

$$20 \times \frac{3}{4} = 15 \text{ jars}$$

[2]

2. Fences are made using vertical posts and 3 horizontal rails between each pair of vertical posts. The fence shown below has 5 vertical posts with 3 horizontal rails between each pair of vertical posts.

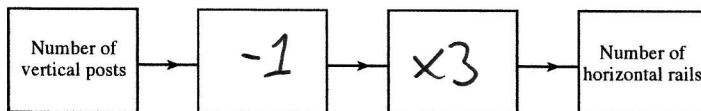


- (a) Complete the following table which shows the relationship between the number of vertical posts and horizontal rails.

Number of vertical posts	Number of horizontal rails
2	3
3	6
4	9
5	12
6	15
10	27

[1]

- (b) Use your table to enter the two missing stages in the number machine.



[2]

Turn over.

4

3. Frankie is offered a choice of two mobile telephone deals.

Mega deal

A free phone and each month have up to 300 minutes of talk time at 4 pence per minute and 200 text messages at 1 pence each

OR

Bonus deal

A free phone and each month have up to 200 minutes of talk time at 3 pence per minute and 250 text messages at 2 pence each

Frankie usually uses 150 minutes of talk time and sends 120 texts each month.

- (a) Work out how much Frankie would pay a month using

- (i) the Mega deal offer,

$$150 \times 4p = £6$$

$$120 \times 1p = £1.20$$

$$£7.20$$

- (ii) the Bonus deal offer.

$$150 \times 3p = £4.50$$

$$120 \times 2p = £2.40$$

$$£6.90$$

[3]

- (b) Frankie will also have to pay tax at a rate of 10% on the total monthly amount charged. Which is the cheapest deal for Frankie to choose and by how much?

$$\text{Mega} = 7.20 + 0.72 = £7.92$$

$$\text{Bonus} = 6.90 + 0.69 = £7.59$$

So Bonus is the best deal by 33p.

[3]

4. (a) Solve
- $8x + 4 = 7 - 4x$
- .

$$8x + 4x = 7 - 4$$

$$12x = 3$$

$$x = \frac{3}{12} = \frac{1}{4}$$

[3]

- (b) Solve
- $5(2x - 3) = 50$
- .

$$10x - 15 = 50$$

$$10x = 65$$

$$x = \frac{65}{10} = 6.5$$

[3]

- (c) (i) Factorise
- $2x^2 - 6x$
- .

$$2x(x - 3)$$

- (ii) Factorise
- $3a - 12$
- .

$$3(a - 4)$$

[3]

- (d) Write down the first three terms of a sequence where the
- n
- th term is
- $n^2 - 3$
- .

$$n = 1 \quad 2 \quad 3$$

$$n^2 - 3 = -2 \quad 1 \quad 6$$

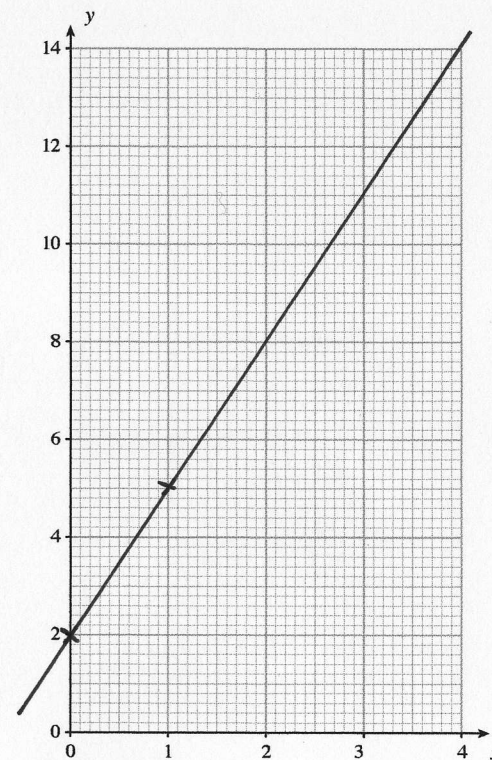
[2]

5. Use the graph paper below to draw the graph of the straight line
- $y = 3x + 2$
- .

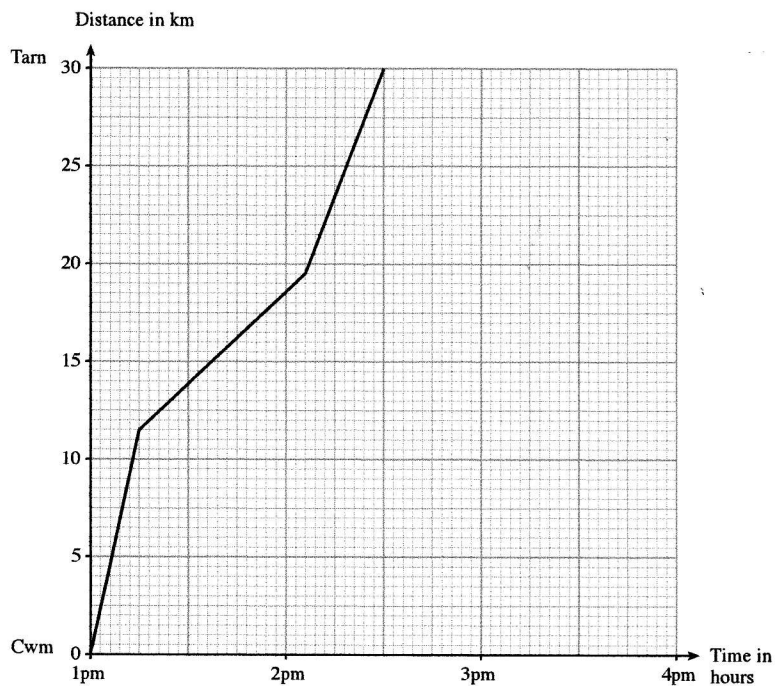
$$x = 0 \quad y = 3 \times 0 + 2 = 2 \quad (0, 2)$$

$$x = 1 \quad y = 3 \times 1 + 2 = 5 \quad (1, 5)$$

[3]



6. The distance – time graph shows a 30 km journey from Cwm to Tarn starting at 1 p.m.



Find the average speed for the 30 km journey.

$$\text{Av Speed} = \frac{\text{Dist}}{\text{Time}} = \frac{30}{1.5} = 20 \text{ km/h}$$

[2]

7. (a) Evaluate $\frac{2.3 \times 4.6}{5.8 - 3.6}$ correct to one decimal place.

$$(2.3 \times 4.6) \div (5.8 - 3.6) = 4.8 \text{ to 1 dp.}$$

[2]

- (b) Evaluate $\frac{1}{4} + \frac{1}{3}$.

$$\frac{3}{4}$$

[1]

- (c) Find $\sqrt{5.23^2 + 3.21^2}$ giving your answer correct to 3 significant figures.

$$6.14 \text{ to 3 sig figs}$$

[2]

- (d) The length of a plank of wood is 950 mm, measured to the nearest 10 mm. Write down the **least** and **greatest** possible values of the length of the plank.

Least value 945 mm

Greatest value 955 mm

[2]

8. Mal invests £5500 for 2 years at 6% per annum compound interest. What is the value of his investment after 2 years?

$$(1.06)^2 \times 5500 = £6179.80$$

[3]

9. The heights of 80 people were measured to the nearest centimetre. The table below shows a grouped frequency distribution of the heights.

Height (h centimetres)	Number of people
$151 \leq h \leq 157$	18
$158 \leq h \leq 164$	37
$165 \leq h \leq 171$	25

Find an estimate for the mean height of these people.

$$(154 \times 18) + (161 \times 37) + (168 \times 25)$$

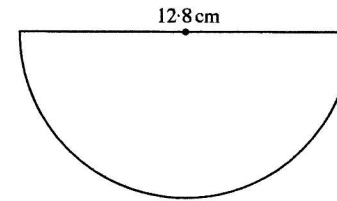
$$= 12929$$

$$\text{Mean} = \frac{12929}{80} = 161.6 \text{ cm}$$

[4]

Turn over.

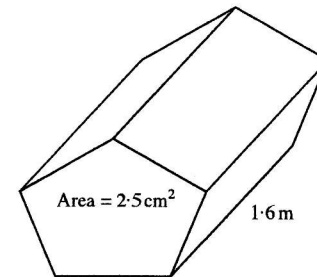
10. (a) Calculate the area of a semicircle with a diameter of 12.8 cm.



$$\text{Area of Semi-circle} = \frac{\pi \times 6.4^2}{2} = 64.3 \text{ cm}^2$$

[2]

- (b) Calculate the volume of a prism with an area of cross-section 2.5 cm^2 and length 1.6 m, giving your answer in cm^3 .



$$\text{Volume} = 2.5 \times 160 = 400 \text{ cm}^3$$

[3]

11. A solution to the equation

$$2x^3 + x - 8 = 0$$

lies between 1 and 2.

Use the method of trial and improvement to find this solution correct to 1 decimal place.

$$x = 1.5 \quad 0.25 \text{ too big}$$

$$x = 1.3 \quad -2.306 \text{ too small}$$

$$x = 1.4 \quad -1.112 \text{ too small}$$

So x lies between 1.4 and 1.5

$$\text{Test using } x = 1.45 \quad -0.45275 \text{ too small}$$

$$\therefore x = 1.5 \text{ to 1 dp.}$$

[4]

12. (a) Factorise the expression
- $x^2 - 9x - 10$
- and hence solve the equation
- $x^2 - 9x - 10 = 0$
- .

$$(x-10)(x+1) = 0$$

$$\text{either } x-10=0 \quad \text{or } x+1=0$$

$$x=10 \quad x=-1$$

[3]

- (b) Solve
- $\frac{24-2x}{5} = 6-x$
- .

$$24-2x = 5(6-x)$$

$$24-2x = 30-5x$$

$$5x-2x = 30-24$$

$$3x = 6$$

$$x = \frac{6}{3} = 2$$

[3]

- (c) Simplify
- $3x^6y^2 \times 4x^2y^5$
- .

$$12x^8y^7$$

[2]

- (d) Factorise
- $x^2 - 9$
- .

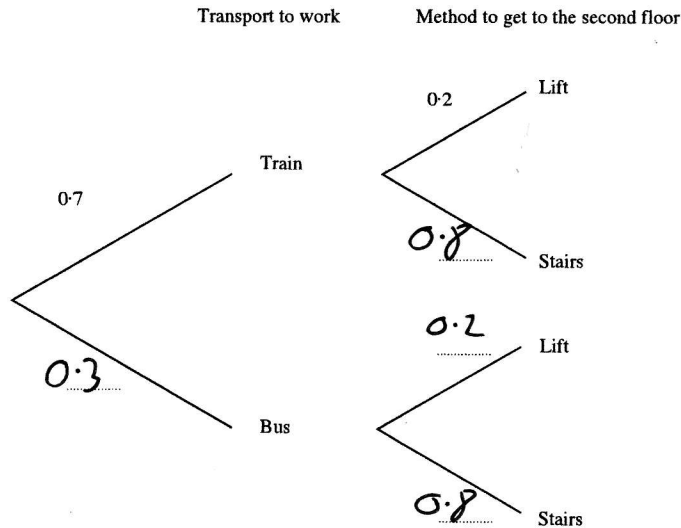
$$(x+3)(x-3)$$

[1]

13. Jayne works in a second floor office, she can either take the train or bus to work. The probability that she takes the train to work is 0.7. When Jayne arrives at the office building where she works she can either use the stairs or the lift to the second floor. The probability that she uses the lift is 0.2.

(a) Complete the following tree diagram.

[2]



- (b) Calculate the probability that Jayne takes the train to work and uses the stairs to get to her office.

$$0.7 \times 0.8 = 0.56$$

[2]

Turn over.

14. (a) Write each of the following numbers in standard form.

(i) 6 million

$$6\,000\,000 = 6 \times 10^6$$

(ii) 0.0043

$$0.0043 = 4.3 \times 10^{-3}$$

[2]

- (b) Find, in standard form, the value of $(8.4 \times 10^3) \times (2 \times 10^5)$.

$$168\,000\,000 = 1.68 \times 10^9$$

[2]

15. (a) The triangle EFG is a right-angled triangle with $\widehat{EFG} = 90^\circ$. The length $EG = 14.8$ cm and $\widehat{GEF} = 39^\circ$.

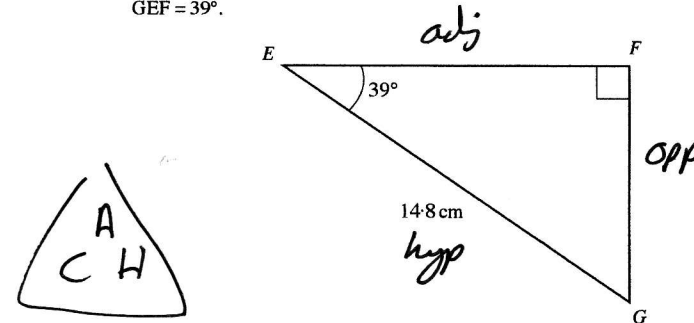


Diagram not drawn to scale.

Calculate the length of EF .

$$\frac{\text{adj}}{\text{hyp}} = \cos 39 \times \text{hyp}$$

$$EF = \cos 39 \times 14.8$$

$$= 11.5 \text{ cm}$$

[3]

- (b) The triangle RST is a right-angled triangle with $\widehat{TRS} = 90^\circ$. The length $RT = 12.8$ cm and the length $RS = 18.5$ cm.

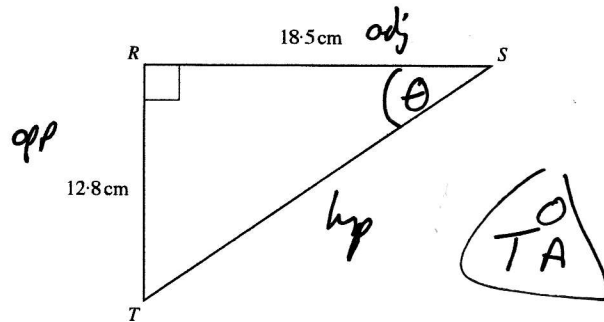


Diagram not drawn to scale.

Calculate the size of the angle \widehat{RST} .

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{12.8}{18.5}$$

$$\theta = \tan^{-1}\left(\frac{12.8}{18.5}\right)$$

$$\theta = 34.7^\circ$$

[3]

16. Use the formula method to solve the equation $3x^2 + 31x + 8 = 0$, giving solutions correct to two decimal places.

$$a=3 \quad b=31 \quad c=8$$

$$x = \frac{-31 \pm \sqrt{31^2 - 4 \times 3 \times 8}}{2 \times 3}$$

$$x = \frac{-31 \pm \sqrt{961 - 96}}{6}$$

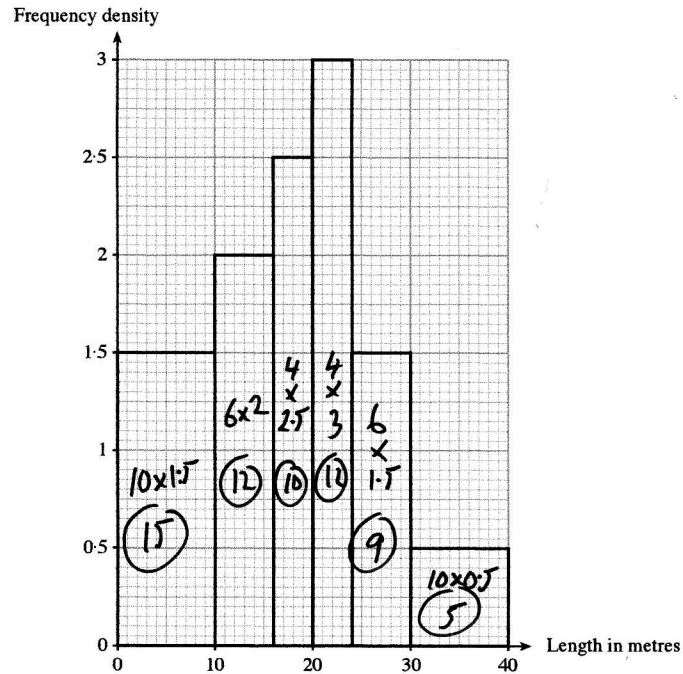
$$x = \frac{-31 \pm \sqrt{865}}{6}$$

[3]

$$\text{either } x = \frac{-31 + \sqrt{865}}{6} = -0.26$$

$$\text{or } x = \frac{-31 - \sqrt{865}}{6} = -10.07$$

17. A survey was carried out to find the distribution of the lengths of boats in a marina. The histogram below illustrates the results of the survey.



Use the histogram to calculate the number of boats measured.

$$15 + 12 + 10 + 12 + 9 + 5 = 63$$

18. Calculate the standard deviation of the following 10 numbers.

5.3, 6.1, 7.3, 4.5, 2.3, 4.4, 9.4, 3.4, 2.3, 7.2

$$\sum x = 52.2$$

$$\sum x^2 = 320.54$$

$$s = \sqrt{\left(\frac{320.54}{10}\right) - \left(\frac{52.2}{10}\right)^2} = 2.2$$

[3]

19. Three points A , B and C lie on the circumference of the circle centre O . The tangent RS meets the circle at A .

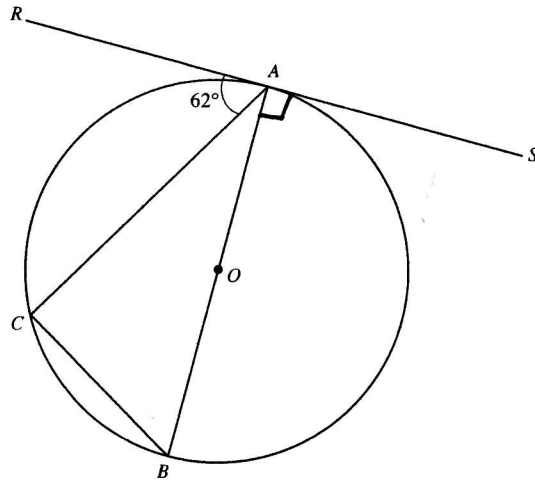


Diagram not drawn to scale.

Given that $\widehat{RAC} = 62^\circ$, find the following angles giving reasons for your answers.

(a) $\widehat{ACB} = 90^\circ$ (Angle in a semi-circle is a right angle).

(b) $\widehat{ABC} = 90 - 62 = 28$ (Tangent to a radius is a right angle)
 $\widehat{ABC} = 180 - 90 - 28 = 62^\circ$.

[3]

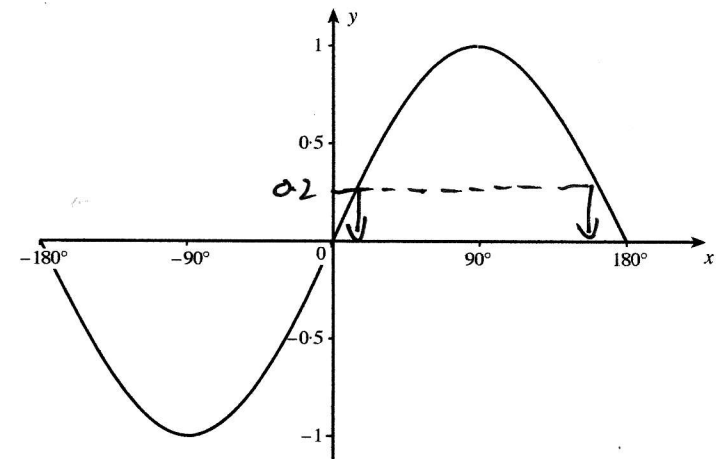
20. The volume of a hemisphere is 34.2 cm^3 . Calculate the radius of the hemisphere.

$$\begin{aligned} \text{Vol of sphere} &= \frac{4}{3}\pi r^3 \\ \text{Vol of hemisphere} &= \left(\frac{4}{3}\pi r^3\right) \div 2 \\ 34.2 &= \left(\frac{4}{3}\pi r^3\right) \div 2 \\ 34.2 \times 2 &= \frac{4}{3}\pi r^3 \\ 68.4 &= \frac{4}{3}\pi r^3 \end{aligned}$$

$$\begin{aligned} \frac{68.4 \times 3}{4\pi} &= r^3 \\ r &= \sqrt[3]{\frac{68.4 \times 3}{4\pi}} \\ r &= \sqrt[3]{16.329...} \\ r &= 2.5 \text{ cm} \end{aligned}$$

[3]

21. The diagram shows a sketch of the graph $y = \sin x$.



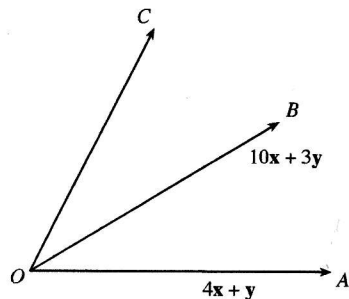
Using your calculator, find the values of x in the range $-180^\circ \leq x \leq 180^\circ$ which satisfy the equation $\sin x = 0.2$.

From rule: $x = \sin^{-1}(0.2) = 11.5^\circ$

From graph: $x = 180 - 11.5 = 168.5^\circ$

[2]

22. Vectors \vec{OA} , \vec{OB} and \vec{OC} are shown in the diagram below.



You are given that $\vec{OA} = 4\mathbf{x} + \mathbf{y}$ and $\vec{OB} = 10\mathbf{x} + 3\mathbf{y}$.

- (a) Express \vec{AB} in terms of \mathbf{x} and \mathbf{y} in its simplest form.

$$\vec{AB} = -\vec{OA} + \vec{OB} = -4\mathbf{x} - \mathbf{y} + 10\mathbf{x} + 3\mathbf{y} = 6\mathbf{x} + 2\mathbf{y}$$

[2]

- (b) Given that $\vec{AC} = 1.5 \vec{AB}$, find \vec{OC} in terms of \mathbf{x} and \mathbf{y} .

$$\vec{AC} = 1.5(6\mathbf{x} + 2\mathbf{y}) = 9\mathbf{x} + 3\mathbf{y}$$

$$\vec{AC} = -\vec{OA} + \vec{OC}$$

$$\vec{OC} = \vec{AC} + \vec{OA}$$

$$= 9\mathbf{x} + 3\mathbf{y} + 4\mathbf{x} + \mathbf{y}$$

$$= 13\mathbf{x} + 4\mathbf{y}$$

[2]

Turn over.

23. The triangles ADB and BDC are such that $BC = 26.2$ cm, $CD = 29.3$ cm, $AB = 14.4$ cm, $\hat{ADB} = 50^\circ$ and $\hat{DCB} = 30^\circ$.

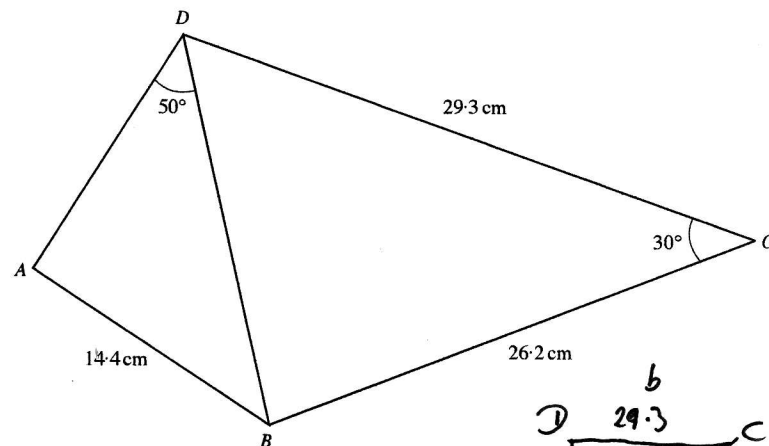


Diagram not drawn to scale.

Find the size of \hat{DAB} .

Find DB from $\triangle DBC$ using cosine rule

$$c^2 = 29.3^2 + 26.2^2 - 2 \times 29.3 \times 26.2 \cos 30$$

$$c^2 = 1544.93 - 1535.32 \cos 30$$

$$c^2 = 1544.93 - 1329.63$$

$$c^2 = 215.3$$

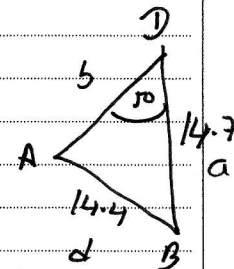
$$c = \sqrt{215.3} = 14.7 \text{ cm}$$

Using Sine Rule in $\triangle ABD$

$$\frac{\sin A}{14.7} = \frac{\sin 50}{14.4}$$

$$\sin A = \frac{\sin 50}{14.4} \times 14.7 = 0.782$$

$$A = \sin^{-1}(0.782) = 51.4^\circ$$



[6]