Ex 2C

(1) Initially: $EPE = \frac{60 \times 0.25^2}{2 \times 1} = 1.875\ J.$

Finally: $KE = \frac{1}{2} \times 2 \times V^2 = V^2$

No PE to consider because particle supported throughout

~~[illegible]~~.

$\therefore\ V^2 = 1.875$

$V = 1.37\ ms^{-1}$

If PE is considered...
gain in $PE = 2 \times 9.8 \times 0.25 = 4.9\ J$

loss in EPE = gain in PE + gain in KE

$1.875 = 4.9 + V^2$

= not possible!

(2) (a) Initial $EPE = \frac{6 \times 1.5^2}{2 \times 1.5} = 4.5\ J.$

Initial $KE = 0$

at pt where string goes slack, $EPE = 0$, $KE = \frac{1}{2} \times 3 \times V^2$

Now gain in KE = loss in EPE

$\frac{1}{2} \times 3 \times V^2 = 4.5$

$V = 1.73\ ms^{-1}$

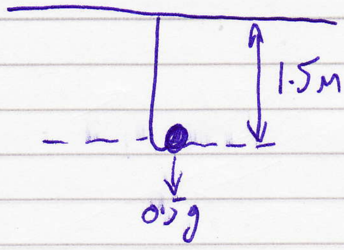
(b) When $OP = 2$, $EPE = \frac{6 \times 0.5^2}{2 \times 1.5} = 0.5\ J.$

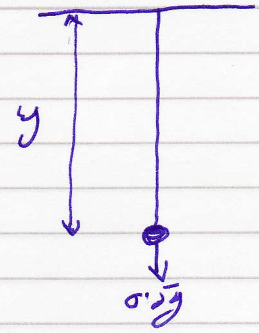
gain in KE = loss in EPE

$\frac{1}{2} . 3 . V^2 = (4.5 - 0.5)$

$V = 1.63\ ms^{-1}$

(3)





(a) length of string @ lowest point $= y$

$\therefore$ extension $(y-2)$ m

At lowest point particle instantaneously at rest

$$EPE = \frac{20 \times (y-2)^2}{2 \times 2}$$

gravitational PE lost $= 0.5g \times (y - 1.5)$

Initial KE = Final KE = 0

$\therefore$ GAIN in EPE = loss in PE

$$\frac{20(y-2)^2}{4} = 0.5g(y-1.5)$$

$$5(y^2 - 4y + 4) = 4.9y - 7.35$$

$$5y^2 - 20y + 20 - 4.9y + 7.35 = 0$$

$$5y^2 - 24.9y + 27.35 = 0$$

$y = 3.34$ or $y = 1.63$ $\Leftarrow$ string still slack

$\therefore$ lowest point $= 3.34$m

(b) @ equilibrium position: $T = 0.5g = 4.9$N

HL: $4.9 = \frac{20 \times x}{2}$

$x = 0.49$ m

$\therefore$ length in equilibrium 2.49m.

3b) contd

$\therefore$ particle has dropped $(2.49 - 1.5) = 0.99\,m$

let speed @ equilibrium be $V\,ms^{-1}$

$PE \text{ lost} = 0.5g \times 0.99 = 4.851\,j$

$KE \text{ gained} = \frac{1}{2} \times 0.5\,V^2$

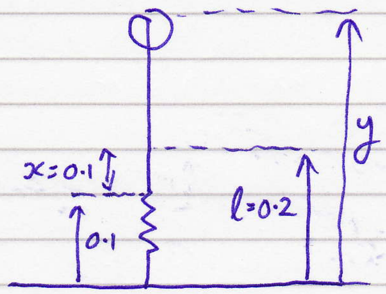
$EPE \text{ at Equilib pos} = \frac{20 \times (0.49)^2}{2 \times 2} = 1.2005$

Now PE lost = KE gained + EPE

$4.851 = 0.25V^2 + 1.2005$

$V = 3.82\,ms^{-1}$

4



let max height above ground = $y\,m$

gain in $PE = 0.5g(y - 0.1) = 4.9y - 0.49$

Final KE = Initial KE = 0

$\text{Initial EPE} = \frac{100 \times 0.1^2}{2 \times 0.2} = 2.5$

$\text{Final EPE} = \frac{100(y-0.2)^2}{2 \times 0.2} = 250(y^2 - 0.4y + 0.04)$
$= 250y^2 - 100y + 10$

Now loss in EPE = gain in PE

$2.5 - (250y^2 - 100y + 10) = 4.9y - 0.49$

$250y^2 - 95.1y + 7.01 = 0$

$y = 0.2804$ or $y = 0.1$ x jack wouldn't have moved

$\therefore y = 0.2804$

Max dist jack has moved $= 0.2804 - 0.1 = 0.180$ metres.

(5) When in equilib $T = Mg$

HL: $Mg = \lambda \dfrac{\left(\frac{5}{3}l - l\right)}{l}$

$Mg = \lambda \dfrac{\frac{2}{3}\cancel{l}}{\cancel{l}}$

$\lambda = \dfrac{3Mg}{2}$

initial vel $= \sqrt{\dfrac{3gl}{2}}$ Final Vel $= 0$

$\therefore$ loss in KE $= \dfrac{1}{2}M\left(\dfrac{3gl}{2}\right) = \dfrac{3Mgl}{4}$

loss in PE $= Mgy$

gain in EPE $= \dfrac{\lambda(y-l)^2}{2l} = \dfrac{3Mg}{2} \times \dfrac{(y-l)^2}{2l} = \dfrac{3Mg}{4l}(y-l)^2$

Now gain in EPE = loss in PE + loss in KE

$\dfrac{3\cancel{Mg}}{4l}(y-l)^2 = \cancel{Mg}y + \dfrac{3\cancel{Mg}l}{4}$

$(y-l)^2 = \dfrac{4ly}{3} + l^2$

$y^2 - 2yl + l^2 = \dfrac{4ly}{3} + l^2$

$y^2 - 2yl - \dfrac{4ly}{3} = 0$

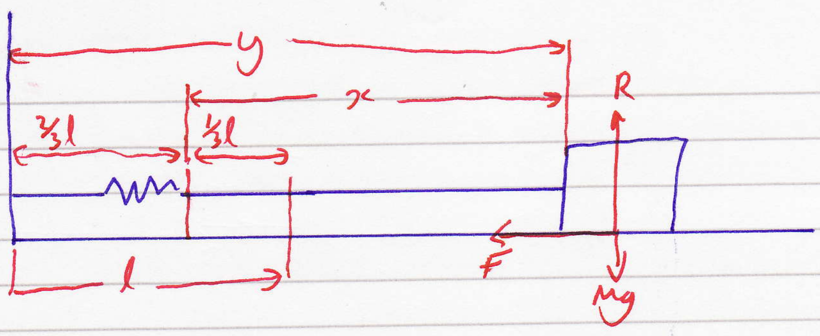
$y^2 - \dfrac{10}{3}yl = 0$

$y\left(y - \dfrac{10l}{3}\right) = 0$

$\therefore$ either $y = 0$ or $y - \dfrac{10}{3}l = 0$

$y = \dfrac{10}{3}l$ as required.

6



Friction Force $F = \mu R_1 = \mu Mg$

wd against friction $= \cancel{Mgx}\ F \times x = \mu Mgx$

Initial EPE $= \dfrac{\lambda(\frac{1}{3}l)^2}{2l} = \dfrac{\lambda l}{18}$

Final EPE $= \dfrac{\lambda(y-l)^2}{2l}$ but $y = \frac{2}{3}l + x$

$$= \frac{\lambda}{2l}\left(\frac{2}{3}l + x - l\right)^2 = \frac{\lambda}{2l}\left(x - \frac{l}{3}\right)^2$$

Now change in EPE = wd v's F

$$\frac{\lambda l}{18} - \frac{\lambda}{2l}\left(x - \frac{l}{3}\right)^2 = \mu Mgx$$

$\times 18l$

$$\lambda l^2 - \cancel{9}\lambda\left(\frac{3x-l}{\cancel{9}}\right)^2 = 18\mu lMgx$$

$$\cancel{\lambda l^2 + \lambda(9x^2 - 18xl + l^2) = 18\mu lMgx}$$

$$\cancel{\lambda l^2 - \lambda x^2 + 2\lambda xl - \lambda l^2 = 18\mu lMgx}$$

$$\lambda l^2 - \lambda(9x^2 - 6xl + l^2) = 18\mu lMgx$$

$$\cancel{\lambda l^2} - 9\lambda x^{\cancel{2}} + 6\lambda \cancel{x} l - \cancel{\lambda l^2} = 18\mu lMg\cancel{x}$$

$$-9\lambda x + 6\lambda l = 18\mu lMg$$

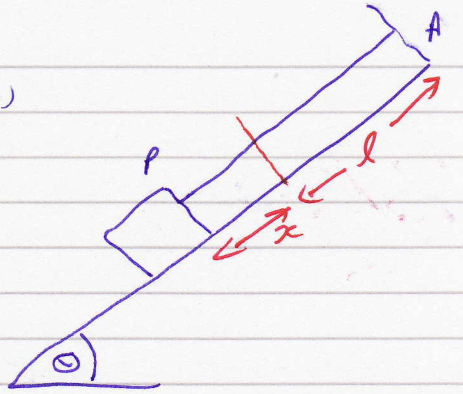
$\div 3$

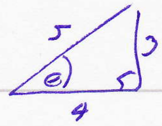
$$-3\lambda x + 2\lambda l = 6\mu lMg$$

$$3\lambda x = 2\lambda l - 6\mu lMg$$

$$x = \frac{2\lambda l - 6\mu lMg}{3\lambda}$$ As required.

(7)(a)





loss in gPE $= mgx\sin\theta = \frac{3}{5}mgx$

$\Delta KE = 0$

gain in EPE $= \frac{\lambda x^2}{2l}$

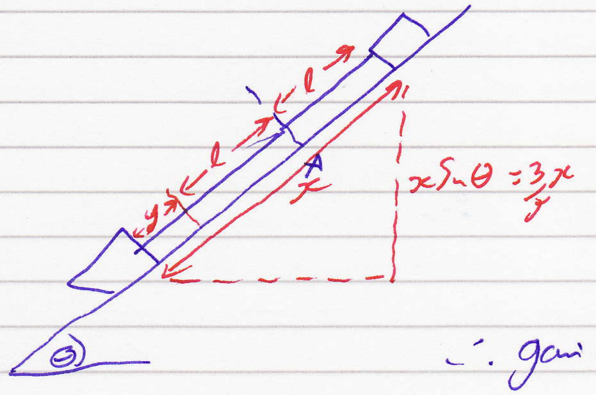
~~Also~~ Smooth plane ∴ No wd v's Fric

Now gain in EPE = loss in gPE

$$\frac{\lambda x^2}{2l} = \frac{3}{5}mgx$$

$$x = \frac{6mgl}{5\lambda}$$ As required.

(b)



loss in gPE $= \frac{3x}{5} \times mg = \frac{3}{5}mgx$

gain in EPE $= \frac{\lambda y^2}{2l}$ but $x = 2l + y$
$y = x - 2l$

∴ gain in EPE $= \frac{\lambda}{2l}(x-2l)^2$

now $\lambda = \frac{6mg}{5}$ ∴ gain in EPE $= \frac{6mg}{5}\cdot\frac{1}{2l}(x-2l)^2$

$$= \frac{3mg}{5l}(x-2l)^2$$

Smooth plane ∴ No wd v's Fric

gain in EPE = loss in gPE

$$\frac{\cancel{3mg}}{\cancel{5}l}(x-2l)^2 = \frac{\cancel{3}}{\cancel{5}}\cancel{mg}x$$

$\times l$

$$(x-2l)^2 = xl$$

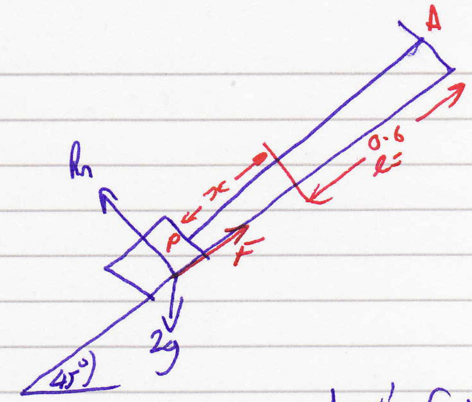
$$x^2 - 4lx + 4l^2 - xl = 0$$

$$x^2 - 5xl + 4l^2 = 0$$

$$(x-l)(x-4l) = 0$$

either $x = l$ not possible, would come to rest @A.
or $x = 4l$

(8)



$F = \mu R_n$

$R_n = 2g\cos 45 = \frac{2g}{\sqrt{2}}$

$F = 0.2 \times \frac{2g}{\sqrt{2}} = \frac{0.4g}{\sqrt{2}}$

wd v's Frict $= \frac{0.4g}{\sqrt{2}} \times x$

No $\Delta KE = 0$

loss in PE $= Mg\, x \sin 45 = \frac{2gx}{\sqrt{2}}$

gain in EPE $= \frac{20(x)^2}{2 \times 0.6} = \frac{20x^2}{1.2}$

~~Now gain in EPE = loss in gPE + wd v's fric~~

$\cancel{\frac{20x^2}{1.2} = \frac{2gx}{\sqrt{2}} + \frac{0.4g\,x}{\sqrt{2}}}$

wd v's fric = loss of PE − gain of EPE   PE lost = gain in EPE + wd v's fric

$\frac{0.4g}{\sqrt{2}}\cancel{x} = \frac{2g\cancel{x}}{\sqrt{2}} - \frac{20x^{\cancel{2}}}{1.2}$

$\frac{20x}{1.2} = \frac{2g}{\sqrt{2}} - \frac{0.4g}{\sqrt{2}}$

$x = 0.665\text{ m}$

Now speed when AP = 0.8 m is $V\text{ ms}^{-1}$

gain in EPE $= \frac{20(0.2)^2}{2 \times 0.6} = \frac{2}{3}$

gain in KE $= \frac{1}{2} \times 2 \times V^2 = V^2$

wd v's Fric $= \frac{0.4g}{\sqrt{2}} \times 0.2 = \frac{0.784}{\sqrt{2}}$

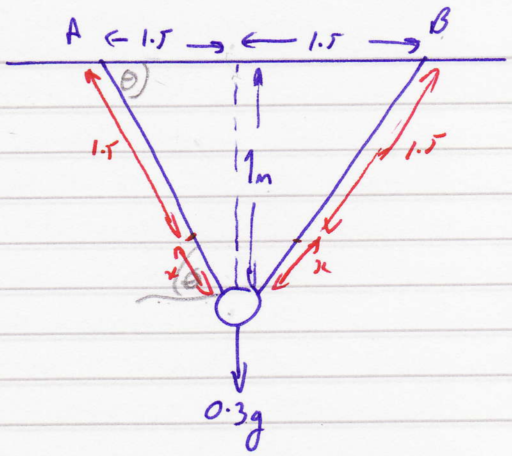
loss in PE $= Mg \times 0.2 \sin 45 = \frac{3.92}{\sqrt{2}}$

Now loss in PE = gain in EPE + gain in KE + wd v's Fric

$\frac{3.92}{\sqrt{2}} = \frac{2}{3} + V^2 + \frac{0.784}{\sqrt{2}}$ $V^2 = 1.55$

$V = 1.25\text{ ms}^{-1}$

9



Using pythag: $(x+1.5)^2 = 1.5^2 + 1^2$

$x^2 + 3x + 2.25 = 2.25 + 1$

$x^2 + 3x - 1 = 0$

$x = 0.303$

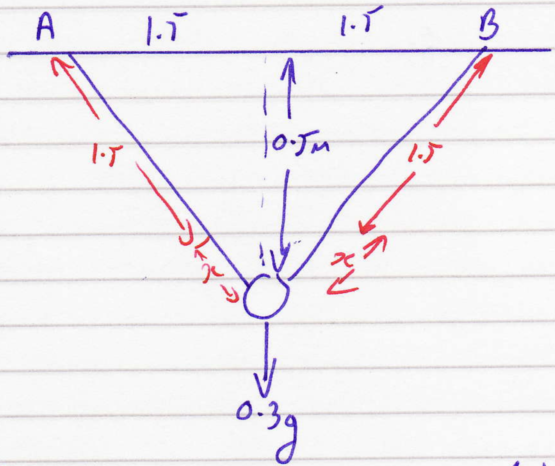
Now loss in PE $= Mg \times 1 = 0.3g$

gain in EPE $= \dfrac{\lambda (0.303)^2}{2 \times 1.5} \times 2$

Now gain in EPE = loss in PE

$$\frac{\lambda (0.303)^2}{2 \times 1.5} \times 2 = 0.3g$$

$$\lambda = 48.1\,N$$



Using pythag: $(x+1.5)^2 = 1.5^2 + 0.5^2$

$x^2 + 3x + 2.25 = 2.25 + 0.25$

$x^2 + 3x - 0.25 = 0$

$x = 0.0811$

Loss in PE $= 0.3 \times 9.8 \times 0.5 = 1.47\,J$

gain in EPE $= \dfrac{48.1 \times (0.0811)^2 \times 2}{2 \times 1.5} = 0.211\,J$

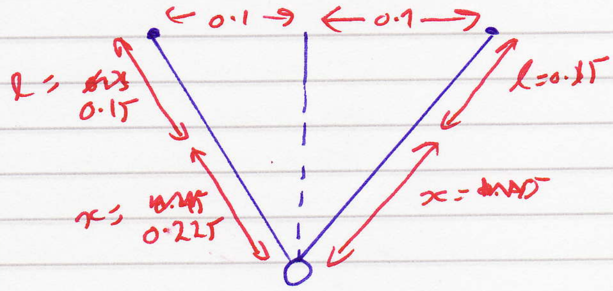
gain in KE $= \frac{1}{2} \times 0.3 \times V^2$

Now loss in PE = gain in KE + gain in EPE

$$1.47 = 0.15V^2 + 0.211$$

$$V = 2.90\,ms^{-1}$$

(10)



(a) Assume stone = particle
elastic = light

(b) $EPE = \dfrac{60 \times (0.225)^2}{2 \times 0.15} \times 2 = 20.25$

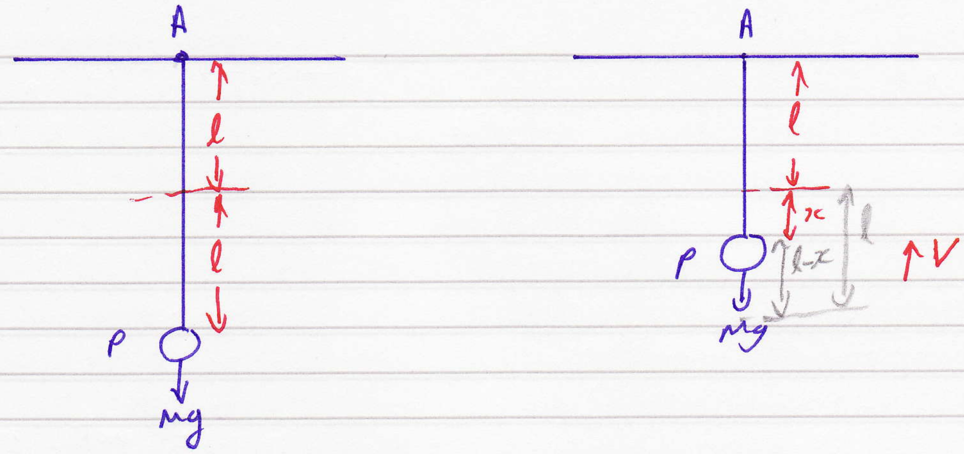
Now gain in KE = loss in EPE

$\frac{1}{2} \times 0.1 \times V^2 = 20.25$

$V = 20.1 \ ms^{-1}$

(c) If elastic were vertical then there would be a change in gravitational potential energy that would need to be included.

(11)



Initial EPE $= \frac{\lambda l^2}{2l} = \frac{\lambda l}{2}$

After release EPE $= \frac{\lambda x^2}{2l}$

loss in EPE $= \frac{\lambda l}{2} - \frac{\lambda x^2}{2l}$

gain in KE $= \frac{1}{2}mv^2$

gain in PE $= mg(l-x)$

Now loss in EPE = gain in KE + gain in PE

$$KE = EPE - PE$$

$$\frac{1}{2}mv^2 = \left(\frac{\lambda l}{2} - \frac{\lambda x^2}{2l}\right) - mg(l-x)$$

$$\frac{1}{2}mv^2 = \frac{\lambda l}{2} - \frac{\lambda x^2}{2l} - mgl + mgx$$

$$\frac{1}{2}mv^2 = \frac{1}{2}l(\lambda - 2mg) + mgx - \frac{1}{2}\frac{\lambda x^2}{l}$$ As required

when $x=0$ $\quad \cancel{\frac{1}{2}}mv^2 = \cancel{\frac{1}{2}}l(\lambda - 2mg)$

$$v^2 = \frac{l(\lambda - 2mg)}{m}$$

for string to remain taught $V<0$ when $x=0$

$\therefore \quad \frac{l(\lambda - 2mg)}{m} < 0 \qquad \lambda - 2mg < 0$

$\lambda < 2mg \Rightarrow 2mg > \lambda$ As required