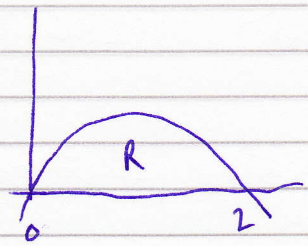
M3 - JUNE 07

Q1 (a)



$y = 2x - x^2$

when $y=0$ $\quad x(2-x)=0$

$x=2$

$$\text{Area of } R = \int_0^2 2x - x^2 \, dx$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= 4 - \frac{8}{3}$$

$$= \frac{4}{3} \quad \text{As required}$$

(b) $M = \int_0^2 \rho y \, dx = \frac{4}{3}\rho$

Curve symmetrical $\therefore \bar{x} = 1$.

$$M\bar{y} = \frac{1}{2}\int_0^2 \rho y^2 \, dx$$

$$\frac{4}{3}\cancel{\rho}\bar{y} = \frac{1}{2}\cancel{\rho} \int_0^2 (2x - x^2)^2 \, dx$$

$$\frac{8}{3}\bar{y} = \int_0^2 4x^2 - 4x^3 + x^4 \, dx$$

$$\frac{8}{3}\bar{y} = \left[\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}\right]_0^2$$

$$\frac{8}{3}\bar{y} = \frac{32}{3} - 16 + \frac{32}{5}$$

$$\frac{8}{3}\bar{y} = \frac{16}{15}$$

$$\bar{y} = \frac{2}{5} \quad \therefore \text{COM @ } \left(1, \frac{2}{5}\right)$$

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open ∴ surface area's not volume.

Q2(a) $(\pi r^2 + 2\pi r h)\rho\bar{y} = \pi r^2(0)\rho + 2\pi r h\left(\frac{h}{2}\right)\rho$

but $r = h$

$(\pi h^2 + 2\pi h^2)\cancel{\rho}\bar{y} = 0 + \cancel{2}\pi h^3$

$3\pi h^2\bar{y} = \pi h^3$

$\bar{y} = \frac{\pi h^3}{3\pi h^2} = \frac{1}{3}h$ As required

(b) $(M+M)\cancel{\rho}\bar{y} = M\left(\frac{1}{2}h\right)\cancel{\rho} + M\left(\frac{1}{3}h\right)\cancel{\rho}$

$2\cancel{M}\bar{y} = \frac{1}{2}\cancel{M}h + \frac{1}{3}\cancel{M}h$

$\bar{y} = \frac{5}{12}h$

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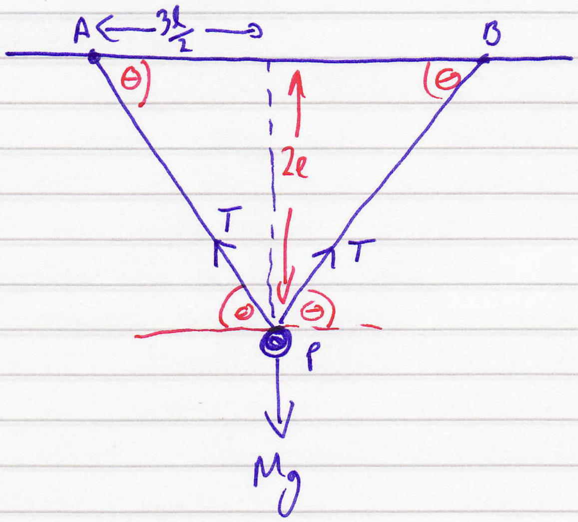
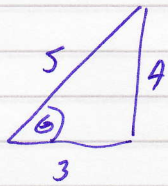
Q7 (a)



$\tan\theta = \dfrac{2l}{\frac{3l}{2}} = \dfrac{4}{3}$



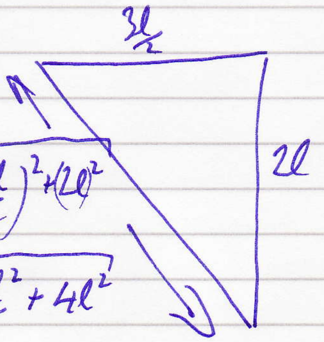
$\sin\theta = \dfrac{4}{5}$

$$T\sin\theta + T\sin\theta = Mg$$

$$2T \times \frac{4}{5} = Mg$$

$$T = \frac{5Mg}{8}.$$

Now Hookes Law $T = \dfrac{\lambda x}{a}$



$$= \sqrt{\left(\frac{3l}{2}\right)^2 + (2l)^2}$$

$$= \sqrt{\frac{9l^2}{4} + 4l^2}$$

$$= \sqrt{\frac{25}{4}l^2} = \frac{5l}{2}$$

Natural Length AP $= \dfrac{3l}{2}$

∴ Hookes Law

$$\frac{5Mg}{8} = \frac{\lambda\left(\frac{5l}{2} - \frac{3l}{2}\right)}{\frac{3l}{2}}$$

$$\frac{5Mg}{8} = \frac{\lambda l}{\frac{3l}{2}}$$

$$\frac{5Mg}{8} = \frac{2\lambda}{3}$$

$$\lambda = \frac{15Mg}{16}$$

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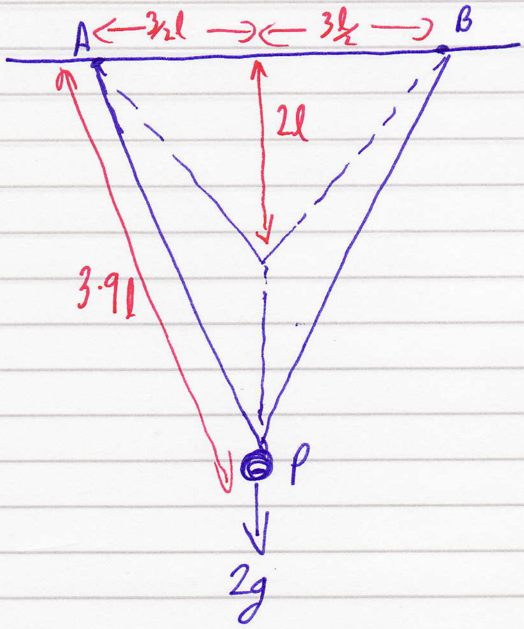
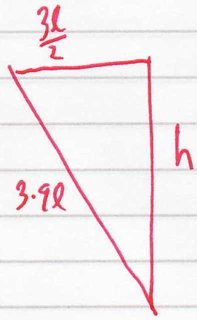
Q7(b)
5





$h^2 = (3.9l)^2 - \left(\frac{3l}{2}\right)^2$

$h = \sqrt{12.96l^2}$

$h = 3.6l$

loss in EPE = gain in KE + gain in PE

let speed of P @ AB be V.

2 elastic strings

$$2\times\left(\frac{\lambda}{2\left(\frac{3l}{2}\right)}\left(3.9l - \frac{3l}{2}\right)^2\right) = \frac{1}{2}MV^2 + Mg\times 3.6l$$

$\cancel{5.76l^2} \; \cancel{\frac{5.76\lambda}{3l}}$

$$2\times\left(\frac{5.76l^2\lambda}{3l}\right) = \frac{1}{2}MV^2 + 3.6Mgl$$

$$2\times\left(1.92l\left(\frac{15Mg}{16}\right)\right) - 3.6Mgl = \frac{1}{2}MV^2$$

$$3.6Mgl - 3.6Mgl = \frac{1}{2}MV^2$$

$$\frac{1}{2}MV^2 = 0$$

$$\therefore V = 0$$