Nov 05 Paper 2 – Solutions

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Arholwr yn unig

1. *(a)* Write down, in terms of n, the nth term of **each** of the following sequences.

(i) 4, 8, 12, 16, 20, ...

$4n$

[1]

(ii) 2, 7, 12, 17, 22, ...

$5n - 3$

[2]

(b) The nth term of another sequence of numbers is $3n^2 - 5$.
Write down the first three terms of this sequence.

$n =$	1	2	3
$3n^2 - 5 =$	-2	7	22

[2]

Turn over.

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2. Calculate the length of the diagonal PR of a rectangular garden $PQRS$ with sides 26·7 m and 18·5 m.

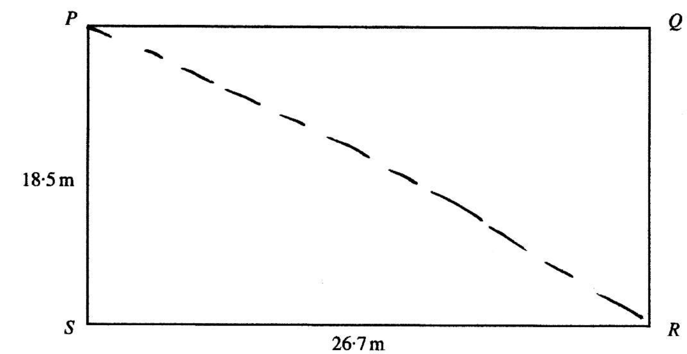

Diagram not drawn to scale.

$PR^2 = 18.5^2 + 26.7^2$

$PR^2 = 1055.14$

$PR = \sqrt{1055.14} = 32.5\text{ m}$

[3]

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3. The times of 80 mobile phone calls were measured. The table shows a grouped frequency distribution of the results.

Time (t seconds)	Number of calls
$0 < t \leqslant 30$	1
$30 < t \leqslant 60$	7
$60 < t \leqslant 90$	15
$90 < t \leqslant 120$	27
$120 < t \leqslant 150$	18
$150 < t \leqslant 180$	12

Find an estimate for the mean time of the calls.

$15 \times 1 = 15$

$45 \times 7 = 315$

$75 \times 15 = 1125$

$105 \times 27 = 2835$

$135 \times 18 = 2430$

$165 \times 12 = 1980$ +

Mean = $8700 \div 80 = 108.75$

[4]

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4. Find the compound interest when £800 is invested for 3 years at 5% per annum.

Amount in account = $(1.05)^3 \times 800$ = £926.10

∴ Interest = £126.10

[4]

5. A prism has a uniform cross-section of $54\,cm^2$ along its length of 22·7 cm and has a mass of 6·5 kg. Calculate the density of the metal from which the prism is made in g/cm^3.

Volume = $54 \times 22.7 = 1225.8\,cm^3$

$D = \frac{M}{V} = \frac{6500}{1225.8} = 5.3\,g/cm^3$

[4]

6. A meal costs £54.05 inclusive of V.A.T. at $17\frac{1}{2}$ %. What was the cost of the meal before V.A.T. was added?

$1.175 \times x = 54.05$

$x = \frac{54.05}{1.175}$ = £46

[3]

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7. A solution to the equation

$$x^3 - 4x + 1 = 0$$

lies between 1·8 and 1·9.

Use the method of trial and improvement to find this solution correct to 2 decimal places.

$x = 1.85$ -0.068 too small

$x = 1.87$ 0.059 too big

$x = 1.86$ -0.005 too small

$\therefore x$ lies between 1.86 and 1.87

test using $x = 1.865$ $0.02688...$ too big

$\therefore x = 1.86$ to 2dp.

[4]

8. *(a)* Write **each** of the following numbers in standard form.

(i) 0·00076

$0.00076 = 7.6 \times 10^{-4}$

[1]

(ii) 415 000 000 000

$415\,000\,000\,000 = 4.15 \times 10^{11}$

[1]

(b) Find, in standard form, the value of

$$(8{\cdot}1 \times 10^{12}) \times (5{\cdot}9 \times 10^{-4}).$$

$4\,779\,000\,000 = 4.779 \times 10^9$

$= 4.8 \times 10^9$

[2]

Turn over.

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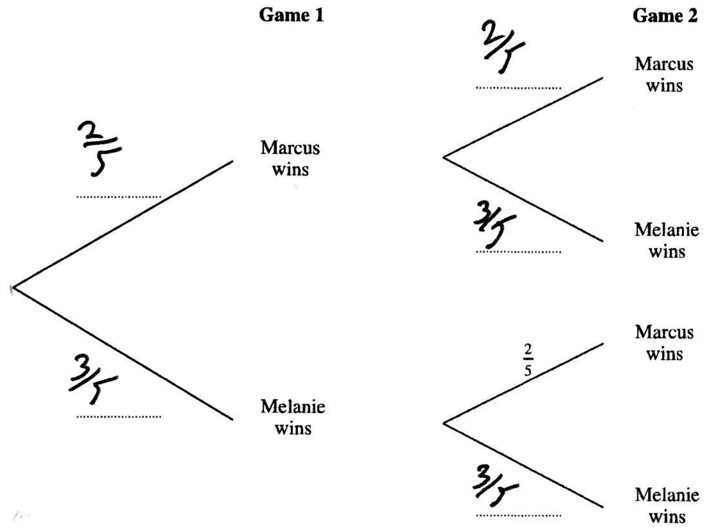
9. Whenever Marcus and Melanie play a game of tennis the probability that Marcus wins the game is $\frac{2}{5}$.

(a) Complete the following tree diagram to show the probabilities of what can happen when Marcus and Melanie play two games of tennis.



[2]

(b) Calculate the probability that Melanie wins both games.

$\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

[2]

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10. In the diagram ABC is a straight line and BDE is a straight line perpendicular to it. It is given that $AD = 36$ m, $BC = 49$ m, $D\hat{A}B = 43°$ and $E\hat{C}B = 54°$.

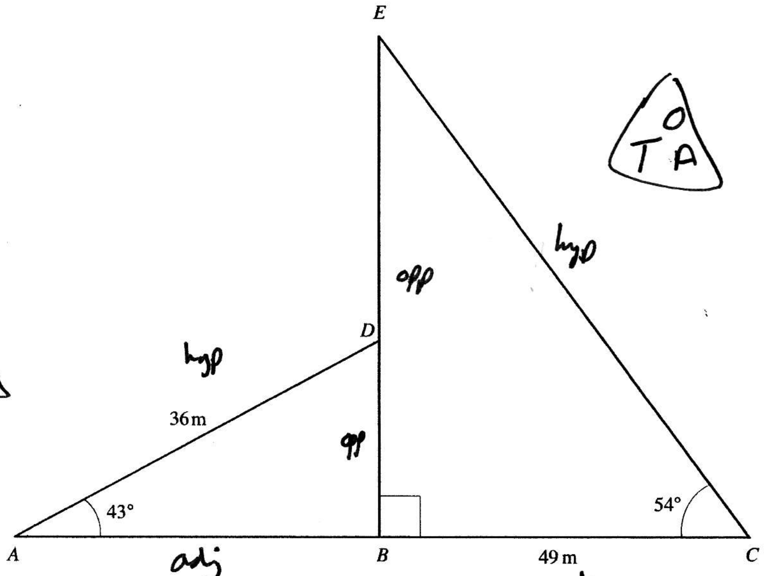

Diagram not drawn to scale.

Calculate the length of DE.

From $\triangle ABD$: $BD = \sin 43 \times 36 = 24.6$ m

From $\triangle BCE$: $BE = \tan 54 \times 49 = 67.4$ m

$\therefore DE = 67.4 - 24.6 = 42.8$ m

[6]

Turn over.

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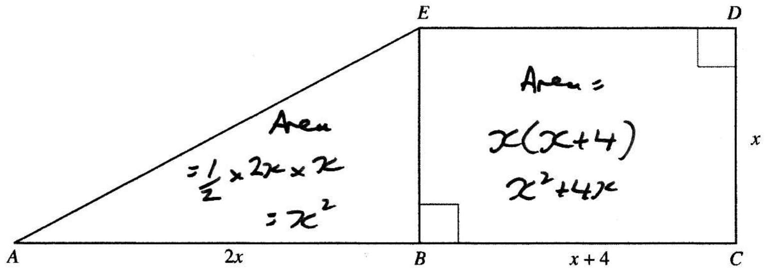
11. *(a)* In the diagram ABC is a straight line and $BCDE$ is a rectangle. The side DC is of length x cm, BC is of length $(x + 4)$ cm and AB is of length $2x$ cm.



The diagram is not drawn to scale and the measurements are in centimetres.

The area of the whole shape $ABCDE$ is 48 cm^2.
Giving full details of all your working, show clearly that x satisfies the equation

$$x^2 + 2x - 24 = 0.$$

Total Area $= x^2 + x^2 + 4x = 48$

$2x^2 + 4x - 48 = 0$

$\div 2$

$x^2 + 2x - 24 = 0$ As required.

[2]

(b) Solve the equation to find the length of DC.

$(x+6)(x-4) = 0$

either $x + 6 = 0$ or $x - 4 = 0$

$x = -6$ $\quad$ $x = 4$ ✓

x can't have -ve length

[2]

$\therefore DC = 4$ cm

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12. Solve the following equation.

$$\frac{4x-1}{4} - \frac{2x-5}{8} = 3$$

$\times 8$ $\quad \frac{\overset{2}{\cancel{8}}(4x-1)}{\underset{1}{\cancel{4}}} - \frac{\overset{1}{\cancel{8}}(2x-5)}{\underset{1}{\cancel{8}}} = 3 \times 8$

$8x - 2 - 2x + 5 = 24$

$6x + 3 = 24$

$6x = 21$

$x = \frac{21}{6} = \frac{7}{2}$

[4]

13. Factorise the expression $12x^2 + 5x - 2$ and hence solve the equation $12x^2 + 5x - 2 = 0$.

-24 $\quad +8x, -3x$

$12x^2 - 3x + 8x - 2$

$3x(4x-1) + 2(4x-1)$

$(3x+2)(4x-1) = 0$

either $3x+2=0$ or $4x-1=0$

$x = -\frac{2}{3}$ $\qquad x = \frac{1}{4}$

[3]

Turn over.

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14. Simplify **each** of the following.

(a) $(x+y)^0$

1

[1]

(b) $\sqrt{x^{16}}$

$(x^{16})^{\frac{1}{2}} = x^8$

[1]

(c) $\dfrac{48 \times a^{\frac{9}{2}} \times a^{-\frac{7}{2}}}{12a^4}$

$\frac{4a^{\frac{9}{2}+-\frac{7}{2}}}{a^4} = \frac{4a^{\frac{2}{2}}}{a^4} = \frac{4a^1}{a^4}$

$= 4a^{1-4} = 4a^{-3}$

[2]

15. Given that y is inversely proportional to x^2, and that $y = 4$ when $x = 5$,

(a) find an expression for y in terms of x,

$y \propto \frac{1}{x^2}$ | when $y=4$ $x=5$ $\quad 4 = \frac{k}{5^2}$

$y = \frac{k}{x^2}$ | $4 \times 25 = k$

$k = 100$

$\therefore\ y = \frac{100}{x^2}$

[3]

(b) calculate

(i) the value of y when $x = \frac{1}{2}$,

$y = \frac{100}{(\frac{1}{2})^2} = \frac{100}{\frac{1}{4}} = 400$

[1]

(ii) a value of x when $y = 10000$.

$10\,000 = \frac{100}{x^2}$

$10\,000\,x^2 = 100$

$x^2 = \frac{100}{10\,000}$

[2]

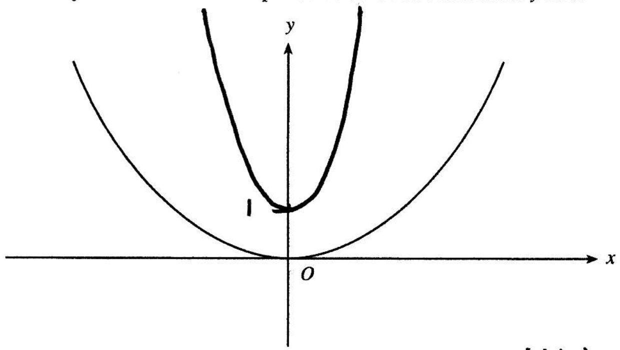
$x = \sqrt{\frac{100}{10\,000}} = \frac{1}{10}$

Turn over.

steeper

move up 1.

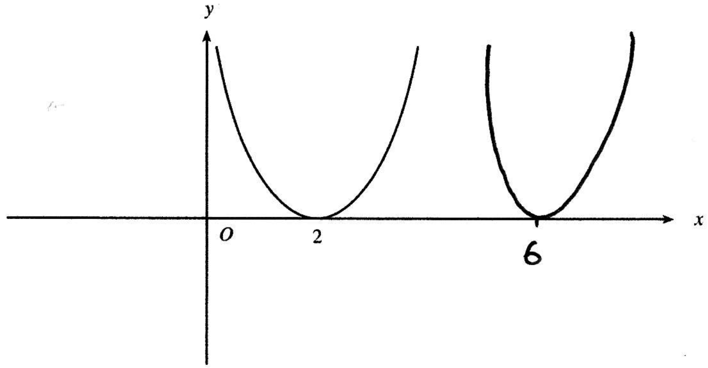
16. (a) The diagram shows a sketch of $y = x^2$.
On the same diagram, sketch the curve $y = 2x^2 + 1$.
Mark clearly the coordinates of the point where the curve crosses the y-axis.



[3]

Move right 4

(b) The diagram shows the sketch of $y = h(x)$.
On the same diagram sketch the curve $y = h(x - 4)$.
Mark clearly the coordinates of the point where the curve crosses the x-axis.



[2]

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17. An international company employs people from around the world.
The number of people employed by the company in each country is given in the following table.

Country	Number of employees
Germany	2355
France	1340
Canada	6867
India	4342
Japan	9843

The company is organising a conference and decides to invite 40 employees to represent the views of the workforce.

Use a stratified sampling method to calculate how many people from each country should be invited to the conference.

Total people = 24747

Germany $= \frac{2355}{24747} \times 40 = 3.8 = 4$

France $= \frac{1340}{24747} \times 40 = 2.2 = 2$

Canada $= \frac{6867}{24747} \times 40 = 11.1 = 11$

India $= \frac{4342}{24747} \times 40 = 7.0 = 7$

JAPAN $= \frac{9843}{24747} \times 40 = 15.9 = 16$

check 40 ✓

[4]

Turn over.

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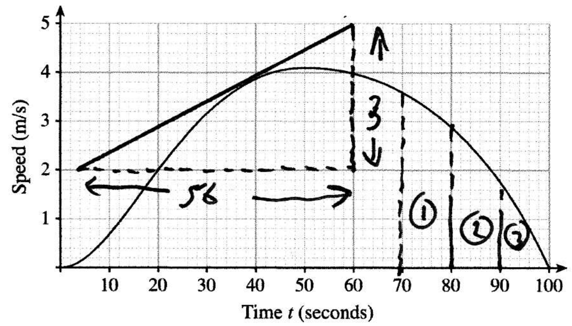
18. The graph below shows the speed of a train, in m/s, over a period of 100 seconds starting at time $t = 0$ seconds.



(a) Estimate the acceleration of the train at time $t = 40$ seconds.

accel = gradient of tangent @ $t=40$

gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{3}{56} = 0.05\ m/s^2$

[3]

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(b) The table below gives the speed of the train between $t = 70$ and $t = 100$.

Time t (seconds)	70	80	90	100
Speed (m/s)	3·6	2·9	1·8	0

Use the trapezium rule with the values taken from the table to estimate the distance, in metres, travelled by the train between $t = 70$ and $t = 100$ seconds.

Area of trapezium (1) $= \frac{1}{2}(3.6+2.9)\times 10 = 32.5$

Area of trapezium (2) $= \frac{1}{2}(2.9+1.8)\times 10 = 23.5$

Area of trapezium (3) $= \frac{1}{2}(1.8+0)\times 10 = 9$

Dist travelled = total Area = 65 metres.

[3]

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19. For the first x seconds of a journey the average speed of a cyclist is 4 m/s. For the next $(5x + 2)$ seconds the average speed is x m/s. The total distance travelled is 128 metres.

(a) Show that x satisfies the equation $5x^2 + 6x - 128 = 0$.

Dist = speed x time

for 1st part of journey Dist $= 4 \times x = 4x$

For 2nd part of journey Dist $= x \times (5x+2) = 5x^2 + 2x$

Tot Dist = 128

$\therefore \quad 5x^2 + 2x + 4x = 128$

$5x^2 + 6x - 128 = 0$ As required

[3]

(b) Use the formula method to solve the equation $5x^2 + 6x - 128 = 0$, giving solutions correct to one decimal place.

$a = 5 \quad b = 6 \quad c = -128$

$$x = \frac{-6 \pm \sqrt{6^2 - 4\times5\times-128}}{2\times5}$$

$$x = \frac{-6 \pm \sqrt{36+2560}}{10} = \frac{-6 \pm \sqrt{2596}}{10}$$

either $x = \frac{-6+\sqrt{2596}}{10} = 4.5$ ✓

or $x = \frac{-6-\sqrt{2596}}{10} = -5.7$ x can't have -ve time

[3]

(c) Hence find the total time for the journey.

total time $= x + 5x + 2$

$= 4.5 + 5(4.5) + 2$

[1]

$= 29$ seconds.

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20. The volume of a hemisphere is 7π cm^3. Calculate the radius of the hemisphere.

Vol of sphere $= \frac{4}{3}\pi r^3$

Vol of hemisphere $= \frac{4}{3}\pi r^3 \times \frac{1}{2}$

$7\pi = \frac{2}{3}\pi r^3$

$\frac{7\pi \times 3}{2\pi} = r^3$

$r^3 = \frac{21}{2}$ $\quad r = \sqrt[3]{\frac{21}{2}} = 2.2$ cm

[4]

21. Express $\sqrt{180}$ in the form $a\sqrt{b}$, where a is a whole number and b is a prime number.

$\sqrt{36 \times 5} = \sqrt{36} \times \sqrt{5} = 6\sqrt{5}$

[2]

Turn over.

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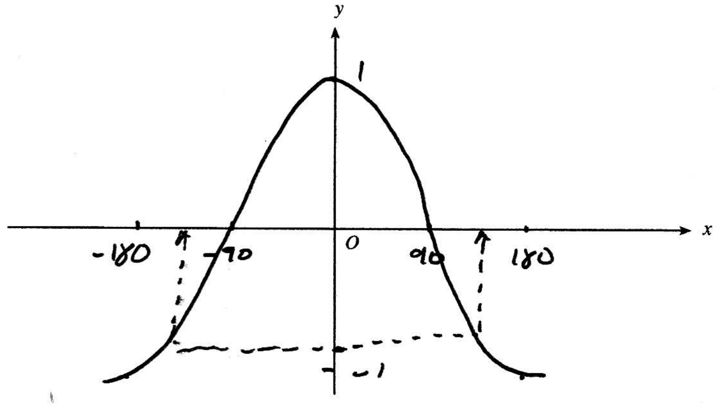
22. *(a)* Using the axes below, **sketch** the graph of $y = \cos x$ for values of x from –180° to 180°. [2]



(b) Find all solutions of the following equation in the range –180° to 180°.

$\cos x = -0{\cdot}829$

From calculator $x = \cos x^{-1}(-0.829) = 146.0^\circ$

From graph symmetry $x = -146.0^\circ$

[2]

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23. The diagram shows triangle *GHK*.

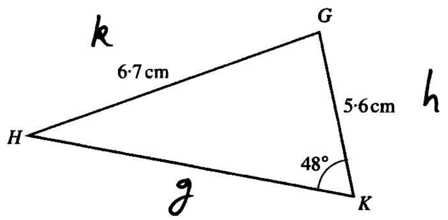

Diagram not drawn to scale.

Given that $GH = 6{\cdot}7$ cm, $GK = 5{\cdot}6$ cm and $G\hat{K}H = 48°$, calculate the area of the triangle *GHK*.

Need to find $H\hat{G}K$

Can find $G\hat{H}K$ using Sine Rule

$$\frac{\sin H}{5.6} = \frac{\sin 48}{6.7}$$

$$\sin H = \frac{\sin 48}{6.7} \times 5.6 = 0.6211$$

$$H = \sin^{-1}(0.6211..) = 38.4°$$

$$\therefore H\hat{G}K = 180 - 38.4 - 48 = 93.6°$$

$$\text{Now Area of } \Delta = \frac{1}{2} \times 6.7 \times 5.6 \times \sin 93.6$$

$$= 18.7\,\text{cm}^2$$

[6]