WELSH JOINT EDUCATION COMMITTEE

General Certificate of Secondary Education

WJEC CBAC

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184/09

MATHEMATICS

HIGHER TIER PAPER 1

P.M. MONDAY, 5 June 2006

(2 Hours)

**CALCULATORS ARE
NOT TO BE USED
FOR THIS PAPER**

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	3	
2	3	
3	4	
4	6	
5	3	
6	4	
7	7	
8	7	
9	4	
10	4	
11	2	
12	4	
13	3	
14	6	
15	4	
16	5	
17	4	
18	3	
19	5	
20	4	
21	5	
22	6	
23	4	
TOTAL MARK		

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Calculators are **not** allowed for this paper.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

No certificate will be awarded to a candidate detected in any unfair practice during the examination.

Formula List

Volume of prism = area of cross-section × length
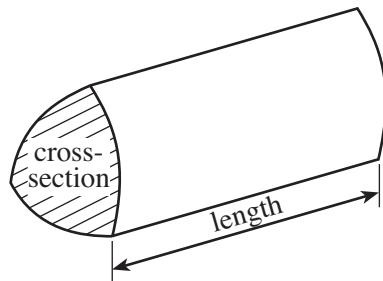
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$
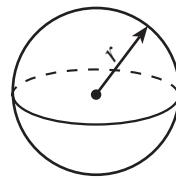
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$
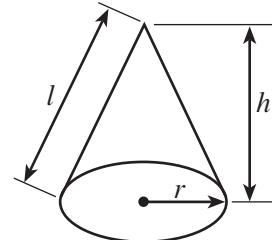
In any triangle *ABC*

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$
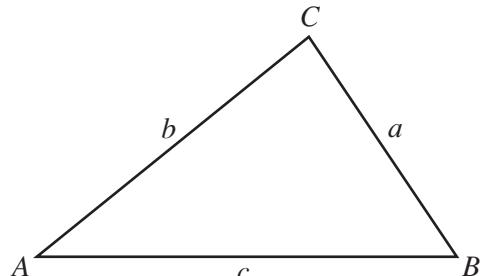
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

(184-09)

1. *(a)* Expand and simplify

$$4(3a - 2b) - 3(2a - 3b).$$

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[2]

(b) Simplify $\dfrac{w^8}{w^4}$.

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[1]

2. The table shows the scores obtained by 7 pupils in a mathematics test and their corresponding scores on a science test.

Mathematics test scores	18	74	56	26	88	81	42
Science test scores	26	65	45	40	73	61	54

The scatter diagram illustrates these results.
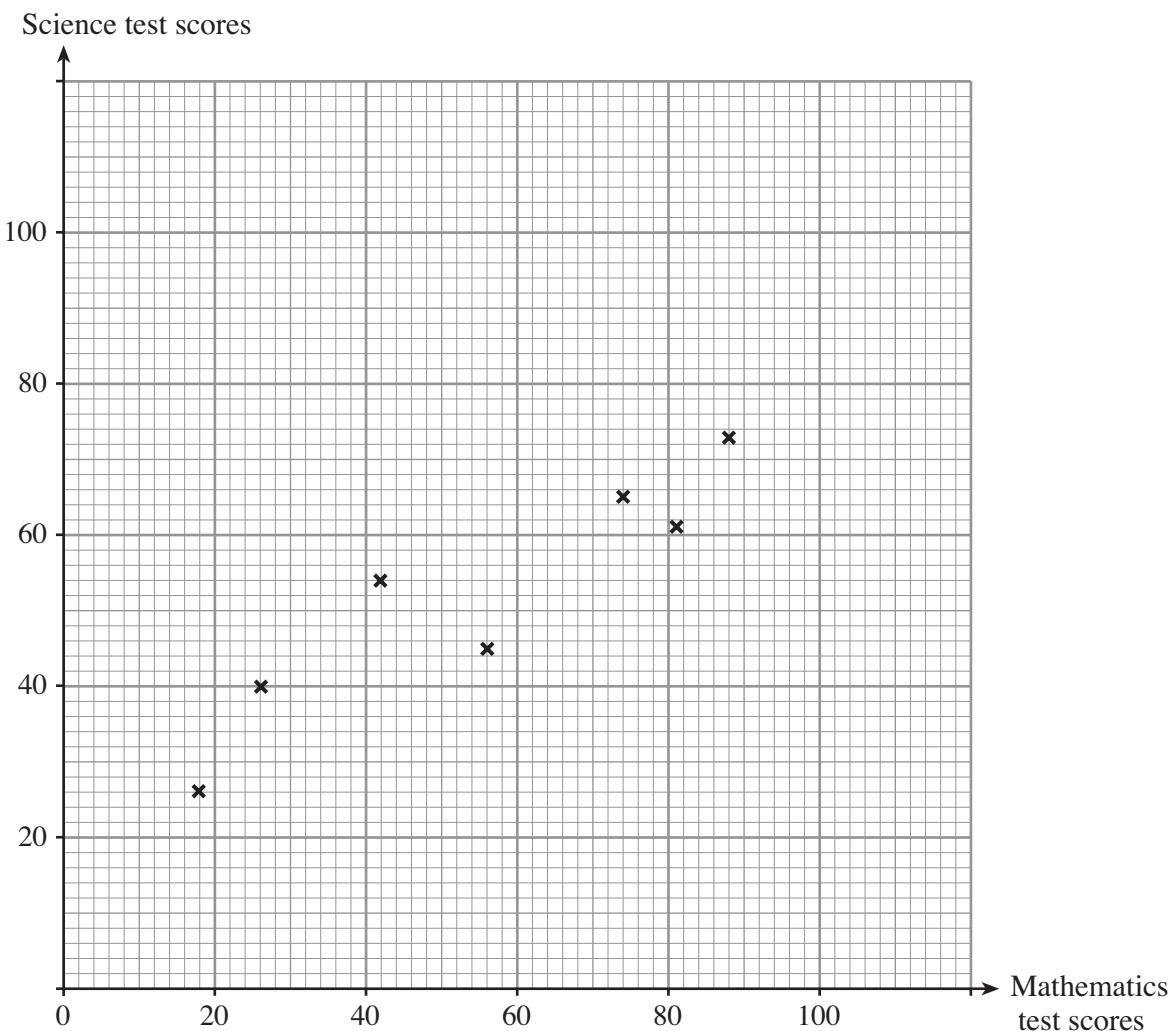
(a) The mean score for the pupils on the mathematics test is 55 and the mean score on the science test is 52.
Draw a line of best fit on your scatter diagram. [2]

(b) John sat the mathematics test and had a score of 68, but he was absent from the science test.
Use your line of best fit to find an estimate for John's score on the science test.

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[1]

3. *(a)* Express 2100 as a product of prime numbers in index form.

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[3]

(b) Write down the least whole number by which 2100 should be divided to make the result a perfect square.

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[1]

Turn over.

4. The table shows some of the values of $y = 2x^2 - 5x - 4$ for values of x from -1 to 4.

(a) Complete the table by finding the value of y for $x = 2$.

x	-1	0	1	2	3	4
$y = 2x^2 - 5x - 4$	3	-4	-7		-1	8

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[1]

(b) On the graph paper opposite, draw the graph of $y = 2x^2 - 5x - 4$ for values of x between -1 and 4. [3]

(c) Draw the line $y = -2$ on your graph paper and write down the x-values of the points where your two graphs intersect.

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[2]

For use with question 4

Turn over.
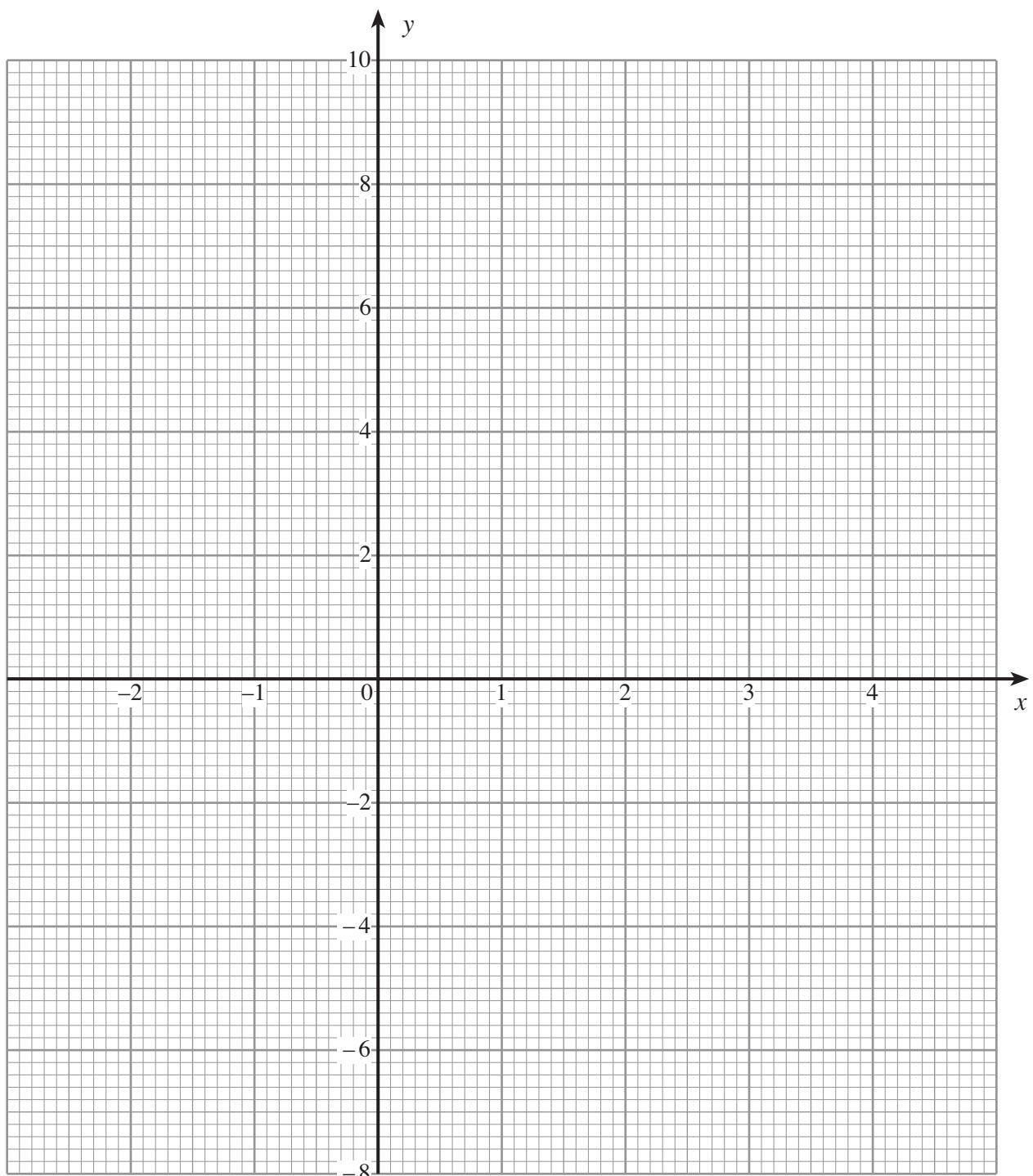
5. The diagram represents an aerial view of a building. A dog is tied, by means of a string, to a side of the building at *X*.
Draw the boundary of the region in which the dog can roam. [3]
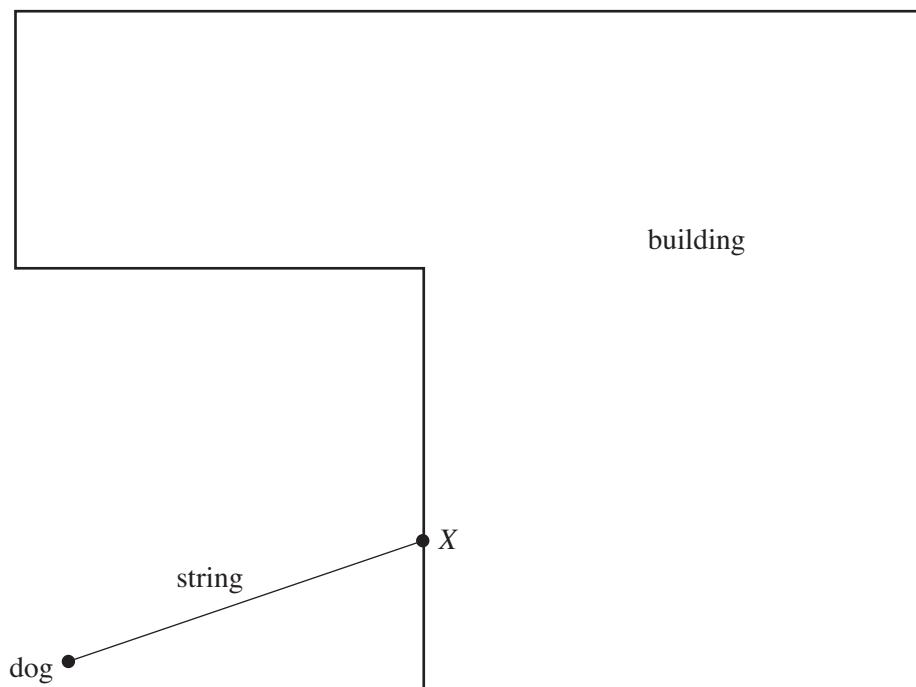
6. *(a)* Draw the image of the triangle **A** after a translation of 4 units in the *x*-direction and –3 in the *y*-direction. Label the image **B**. [2]
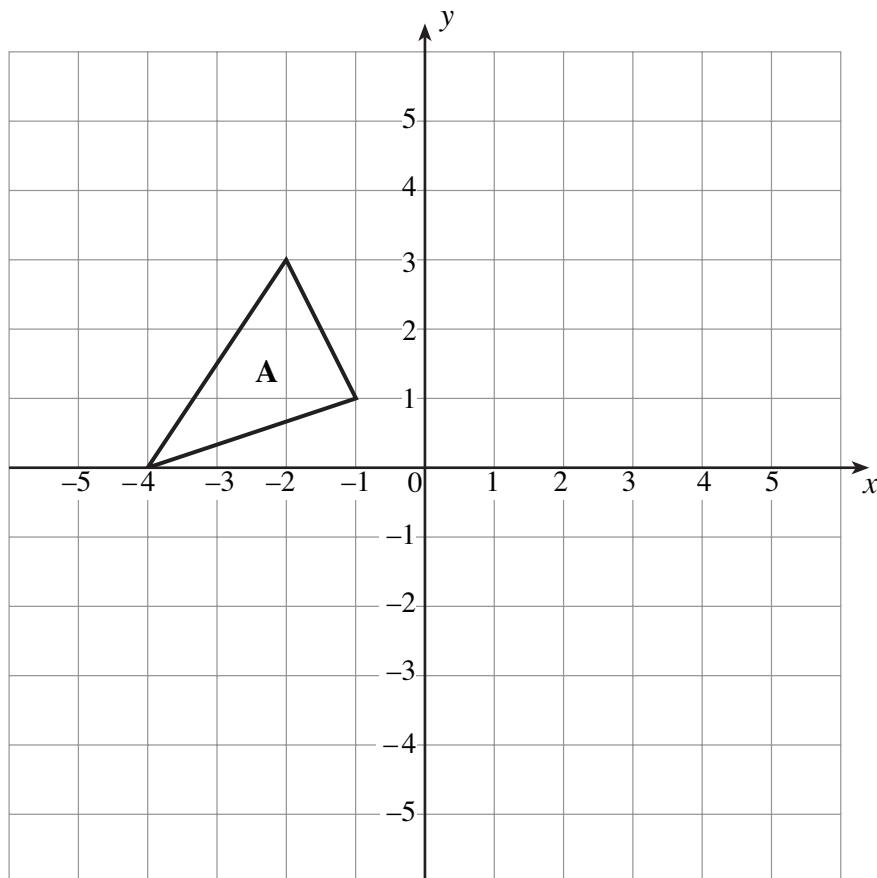
(b) Rotate the triangle **C** through 90° clockwise about the point (–1, 2).
Label the image **D**. [2]

(184-09)

Turn over.
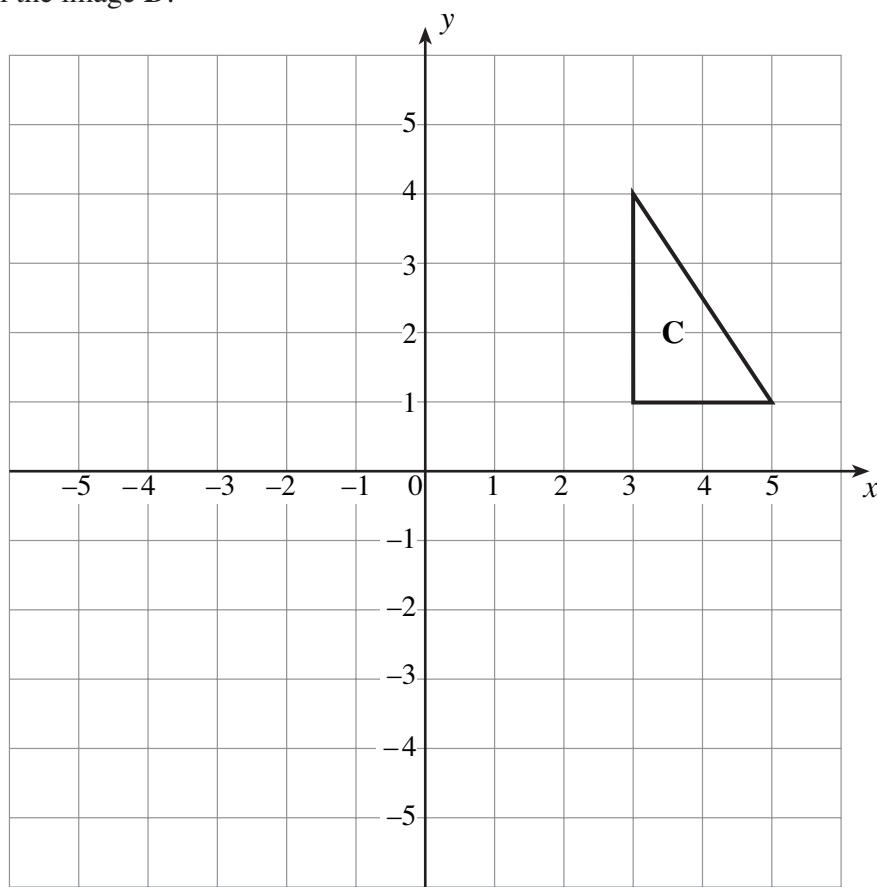
7. Square tiles are manufactured. The length of each tile is 300mm, measured correct to the nearest millimetre.

    (a) Write down the least and greatest possible values of the length of the tile.

    Least length .... mm   Greatest length .... mm

    [2]

    (b) The distance between two walls in a passageway is 302 cm, measured correct to the nearest centimetre.
    The tiles are laid end to end between the walls. Explain, showing all your calculations and reasoning, why it is always possible to lay 10 tiles between the walls.

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    [5]

8. *(a)* Expand the following expression, simplifying your answer as far as possible.

$$(x - 7)(x + 4)$$

[2]

(b) Make r the subject of the formula

$$3d - 2r = 4(7 - 2r).$$

[3]

(c) Factorise $6x^2 + 2x$.

[2]

Turn over.

9. *(a)* Explain clearly why the following triangles are **not** similar.
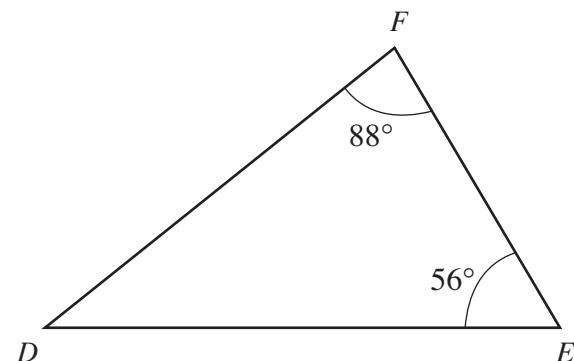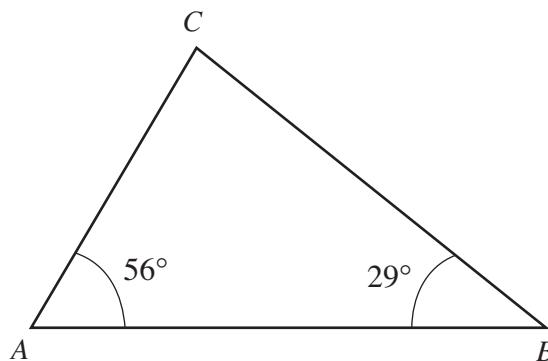
Diagrams not drawn to scale.

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[2]

(b) Explain clearly why the following triangles are **not** similar.
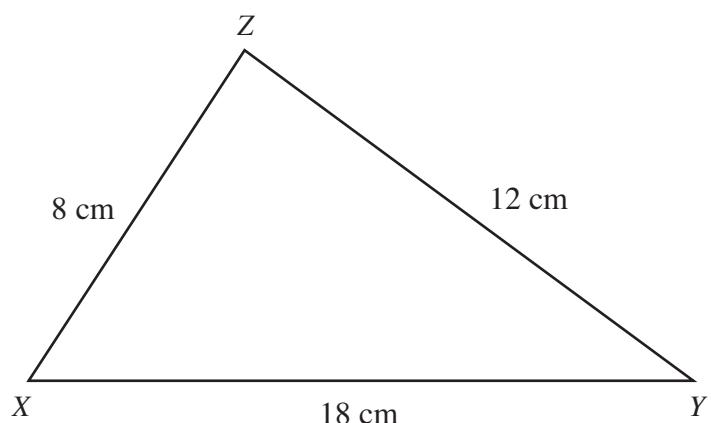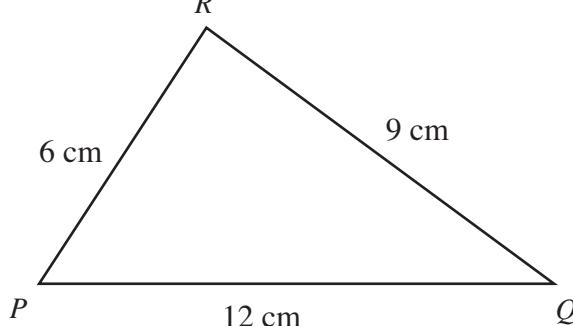
Diagrams not drawn to scale.

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[2]

10. When Kate has lunch, the probability of her having a starter course is $\frac{1}{4}$. Whether or not she has a starter, the probability of her having a dessert is $\frac{2}{3}$.

(a) Complete the following tree diagram.

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                                                    ............      Kate has a
                                                                      dessert
                           Kate has              ╱
                           a starter    ────────<
              1                          ╲        ............      Kate does
              ─                                                     NOT have
              4                                   ╲                 a dessert
         ────<
             ╲                                                     Kate has a
              ............                         ............     dessert
                                         ╱       ╱
                           Kate does    ────────<
                           NOT have              ╲
                           a starter              ............      Kate does
                                                                    NOT have
                                                                    a dessert
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[2]
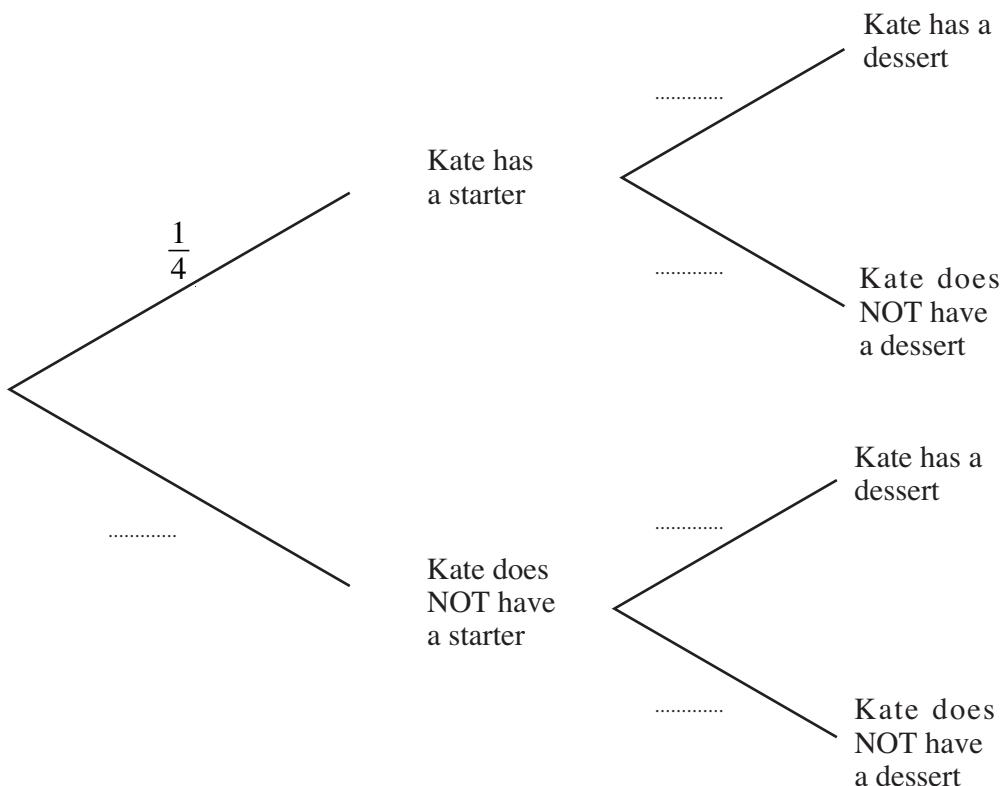
(b) Calculate the probability that Kate has a starter or a dessert, but not both.

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[2]

Turn over.

11. Each of the following quantities has a particular number of dimensions. Give the number of dimensions of each quantity. The first one has been done for you.

Quantity	Number of dimensions
The distance travelled in two laps of a circuit.	1
The volume of a cuboid.	
The area of the curved surface of a cylinder.	
The perimeter of a rectangle.	
The area of a circle.	

[2]

12. Solve the following equation.

$$\frac{5x + 3}{6} - \frac{x + 10}{3} = \frac{2}{3}$$

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[4]

13. Using the origin as the centre, enlarge the triangle *ABC* by a scale factor of $-\dfrac{1}{2}$.

[3]

Turn over.
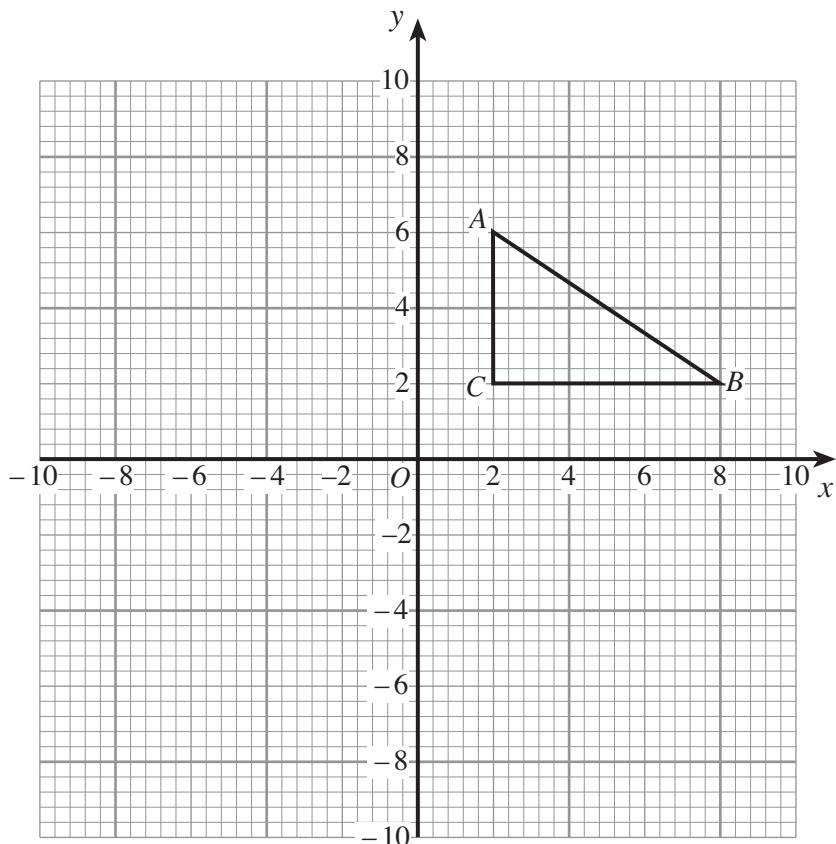
14. Two consecutive whole numbers are x and $x + 1$. When the product of these numbers is subtracted from 47 times the first number x, the answer is 360. Form an equation in x and solve it to find the two possible pairs of consecutive numbers.

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[6]

15. *(a)*
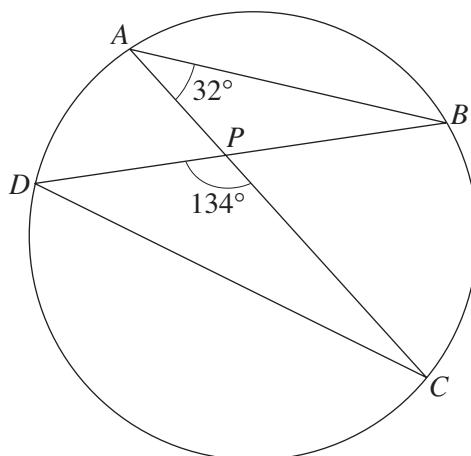
Diagram not drawn to scale.

Four points A, B, C and D lie on the circumference of the circle.
The lines AC and BD intersect at the point P.

Given that $B\hat{A}C = 32°$ and $D\hat{P}C = 134°$, find the size of $A\hat{C}D$ giving a reason for your answer.

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[2]

(b)
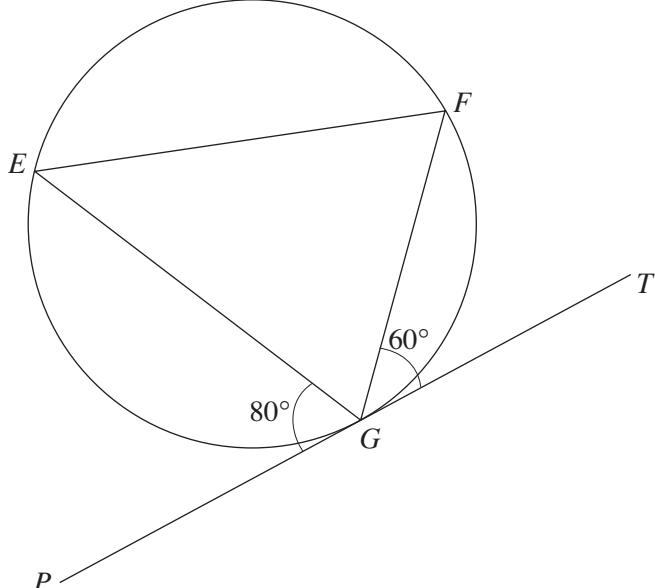
Diagram not drawn to scale.

Three points E, F and G lie on the circumference of the circle.
The tangent PT touches the circle at G.

Given that $E\hat{G}P = 80°$ and $F\hat{G}T = 60°$, find the size of $F\hat{E}G$ giving a reason for your answer.

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[2]

Turn over.

16. Given that y is inversely proportional to x, and that $y = 2$ when $x = 5$,

(a) find an expression for y in terms of x,

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[3]

(b) complete the following table for values of x and y.

x	5	10	
y	2		20

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[2]

17. Make n the subject of the following formula.

$$\frac{4n(3 + g)}{5 + n} = 7$$

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[4]

18. Factorise the expression $15x^2 - 19x - 10$ and hence solve the equation $15x^2 - 19x - 10 = 0$.

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[3]

Turn over.

19. A bag contains 5 nut chocolates, 6 raspberry chocolates and 10 caramel chocolates. Two chocolates are selected at random from the bag.

(a) Calculate the probability that both selected chocolates are raspberry chocolates.

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[2]

(b) Calculate the probability that at least one of the selected chocolates is a caramel chocolate.

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[3]

20. Express the following as a single fraction in its simplest form.

$$\frac{3x - 2}{x} + \frac{4}{4x + 3}$$

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[4]

Turn over.

21. In the diagram below *AD* and *BE* insersect at the point *C*.
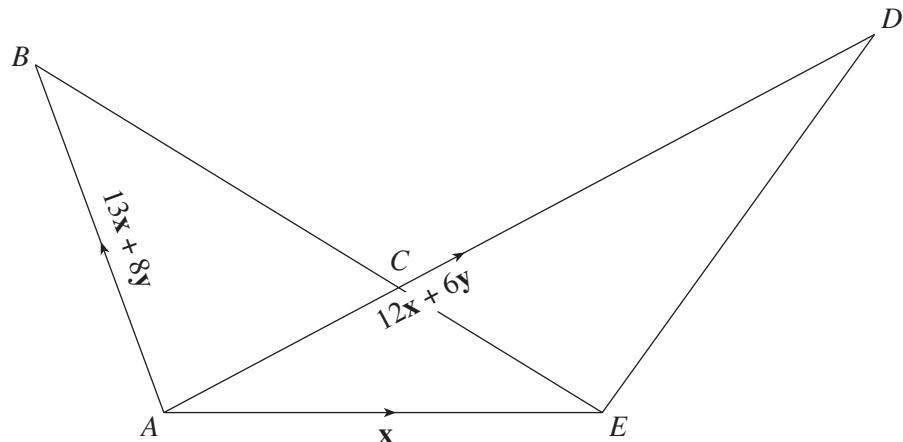
Diagram not drawn to scale.

(a) Given that **AB** = 13**x** + 8**y**, **AD** = 12**x** + 6**y**, **AE** = **x** and *AC* : *AD* = 1 : 3, express each of the following vectors in terms of **x** and **y** in its simplest form.

(i)   **EB**

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[1]

(ii) **AC**

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[1]

(b)   (i)   Show that **EB** = *k***EC**, where *k* is a constant whose value is to be found.

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[2]

(ii) Hence complete the following ratio:

$$EC : CB = 1 : .....$$

[1]

22. *(a)* The diagram shows a sketch of $y = x^3$.
On the same diagram, sketch the curve $y = \frac{1}{2} x^3$. [1]
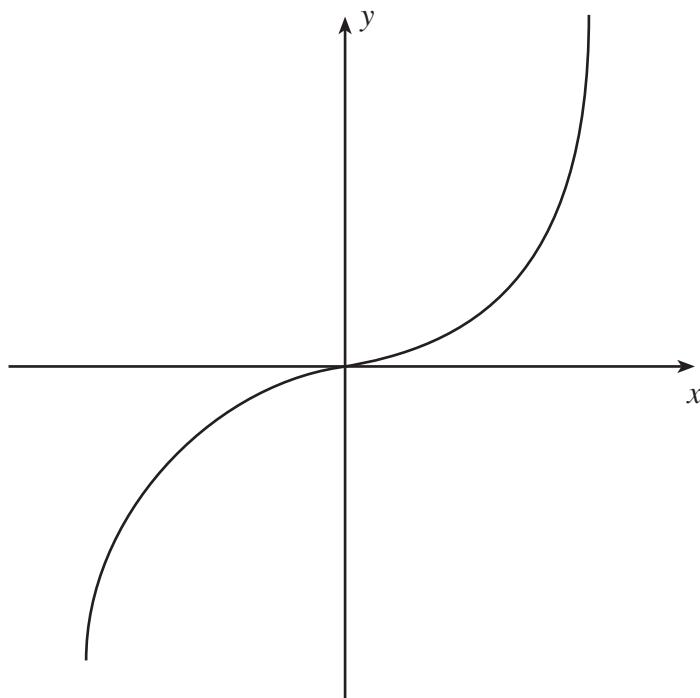
(b) The diagram shows a sketch of $y = x^2$.
On the same diagram, sketch the curve $y = (x - 2)^2$. Mark clearly the coordinates where the curve meets the *x*-axis. [2]

Turn over.
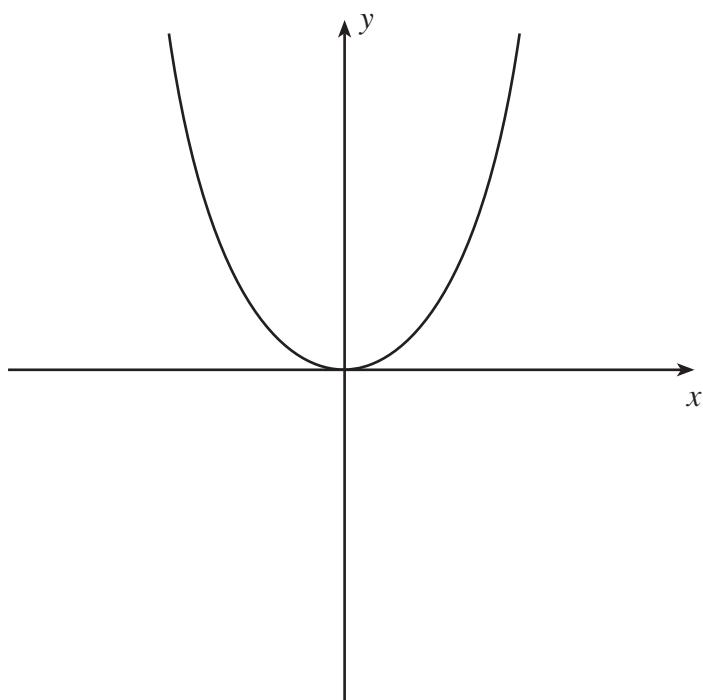
(c) The diagram shows a sketch of $y = x^2$.
On the same diagram sketch the curves
 (i) $y = -x^2$, and
  (ii) $y = -x^2 + 3$, mark clearly the coordinates of the point where the curve crosses the
        y-axis.   [3]

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Turn over.
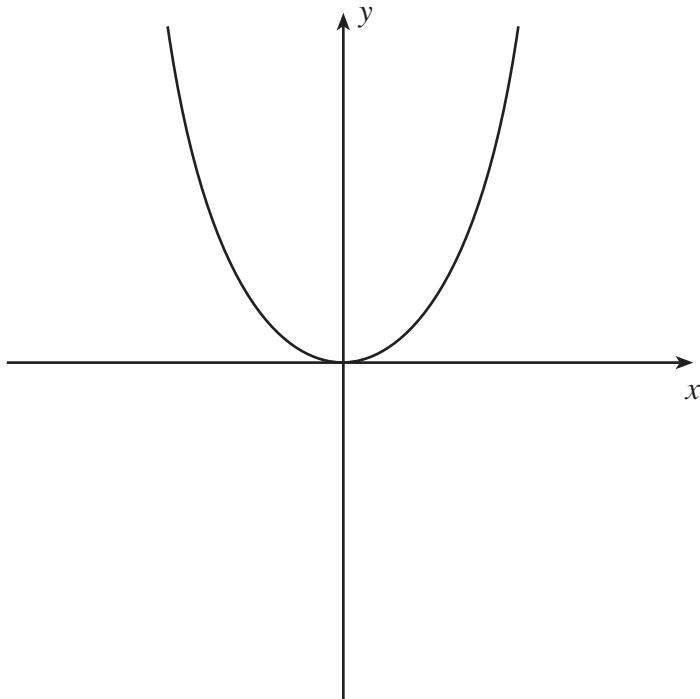
23. The diagram shows the graph of $y = bc^x$.
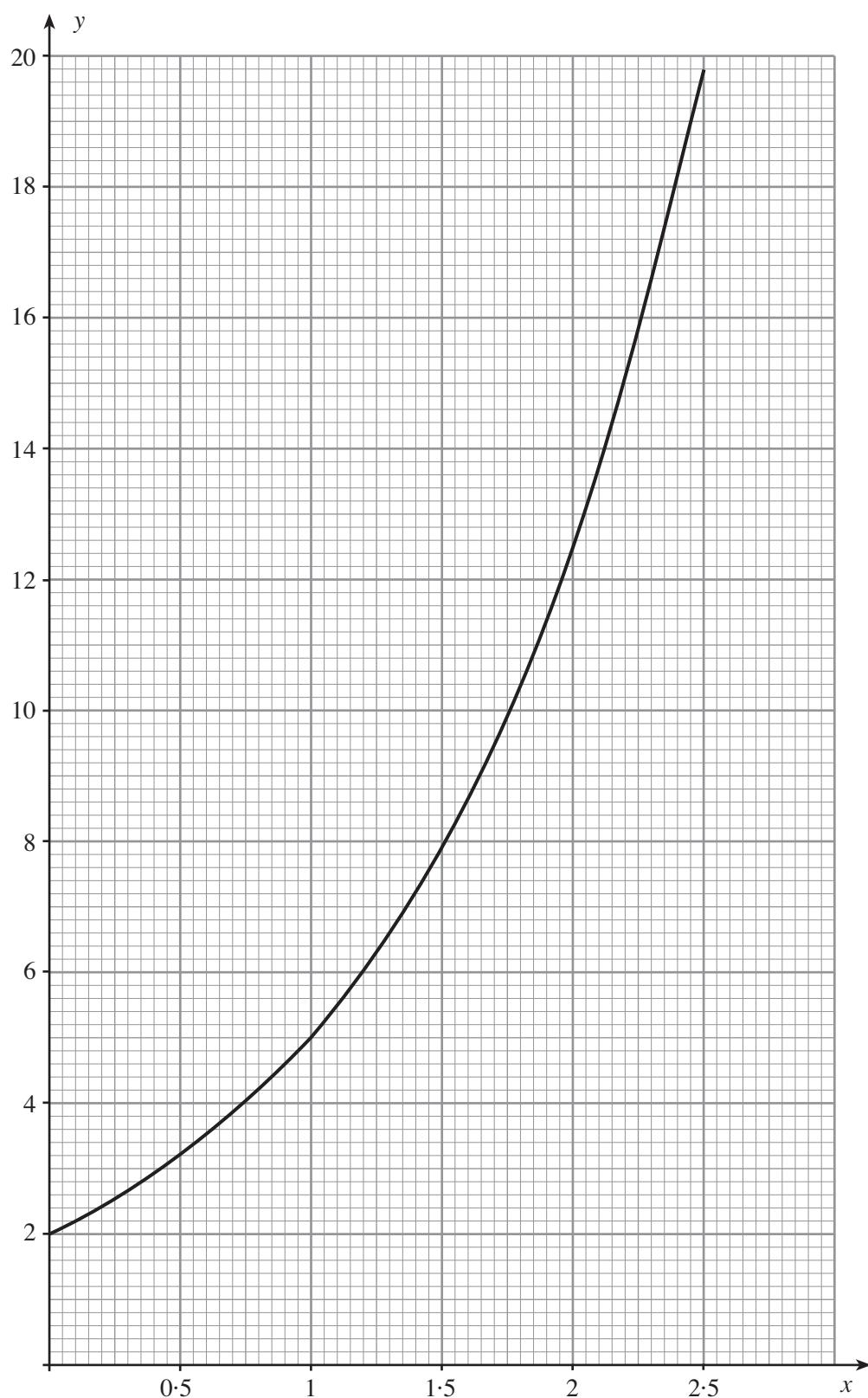
Use the graph opposite of $y = bc^x$ to find the values of b and c.

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[4]