Candidate Name	Centre Number	Candidate Number

WELSH JOINT EDUCATION COMMITTEE

General Certificate of Secondary Education

WJEC CBAC

CYD-BWYLLGOR ADDYSG CYMRU

Tystysgrif Gyffredinol Addysg Uwchradd

184/09

MATHEMATICS

HIGHER TIER PAPER 1

A.M. TUESDAY, 7 November 2006

(2 Hours)

CALCULATORS ARE NOT TO BE USED FOR THIS PAPER

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

No certificate will be awarded to a candidate detected in any unfair practice during the examination.

Formula List

Volume of prism = area of cross-section × length
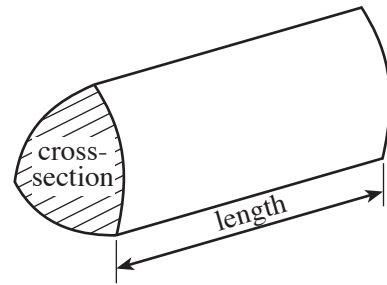
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
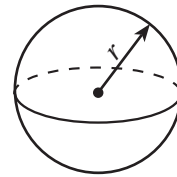
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
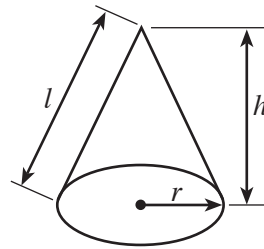
In any triangle *ABC*

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$
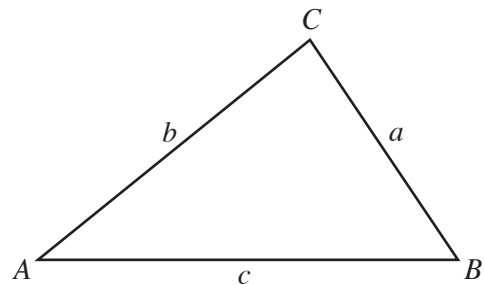
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n,$ having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

(184/09)

BLANK PAGE

1. The table shows some of the values of $y = 3x^2 - 4x - 10$ for values of x between -2 and 3.

(a) Complete the table by finding the value of y for $x = -1$.

x	-2	-1	0	1	2	3
$y = 3x^2 - 4x - 10$	10		-10	-11	-6	5

....

....
[1]

(b) On the graph paper opposite, draw the graph of $y = 3x^2 - 4x - 10$ for values of x between -2 and 3. [3]

(c) Draw the line $y = -5$ on your graph paper and write down the x-values of the points where your two graphs intersect.

....

....

....

....
[2]

For use with question 1.

Turn over.
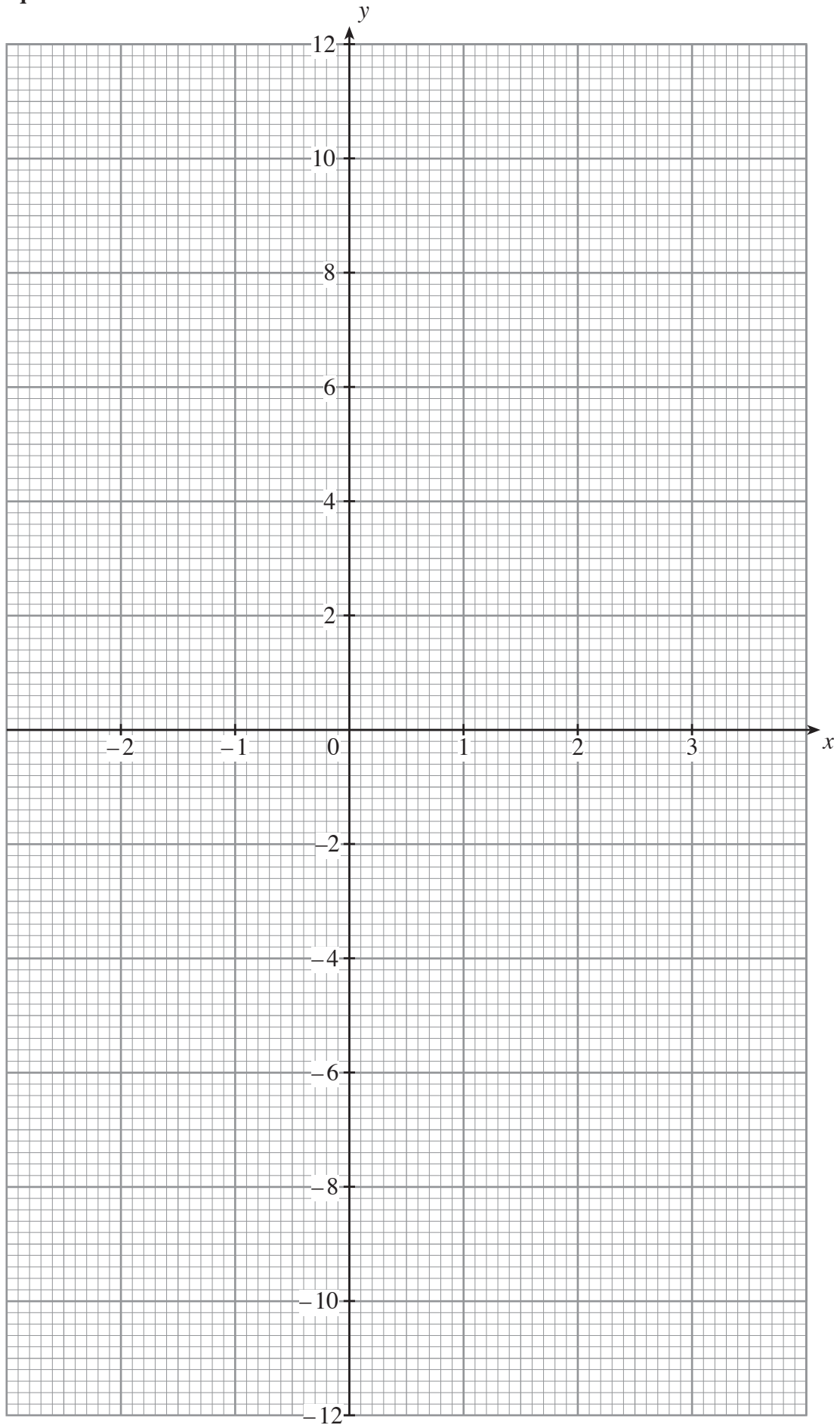
2. Expand and simplify

$$5(3m + 7) - 3(4m - 1).$$

....

....

....

....

[2]

3. *(a)* Express 120 as a product of prime numbers in index form.

....

....

....

....

....

....

....

[3]

(b) The number 168 can be expressed as $2^3 \times 3 \times 7$. Using this fact and your answer to *(a)* write down the Highest Common Factor (HCF) of the numbers 120 and 168.

....

....

....

....

[1]

4. An unbiased cubical dice has the numbers from 1 to 6 denoted by spots on its faces as shown below. The dice is rolled twice. What is the probability that a 3 is thrown both times?

....

....

....

....

[2]

Turn over.

5. *(a)* Describe fully the transformation that transforms triangle *ABC* into triangle *PQR*. [2]

....

....

....
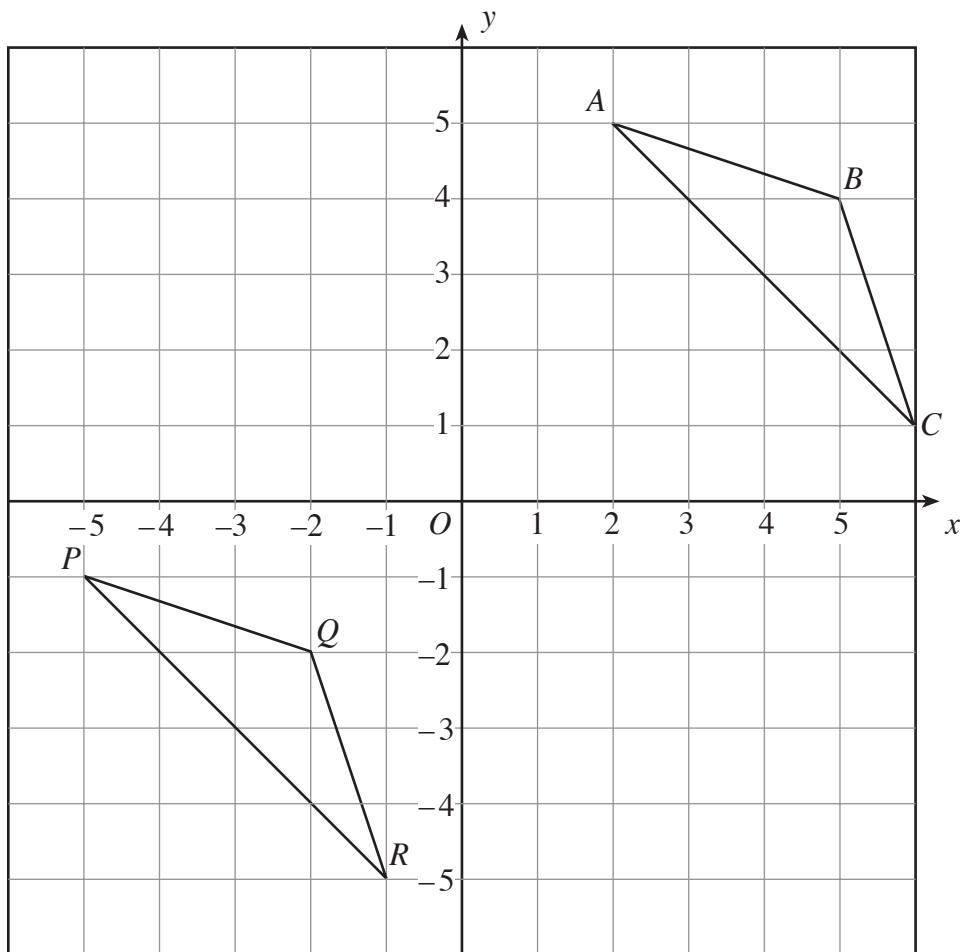
(b) Rotate the triangle **D** through 270° anti-clockwise about the point (–2, –1). Label the image **E**. [2]

Turn over.
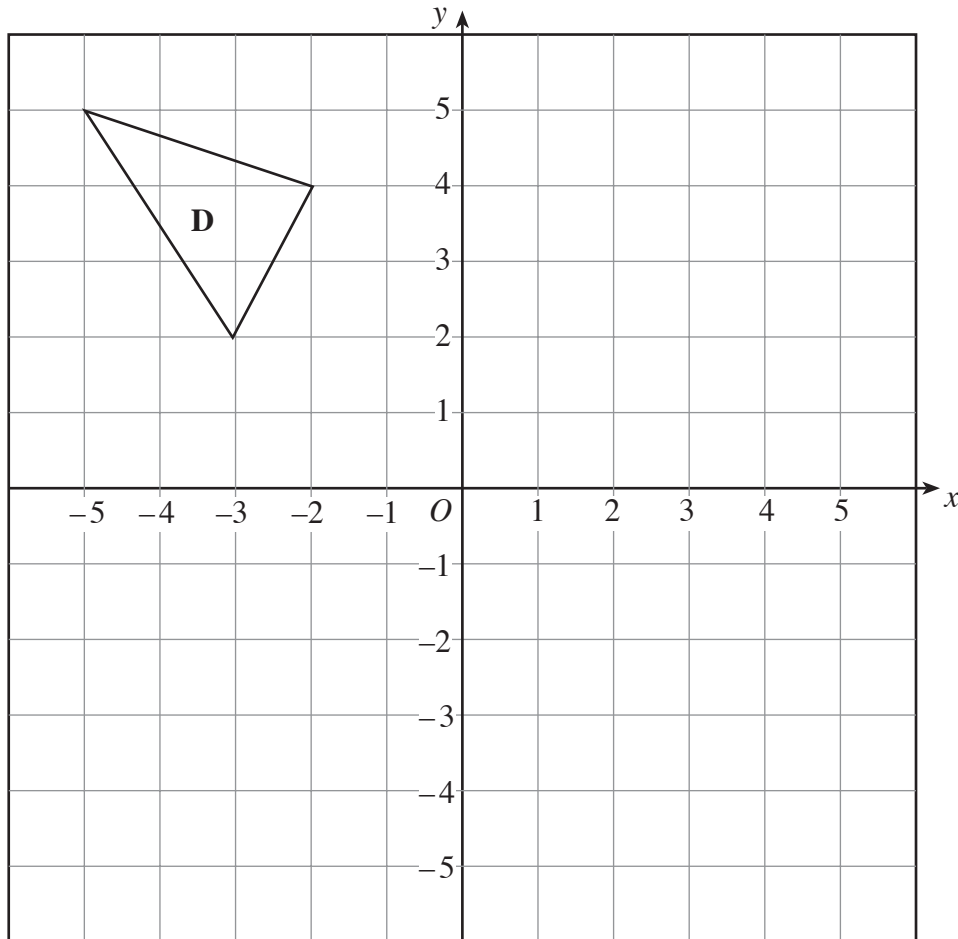
6. The diagram represents a plan of a plot of land *PQRS*. There is a tree at the point *T*. The scale used is 1cm represents 5m.

Find the region that satisfies all of the following conditions.

All the points in the region are

(i)   nearer to *PS* than to *PQ*,
(ii)   further than 15m from *PS*,
(iii) within 25m of the tree marked as *T*.   [4]

S ┌──┐ R

                                    •
                                    T

P └───────────────────────────────────────┘ Q

┌─────────────────┐
│ Scale   │
│ 1 cm = 5m   │
└─────────────────┘
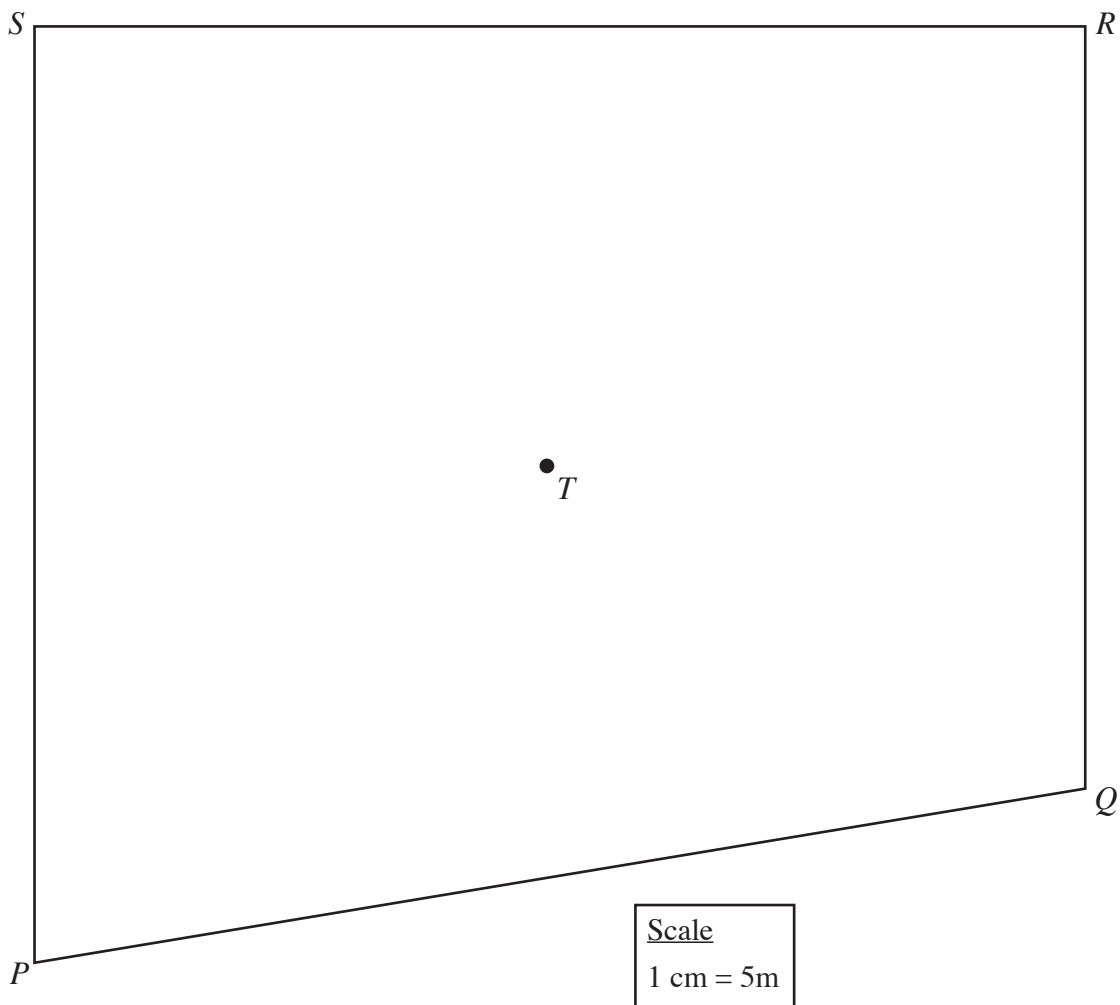
7. A cup has a capacity of 300ml measured to the nearest 10ml.

Write down the least and greatest possible values of the capacity of the cup.

Least capacity ....ml   Greatest capacity ....ml

[2]

8. Solve the following simultaneous equations by an algebraic (not graphical) method. Show all your working.

$$4x + 3y = 1$$
$$3x + 2y = 2$$

....

....

....

....

....

....

....

....

....

....

[4]

Turn over.

9. In **each** of the following formulae, every letter stands for the measurement of a length. By considering the dimensions implied by the formulae, write down, for **each** case, whether the formula could be for a length, an area, a volume or none of these.

The first one has been done for you.

<u>Formula could be for</u>

$2ab + c^2$ area

$4a + 3b - c$ ....

$5ab^2$ ....

$ab + c^3$ ....

$(a + 2b)c$ ....

[2]

10. In triangles *ABC* and *PQC*, $A\hat{B}C = P\hat{Q}C$ and $B\hat{A}C = Q\hat{P}C$. The lengths $QC = 9$cm, $PC = 7\cdot2$cm, $BQ = 11$cm, and $AB = 12$cm.
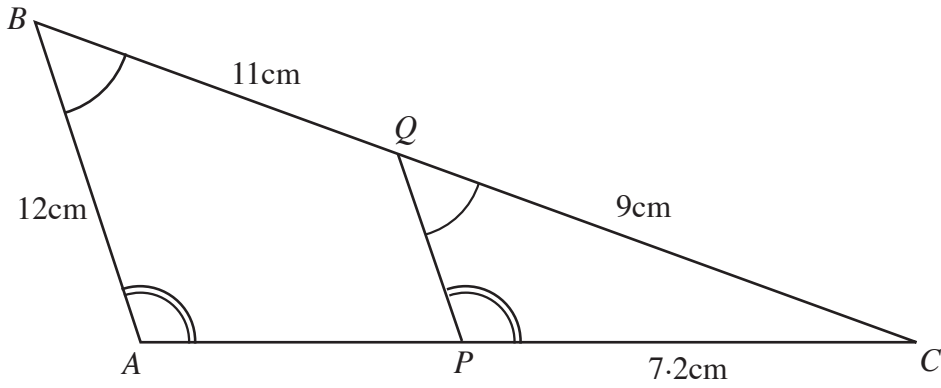
Diagram not drawn to scale.

Showing all your working, find the length of

(a) QP,

....

....

....

....

....

[2]

(b) AC.

....

....

....

....

....

[2]

Turn over.

11. The waist measurements of 100 people were measured. Below is a cumulative frequency polygon based on the results.

Cumulative frequency
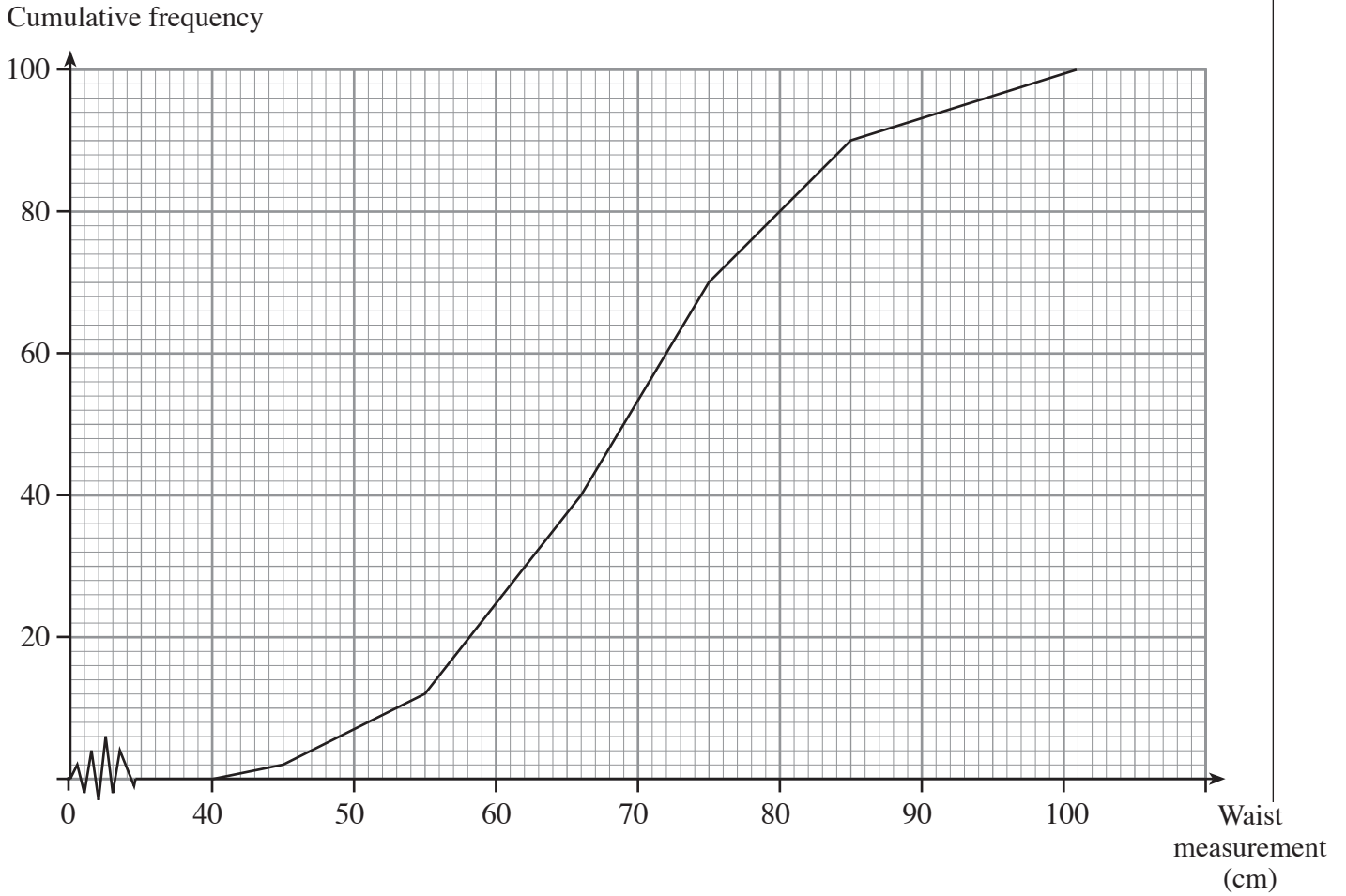
Use the cumulative frequency polygon to find an estimate for

(a) the inter-quartile range,

....

....

[2]

(b) how many people have a waist measurement of more than 88 cm.

....

....

....

[2]

12. *(a)*   Expand the following expression, simplifying your answer as far as possible.

$$(x + 5)(x - 3)$$

....

....

....

[2]

(b)   Simplify the expression

$$5a^4b^3 \times 2a^3b^2.$$

....

....

[2]

(c)   Make *r* the subject in the following formula.

$$3(r + 2) = 7d - 2r$$

....

....

....

....

[3]

Turn over.

13. Evaluate $2\frac{1}{4} \times 3\frac{1}{3}$ giving your answer in its simplest form.

....

....

....

....

[3]

14. Simplify **each** of the following.

(a) 8^0

....

[1]

(b) $(3x^5y^3)^4$

....

....

....

[2]

(c) $\dfrac{24 \times a^{\frac{13}{2}} \times a^{-\frac{3}{2}}}{6a^3}$

....

....

....

....

[2]

15. Given that y is inversely proportional to x^2, and that $y = 3$ when $x = 2$,

(a) find an expression for y in terms of x,

....

....

....

....

[3]

(b) use the expression you found in *(a)* to complete the following table.

x	2	10	
y	3		12

....

....

....

[2]

Turn over.

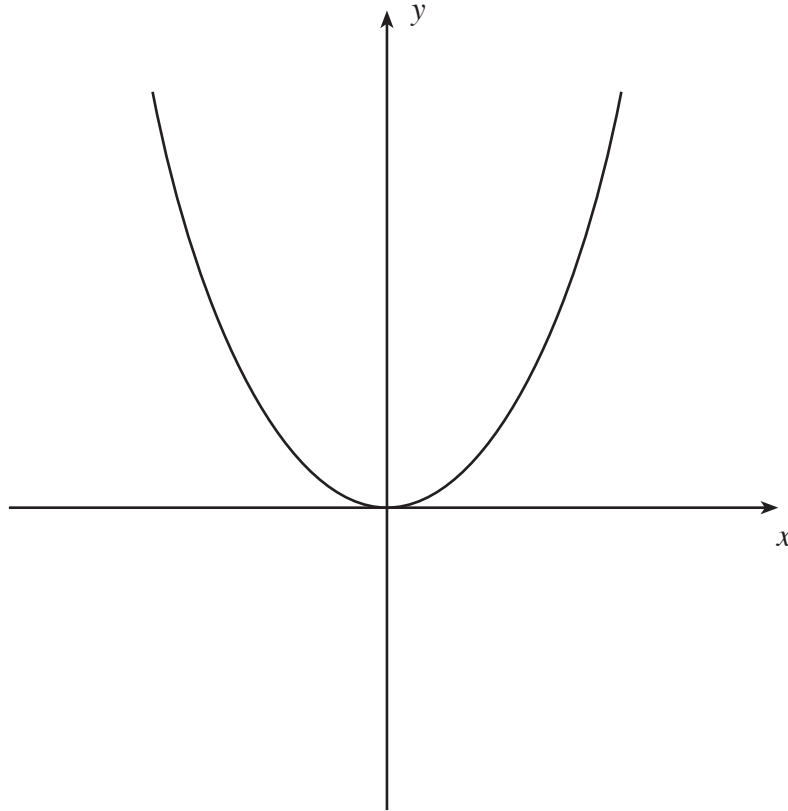**16.** *(a)* The diagram shows a sketch of $y = f(x)$.

On the same diagram, sketch the curve $y = \dfrac{1}{2}f(x) + 3$.

Mark clearly the coordinates of the point where the curve crosses the y-axis.

[3]

(b) The diagram shows the sketch of $y = h(x)$.
On the same diagram sketch the curve $y = h(x - 5)$.
Mark clearly the coordinates of the point where the curve crosses the x-axis.

[2]
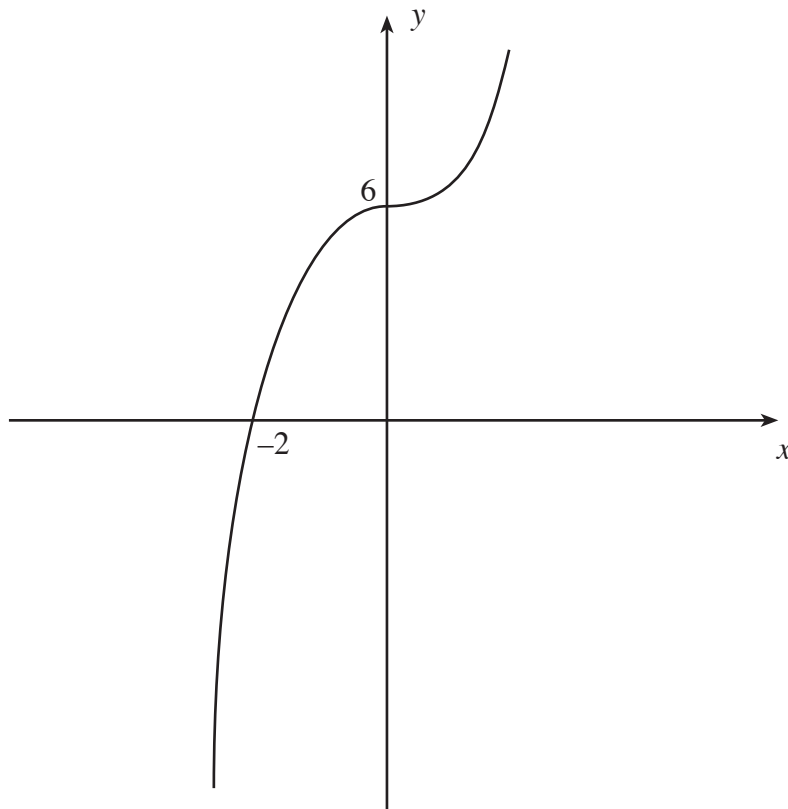
17. A bag contains 12 marbles. There are 2 yellow, 3 blue and 7 red marbles in the bag.
Two marbles are selected at random from the bag.

(a) Calculate the probability that both the selected marbles are red.

....

....

....

....

....

....

[2]

(b) Calculate the probability that exactly one of the selected marbles is blue.

....

....

....

....

....

....

....

....

....

....

....

[3]

Turn over.

18. The diagram shows two regular pentagons, *ABCDE* and *PQRST*.
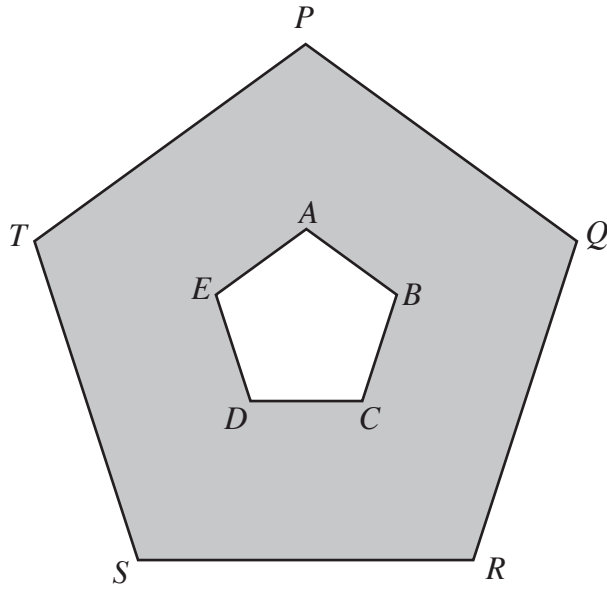
Diagram not drawn to scale.

Pentagon *PQRST* is an enlargement of *ABCDE* with a scale factor 3. The area of *ABCDE* is $4\,\text{cm}^2$. Calculate the area shaded in the diagram.

....

....

....

....

....

[4]

19. There are 100 pupils in Year 9. The time taken by each pupil to answer a mental mathematics question was recorded. The following grouped frequency distribution was obtained.

Time, t seconds	$0 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 60$
Number of pupils	8	17	25	38	12

(a) Find an estimate for the median of this distribution.

....

....

[1]

(b) Draw a histogram to illustrate the distribution on the graph paper below.

....

....

....

....

....

[2]

Time taken to answer in seconds

Turn over.
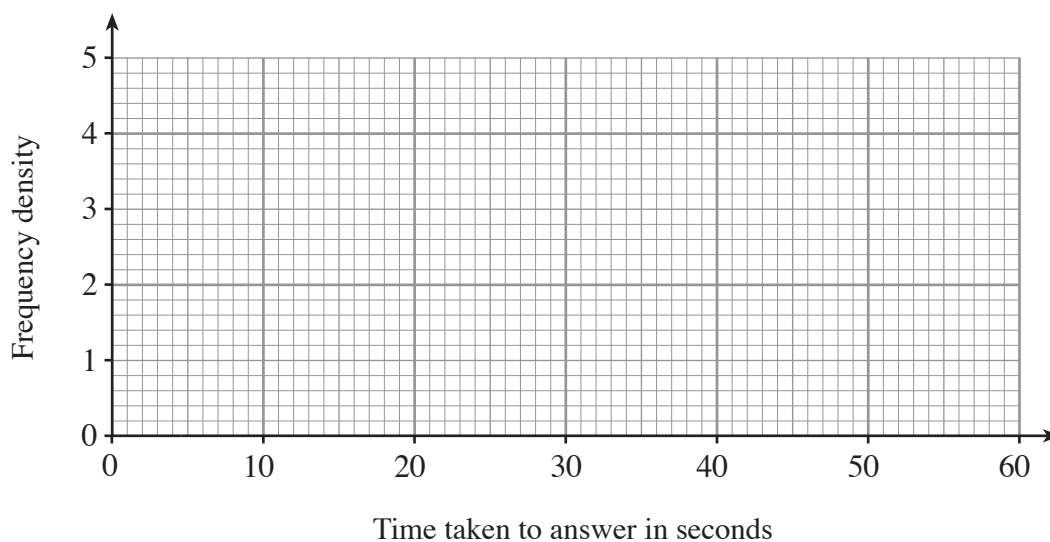
20. The volume of a cone with height h cm and radius r cm is 5πcm^3. Find an expression for r in terms of h.

....

....

....

....

....

....

....

....

[4]

21. Given that $a = \sqrt{5}$, $b = \sqrt{7}$ and $c = \sqrt{35}$, simplify abc and state whether your answer is rational or irrational.

....

....

....

....

....

[2]

22. Express the following as a single fraction in its simplest form.

$$\frac{7}{3-4x} + \frac{x}{3x+1}$$

....

....

....

....

....

....

....

....

....

[3]

Turn over.

23. The length of each edge of a square-based pyramid is $2x$ cm.

$2x$ cm

$2x$ cm

$2x$ cm

$2x$ cm

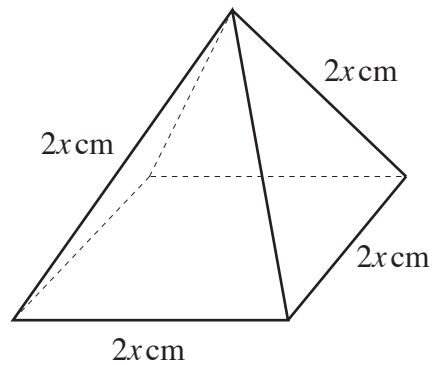*Diagram not drawn to scale.*

(a) Show that the area of one of the triangular faces is $x^2\sqrt{3}$ cm^2.

....

....

....

....

....

....

....

[4]

(b) Write down an expression for the total surface area of the square-based pyramid.

....

....

....

[1]

24. Vectors **OM**, **ON** and **OP** are shown in the diagram below.

M

$6\mathbf{a} + 9\mathbf{b}$

O

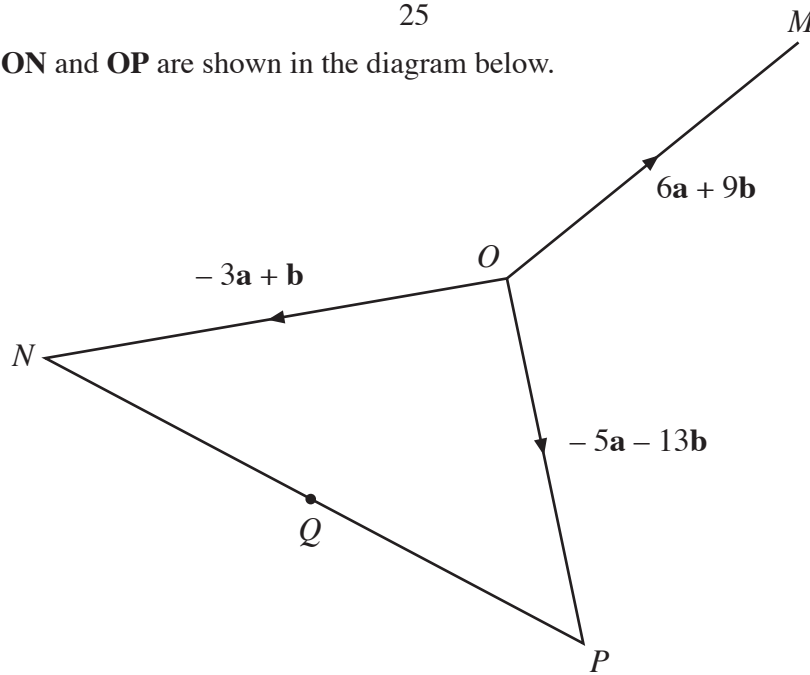$-3\mathbf{a} + \mathbf{b}$

N

$-5\mathbf{a} - 13\mathbf{b}$

Q

P

Diagram not drawn to scale.

Given that the point Q is the mid-point of NP and **OM** $= 6\mathbf{a} + 9\mathbf{b}$, **ON** $= -3\mathbf{a} + \mathbf{b}$ and **OP** $= -5\mathbf{a} - 13\mathbf{b}$.

(a) Find **PN** in terms of **a** and **b**.

....

....

[1]

(b) Find **QO** in terms of **a** and **b** in its simplest form.

....

....

....

[2]

(c) Show that **QO** $= k$**OM** where k is a constant.

....

....

....

[1]

(d) State **two** geometrical relationships between QO and OM.

....

....

....

....
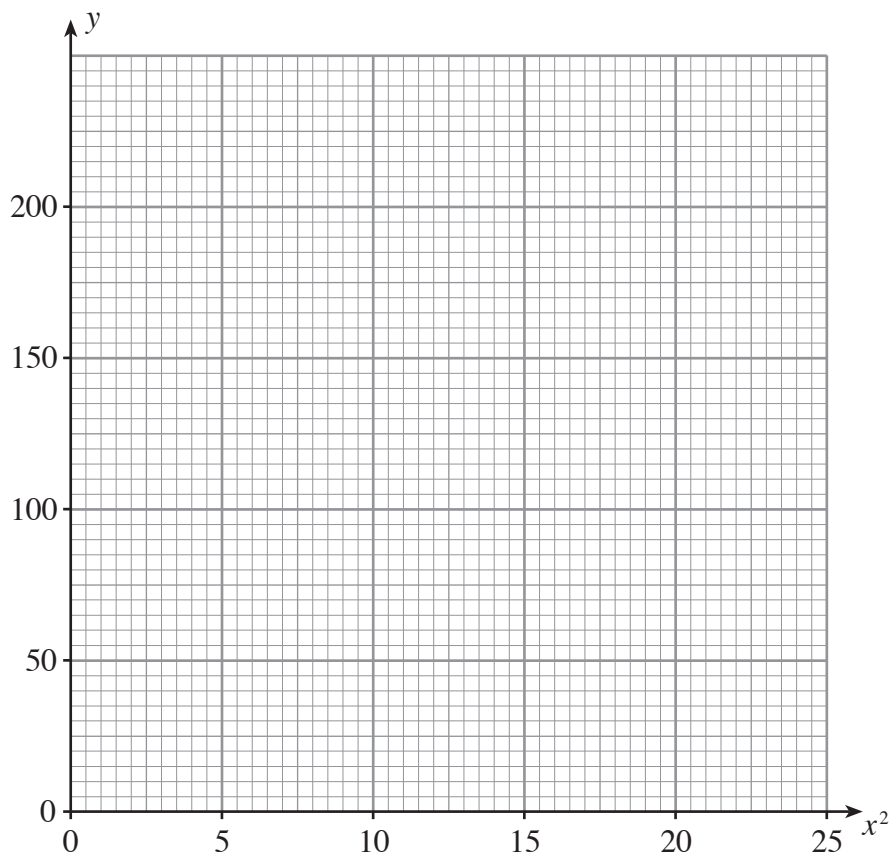
[2]

(184/09)

Turn over.

25. The data in the table was recorded during an experiment. Results were recorded for the two variables x and y.

x	1	2	3	4	5
y	31	52	87	136	199

(a) On the graph paper plot the values of y against the values of x^2.

[2]

(b) It is known that y is approximately equal to $ax^2 + b$.
Use your graph to estimate the values of a and b.

....

....

....

....

....

....

[3]