WELSH JOINT EDUCATION COMMITTEE

General Certificate of Secondary Education

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184/09

MATHEMATICS

HIGHER TIER PAPER 1

A.M. TUESDAY, 6 November 2007

(2 Hours)

CALCULATORS ARE NOT TO BE USED FOR THIS PAPER

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

No certificate will be awarded to a candidate detected in any unfair practice during the examination.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	4	
2	5	
3	4	
4	2	
5	3	
6	2	
7	4	
8	7	
9	2	
10	4	
11	4	
12	5	
13	4	
14	5	
15	5	
16	5	
17	3	
18	7	
19	3	
20	5	
21	4	
22	4	
23	5	
24	4	
TOTAL MARK		

Formula List

Volume of prism = area of cross-section × length
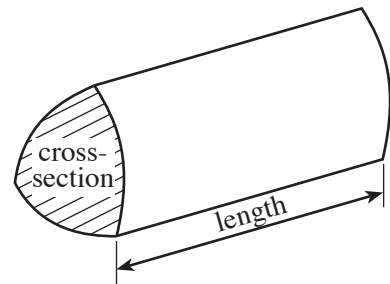
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
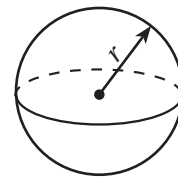
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
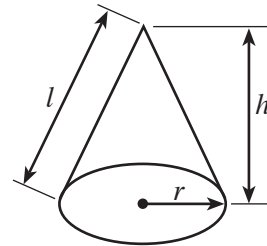
In any triangle *ABC*

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$
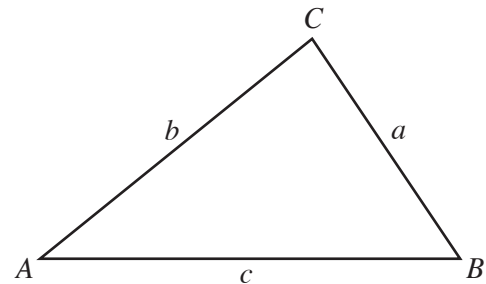
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n,$ having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

1. Write down, in terms of n, the nth term of **each** of the following sequences.

(a) 9   16   23   30   ....

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[2]

(b) 3×2 4×4 5×6 6×8 ....

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[2]

2. *(a)* Solve $11x - 8 = 4(2x - 5)$.

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[3]

(b) Simplify $4(2x + 7y) + 3(2x - 3y)$.

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[2]

Turn over.

3. *(a)* Describe fully the transformation that transforms triangle *ABC* into triangle *PQR*.

[2]

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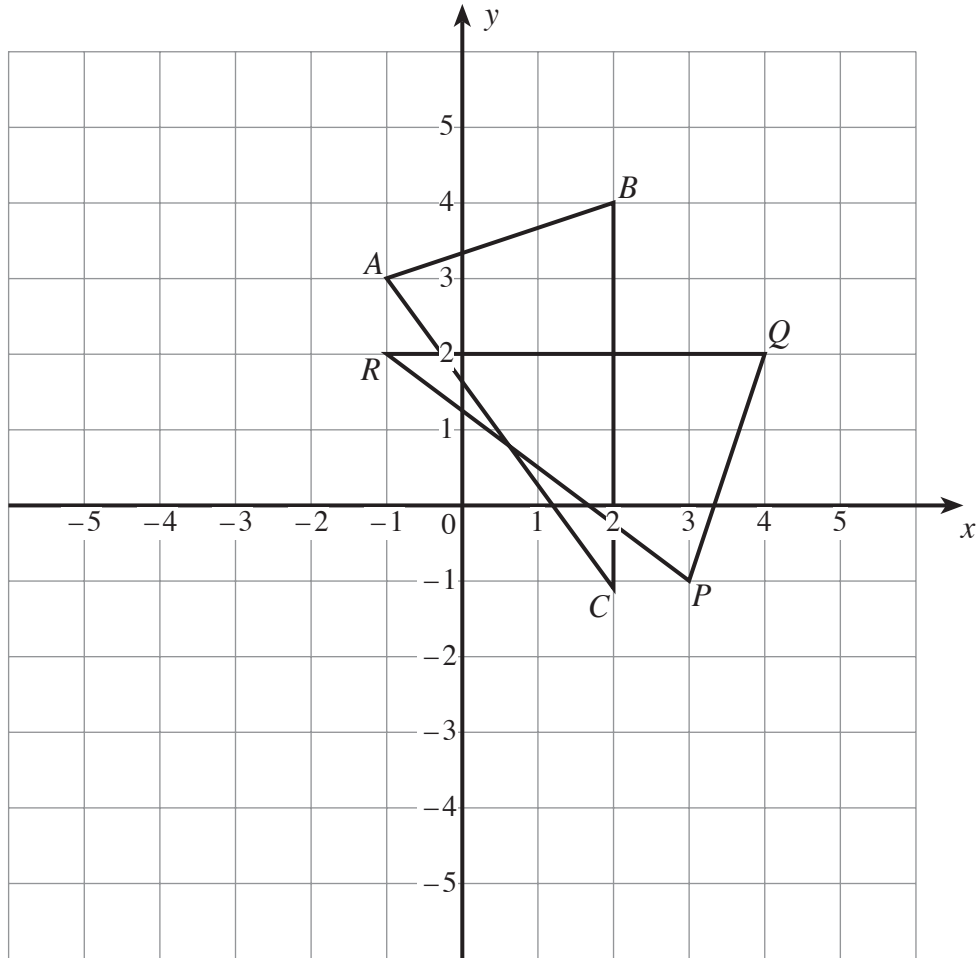
(b) Rotate the triangle D through 90° clockwise about the point (3, 1).
Label the image E.

[2]

Turn over.
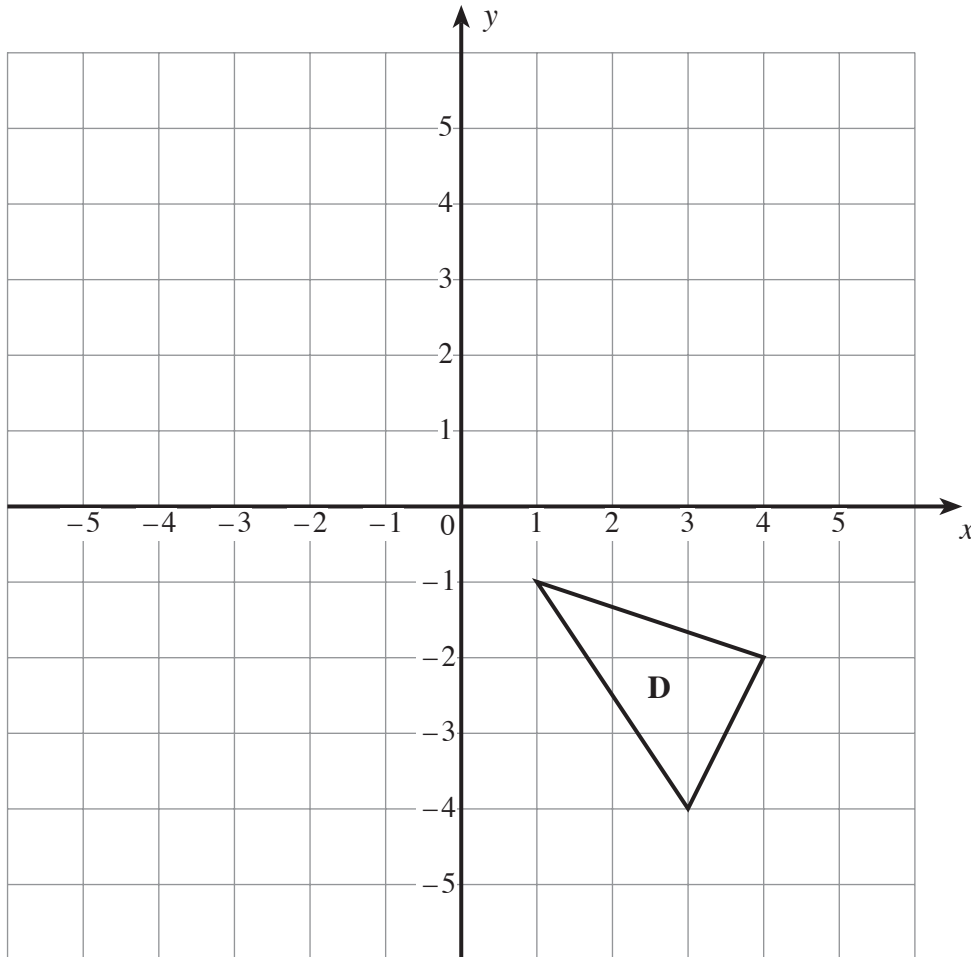
4. A machine is used to display one digit at random between 1 and 8 inclusive.
The machine is to be used twice. Find the probability that the digits displayed will be 5 and 5.

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[2]

5. Find and shade the region of points **inside** the parallelogram *ABCD* that satisfy **both** of the following conditions.

(i)   The points are within 4 cm of *BC*.

(ii) The points are further than 8 cm from *D*.   [3]
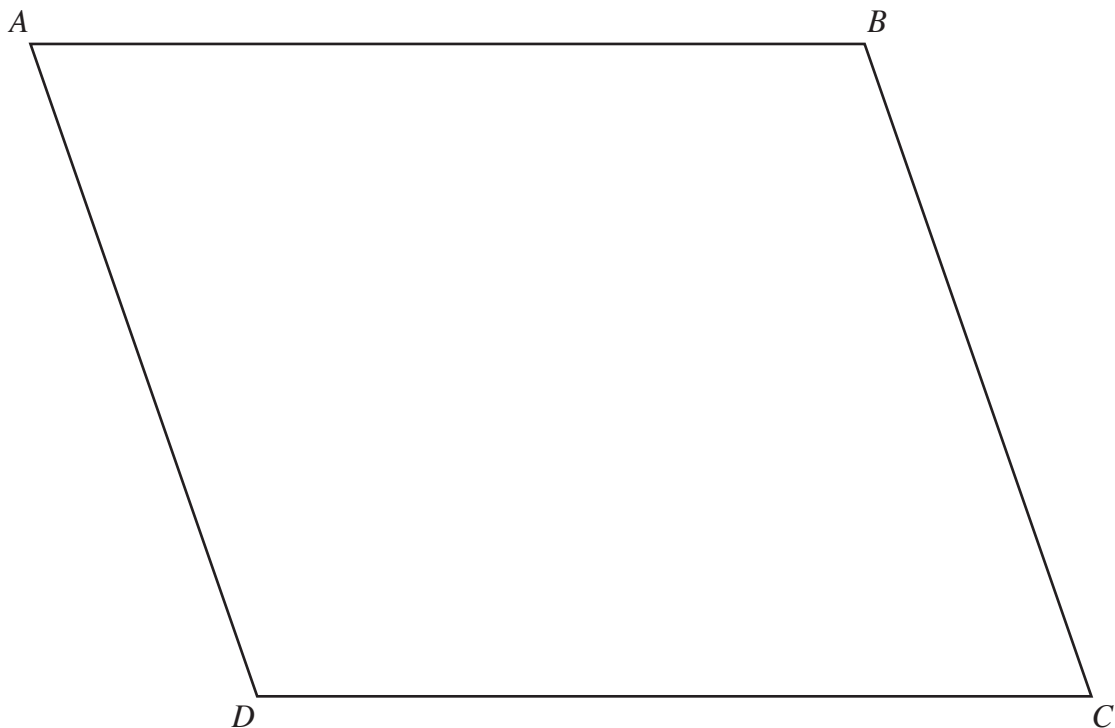
6. A regular polygon has exterior angles of 45°. How many sides has the polygon?

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[2]

7. Solve the following simultaneous equations by an algebraic (not graphical) method.
Show all your working.

$$5x + 2y = 11$$
$$4x - 3y = 18$$

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[4]

Turn over.

8. Sacks are filled with 100 kg of sand correct to the nearest 10 kg.

(a) Write down the least possible amount of sand and the greatest possible amount of sand in a sack.

Least possible amount of sand .... kg   Greatest possible amount of sand .... kg
[2]

(b) Bob wishes to have at least 5000 kg of sand.
Find, showing all your calculations, the least number of sacks needed.

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[3]

(c) Bob has the number of sacks you found in *(b)*. What is the greatest total amount of sand that Bob could have?

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[2]

9. In each of the following formulae, every letter stands for the measurement of a length. By considering the dimensions implied by the formulae, write down, for each case, whether the formulae could be for a length, an area, a volume or none of these.
The first one has been done for you.

<u>Formula could be for</u>

$d^2 - hd$ area

$(d - 3r)r^2$ ....

$7d + 6h + 2r$ ....

$4rh - 3r^2 + dh$ ....

$3rh + 2rd - h$ ....

[2]

10. *(a)* Explain clearly why triangles *ABC* and *PQR* are **NOT** similar.

C

78°

56°

A   B

P

36°

Q   R
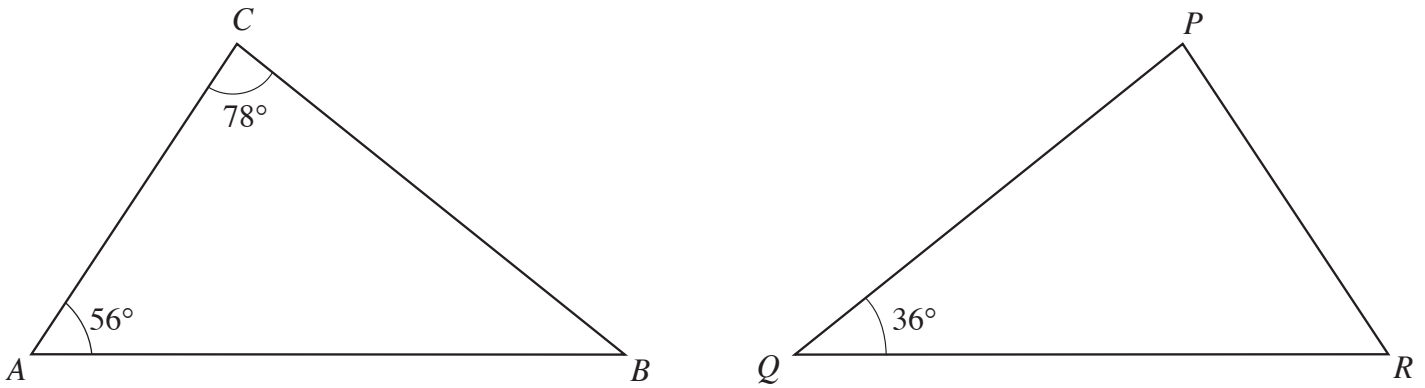
Diagrams not drawn to scale.

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[2]

(b) Triangles *DEF* and *XYZ* **are** similar. Their corresponding sides are in the ratio 3:5. Calculate the length of *ZY*.

X

D

10 cm   15 cm

E   12 cm   F

Y   Z

Diagrams not drawn to scale.

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[2]

Turn over.
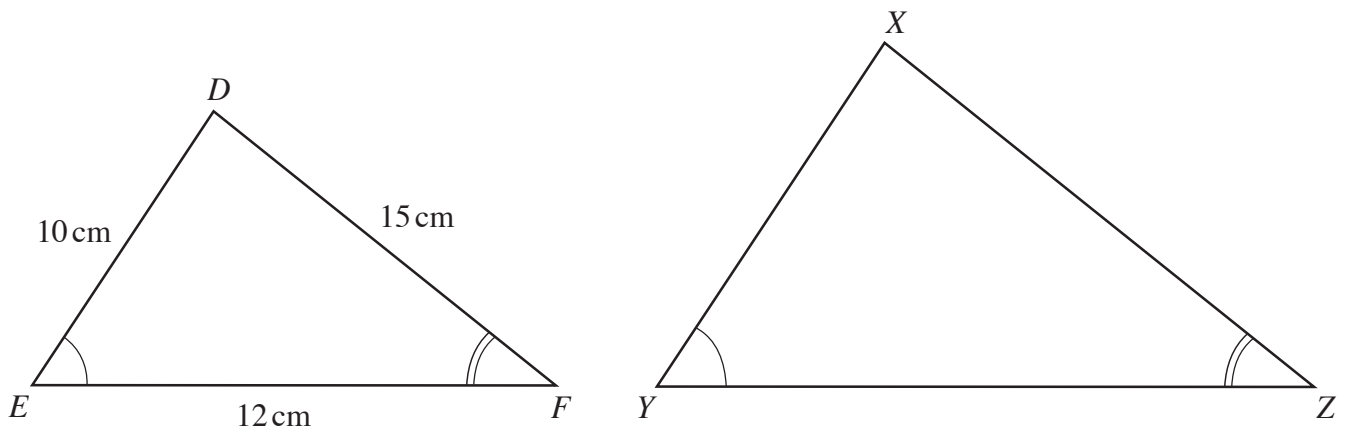
11. The probability that Tony wins a game of badminton against Cliff is $\frac{1}{3}$.
Tony plays two games of badminton against Cliff.
In the space below, draw and fully label a tree diagram for these two games.

[4]

12. *(a)* Expand the following expression, simplifying your answer as far as possible.

$$(x - 7)(x + 3)$$

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[2]

(b) Factorise $x^2 - 25$.

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[1]

(c) Factorise $4xy + 6x^2$.

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[2]

Turn over.

13.
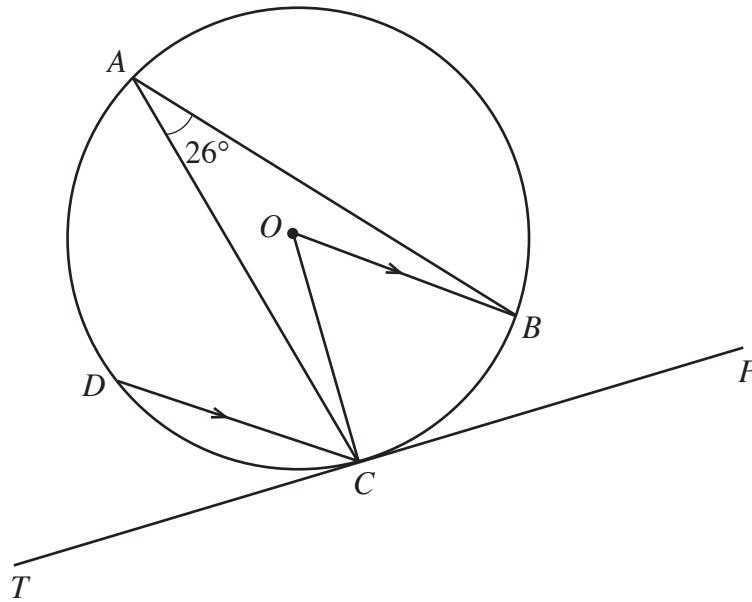
Diagram not drawn to scale.

Four points *A, B, C* and *D* lie on the circumference of the circle centre *O*.
The tangent *TP* touches the circle at *C*. The radius *OB* is parallel to *DC*.
Given that $B\hat{A}C = 26°$, find **each** of the following angles, giving reasons for your answers.

(a) $B\hat{O}C$

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....
[2]

(b) $D\hat{C}T$

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....
[2]

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Turn over.

14. *(a)* The diagram shows a sketch of $y = f(x)$.
On the same diagram, sketch the curve $y = f(x) - 2$.
Mark clearly the coordinates of the point where the curve crosses the y-axis.

[2]
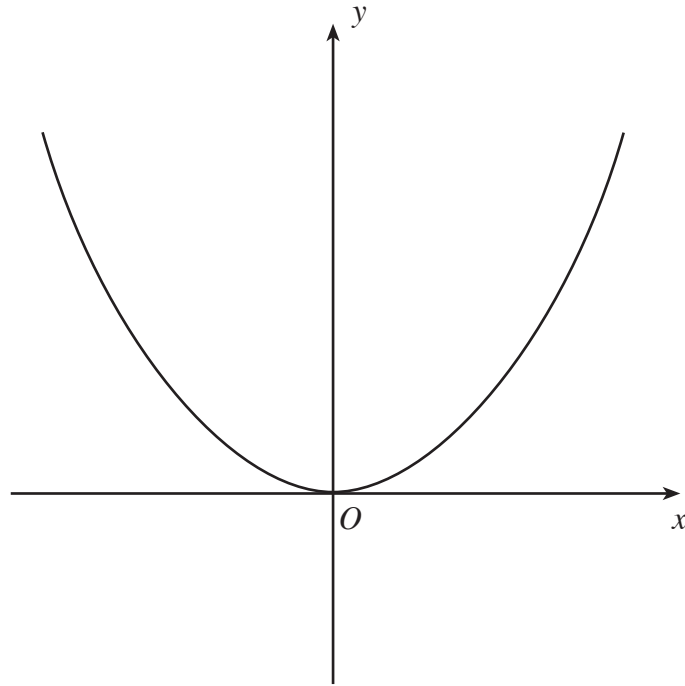
(b) The diagram shows the sketch of $y = g(x)$.
On the same diagram sketch the curve $y = g(3x)$.

[1]
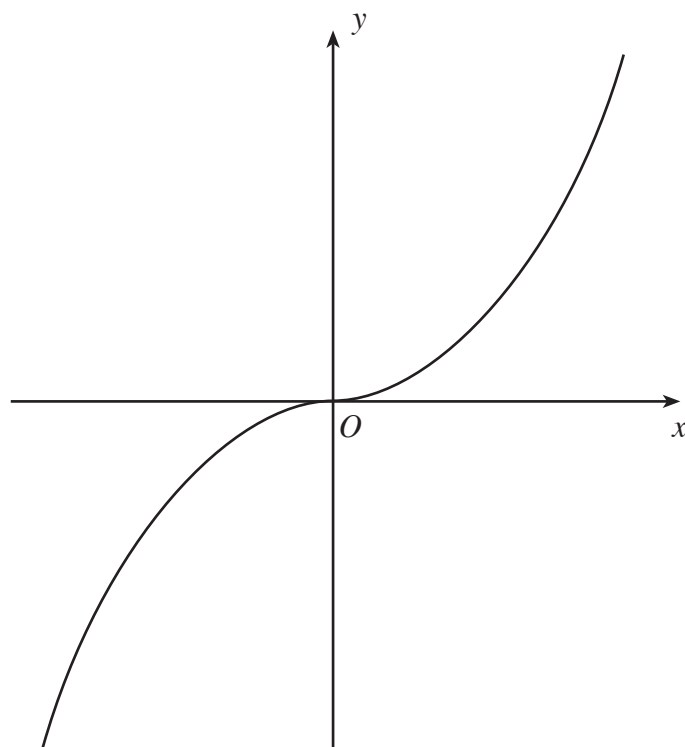
(c) The diagram shows the sketch of $y = h(x)$.
On the same diagram sketch the curve $y = h(x - 4)$.
Mark clearly the coordinates of the point where the curve crosses the *x*–axis.

[2]

Turn over.
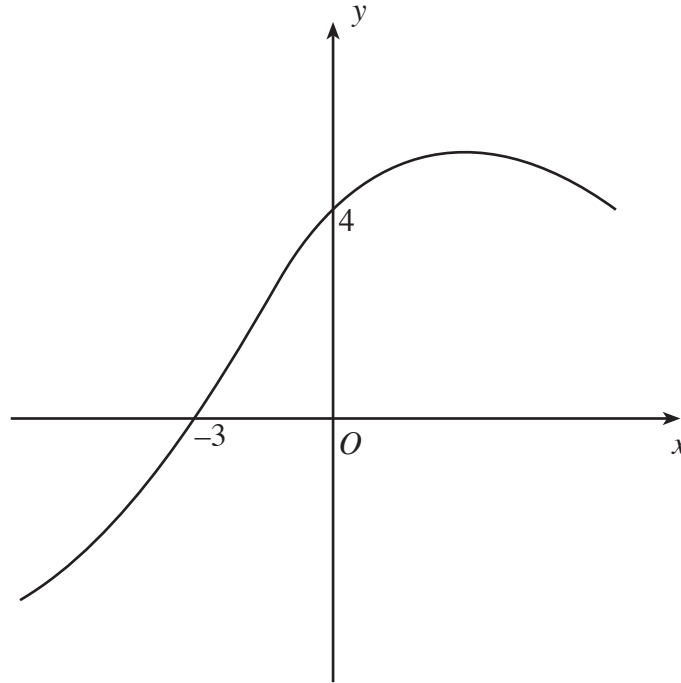
15. Simplify **each** of the following.

   (a) 25^0

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   [1]

   (b) $(2x^3y^4)^5$

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   [2]

   (c) $\dfrac{28 \times a^{\frac{19}{2}} \times a^{-\frac{5}{2}}}{7a^4}$

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   [2]

16. Given that y is inversely proportional to x, and that $y = 8$ when $x = 0.5$,

   (a) find an expression for y in terms of x.

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   [3]

   (b) use the expression you found in *(a)* to complete the following table.

x	0·5	2	
y	8		0·8

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   [2]

17. As part of an investigation, the time taken to solve a logic problem was measured for each pupil in a group of fourteen-year-olds. The histogram below illustrates the results obtained.

Times for the 14 year olds

Time in seconds
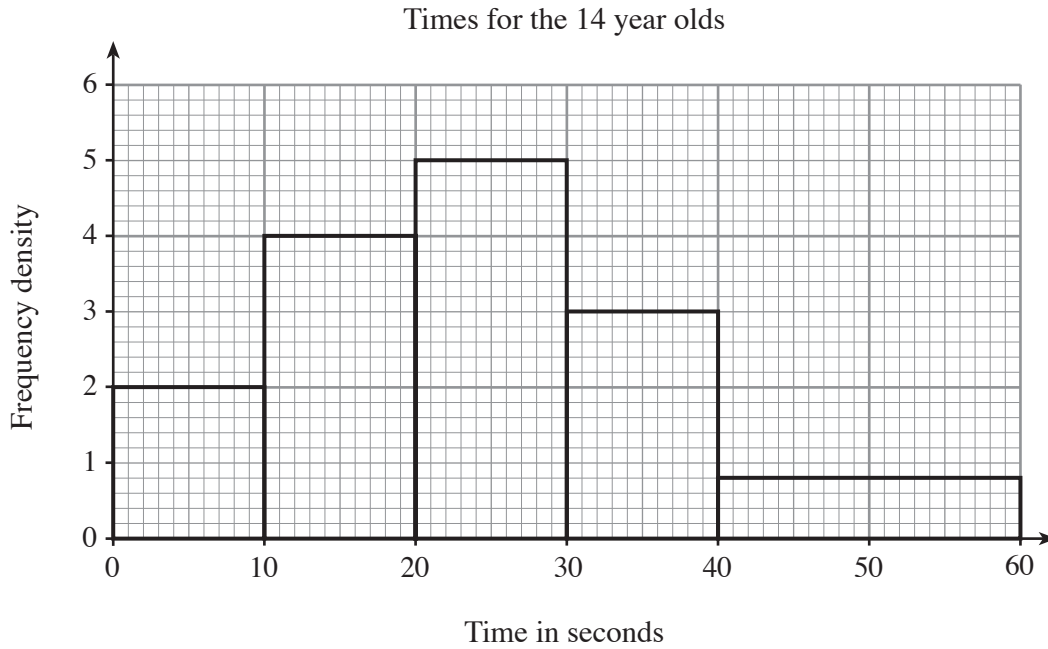
Use the histogram to calculate the number of fourteen-year-olds in this group.

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[3]

Turn over.

18.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |

Seven cards are numbered 1, 2, 3, 4, 5, 6, 7 respectively. The cards are shuffled and two cards are chosen at random.

(a) Calculate the probability that the numbers on the chosen cards are both even.

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[2]

(b) Calculate the probability that the **sum** of the numbers on the chosen cards is odd.

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[2]

(c)   Calculate the probability that the **product** of the numbers on the chosen cards is odd.

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[3]

Turn over.

19. Express the following as a single fraction in its simplest form.

$$\frac{7}{4-5x} - \frac{2}{x+3}$$

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[3]

20. *(a)* Find the value of $\left(\sqrt{32} - \sqrt{2}\right)^2$.

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[3]

(b) Given that $p = \sqrt{7}, q = \sqrt{11}$ and $r = \sqrt{154}$, simplify *pqr*.

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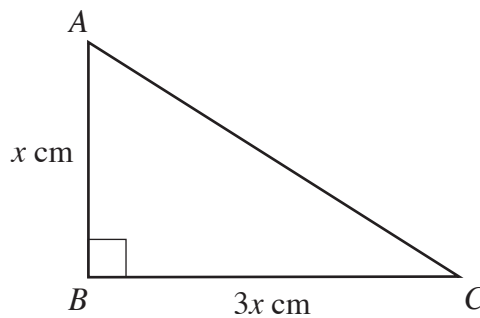
[2]

21. *ABC* is a right-angled triangle with $AB = x$ cm, $BC = 3x$ and $\hat{B} = 90°$.

Diagram not drawn to scale.

Find an expression, in terms of *x*, for the perimeter of triangle *ABC*. Simplify your answer.

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[4]

22. The diagram shows a quadrilateral *OABC*.
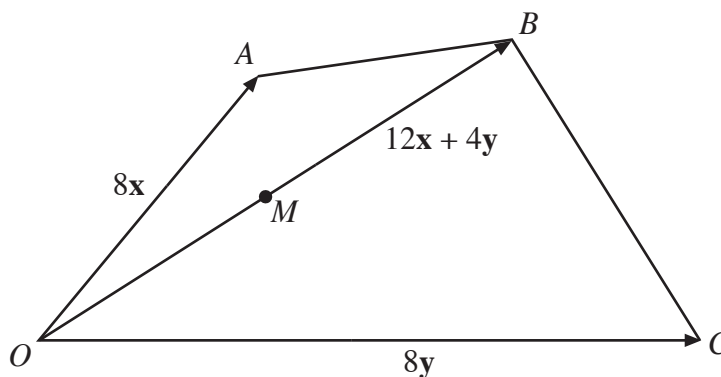
Diagram not drawn to scale.

In the quadrilateral *OABC*, the vectors **OA**, **OB** and **OC** are given by **OA** = 8**x**, **OB** = 12**x** + 4**y** and **OC** = 8**y**.

(a) Given that *M* is the mid-point of *OB*, express each of the following in terms of **x** and **y** in their simplest form,

(i) **OM**

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....

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[1]

(ii) **CA**

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....
[1]

(iii) **MA**

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[1]

(b) Does the point *M* lie on the line *CA*? Give a reason for your answer.

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[1]

23. The data in the table was recorded during an experiment. Results were recorded for the two variables x and y.

x	1	2	3	4	5
y	170	320	575	925	1370

(a) On the graph paper plot the values of y against the values of x^2. [2]
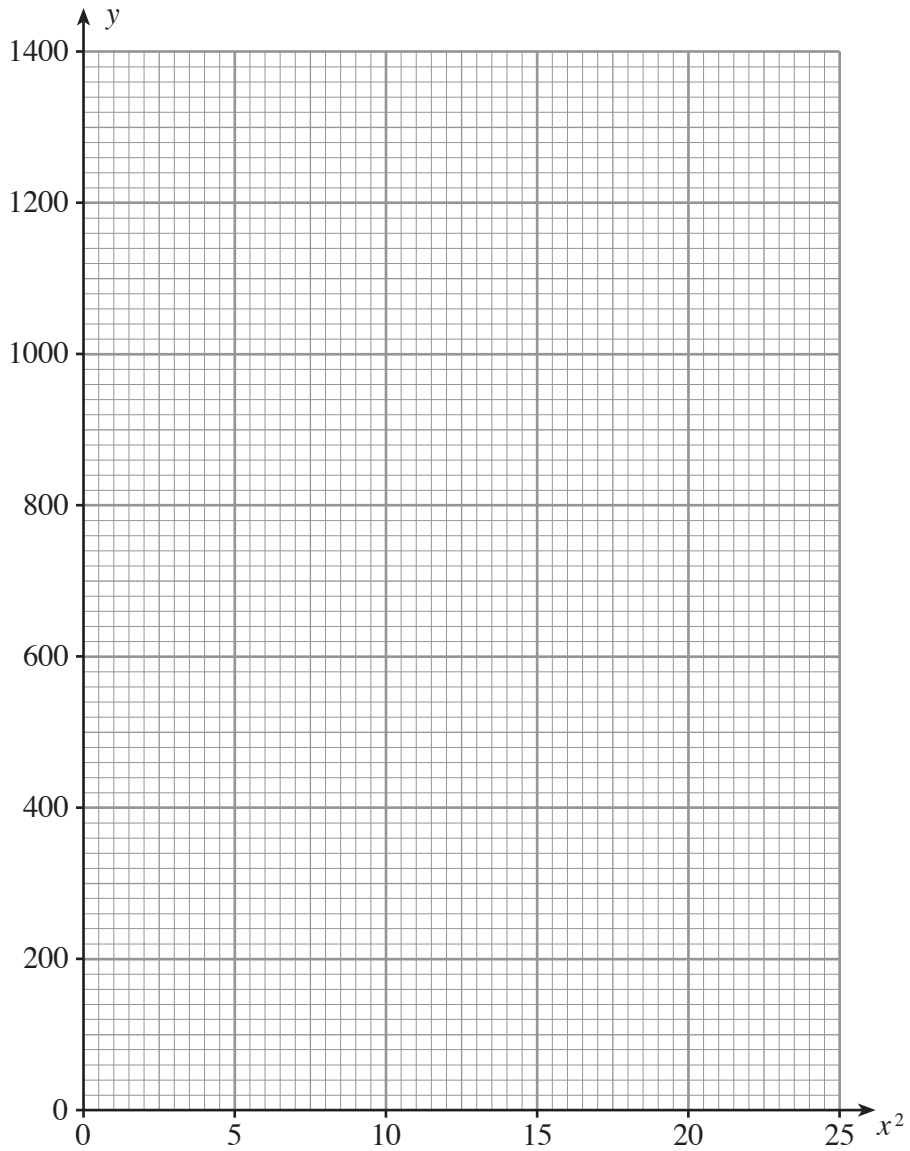
(b) It is known that y is approximately equal to $ax^2 + b$.
Use your graph to estimate the values of a and b.

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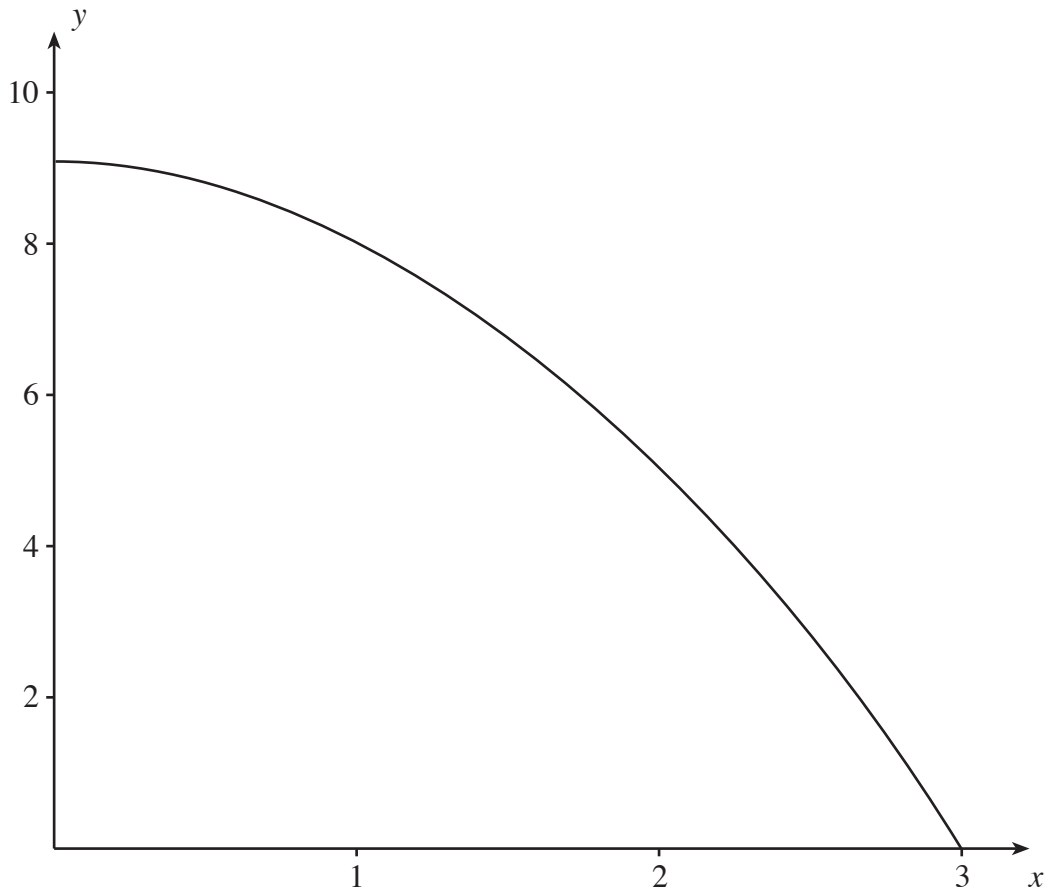
[3]

Turn over.

24. A sketch of $y = 9 - x^2$ is shown below for values of x from 0 to 3.

Use the trapezium rule, with the four ordinates $x = 0$, $x = 1$, $x = 2$ and $x = 3$, to estimate the area of the region bounded by the curve, the x-axis and the y-axis.

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[4]