**WJEC CBAC**

GCSE

185/04

## MATHEMATICS (2 Tier)
**HIGHER TIER**
**PAPER 1**

A.M. THURSDAY, 6 November 2008

2 hours

**CALCULATORS ARE NOT TO BE USED FOR THIS PAPER**

### INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

### INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

| For Examiner's use only | | |
|---|---|---|
| Question | Maximum Mark | Mark Awarded |
| 1 | 2 | |
| 2 | 3 | |
| 3 | 2 | |
| 4 | 8 | |
| 5 | 4 | |
| 6 | 4 | |
| 7 | 5 | |
| 8 | 3 | |
| 9 | 4 | |
| 10 | 4 | |
| 11 | 3 | |
| 12 | 4 | |
| 13 | 7 | |
| 14 | 4 | |
| 15 | 4 | |
| 16 | 4 | |
| 17 | 4 | |
| 18 | 6 | |
| 19 | 3 | |
| 20 | 3 | |
| 21 | 3 | |
| 22 | 3 | |
| 23 | 5 | |
| 24 | 3 | |
| 25 | 2 | |
| 26 | 3 | |
| **TOTAL MARK** | | |

## Formula List

**Volume of prism** = areas of cross-section × length
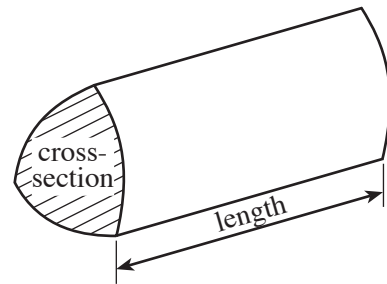
**Volume of sphere** = $\frac{4}{3}\pi r^3$

**Surface area of sphere** = $4\pi r^2$
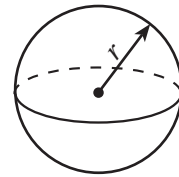
**Volume of cone** = $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** = $\pi r l$
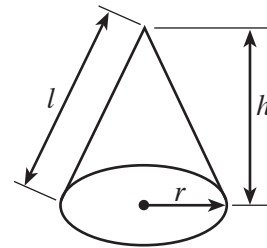
**In any triangle** $ABC$

**Sine rule** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

**Cosine rule** $a^2 = b^2 + c^2 - 2bc\cos A$

**Area of triangle** = $\frac{1}{2}ab\sin C$
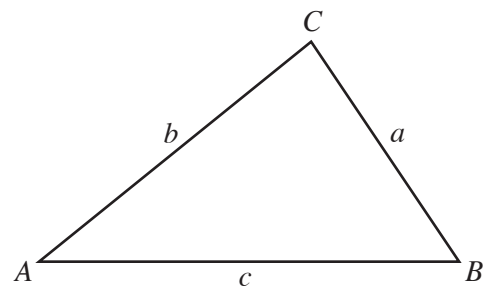
**The Quadratic Equation**

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

**Standard Deviation**

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \text{ or } s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

1. Norah and Janice share £300 in the ratio 9:1. Calculate the share of the money Norah and Janice will each receive.

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2. William goes on holiday for two weeks. During his holiday William uses $\frac{3}{4}$ of a large bottle of water every day. What is the least number of large bottles of water William needs to buy to last for the two weeks he is on holiday?

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3. The diagram shows a regular hexagon. Showing all your working, calculate the size of the angle marked $x$.



*Diagram not drawn to scale*.

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**Turn over.**
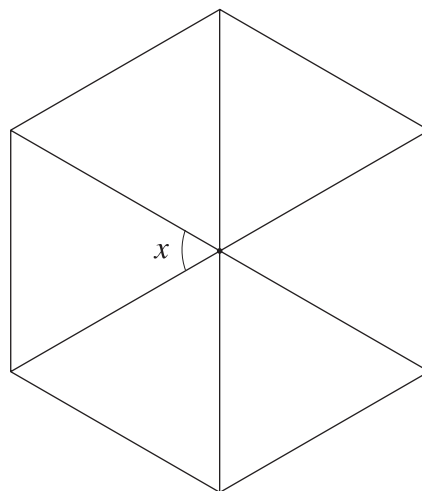
**4.** Solve **each** of the following equations.

(a)    $6x - 11 = 17 + 2x$

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(b)    $3(x - 7) = 27$

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(c)    $\dfrac{2x}{3} = 6$

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**5.** In the diagram below, *ABCD* and *DCEF* are parallelograms with $B\hat{C}E = 135°$ and $A\hat{B}C = 80°$.
Find the size of the angle marked *x*.

*A*

*B*

80°

*D*

135°

*C*

*x*

*F*

*E*
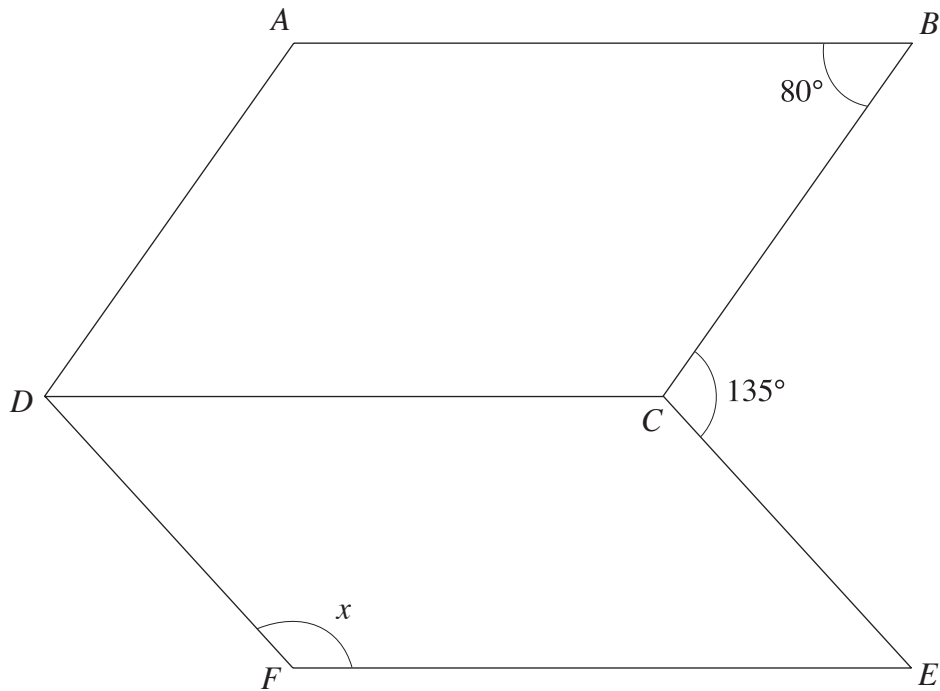
*Diagram not drawn to scale.*

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**Turn over.**

**6.** *(a)* Two towns are represented by the points *A* and *B* on the grid below. Write down the bearing of *A* from *B*.

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N

•*A*

•*B*
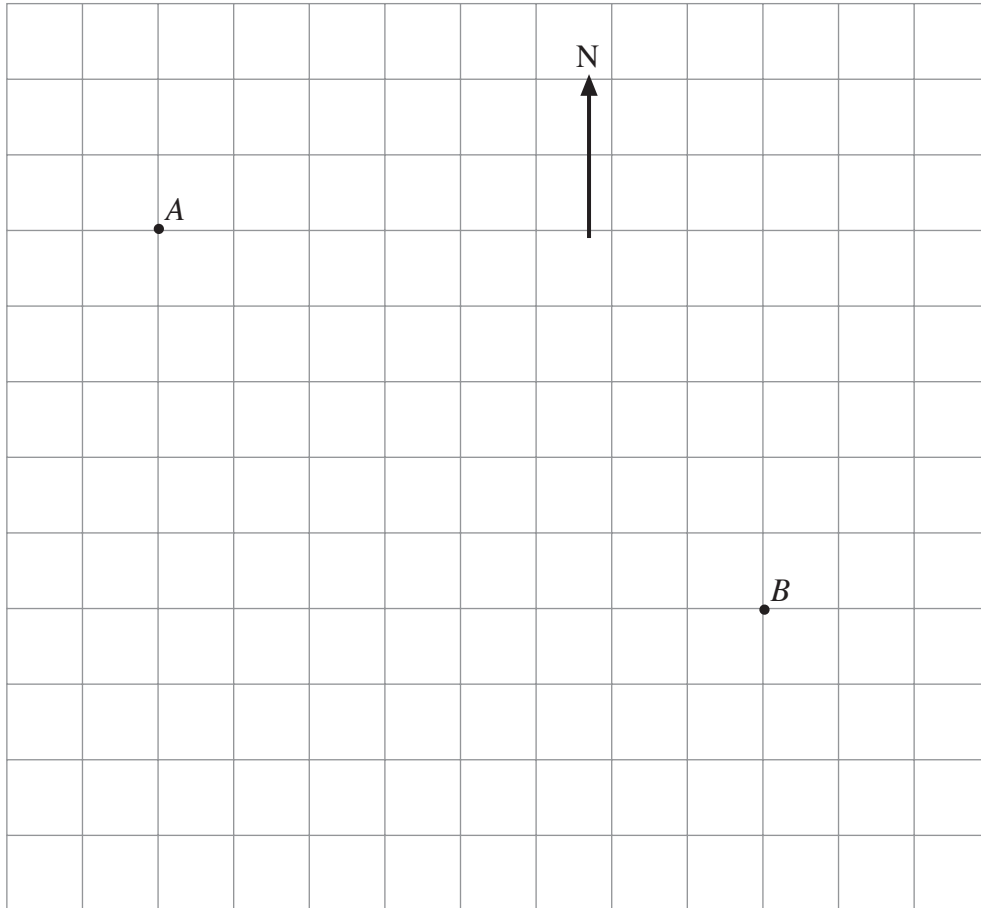
*(b)* Another town, *C*, is on a bearing of 145° from *A* and on a bearing of 243° from *B*. Plot as accurately as you can, the position of this town.

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**7.** *(a)*   In an examination, a pupil scores 165 marks out of a total of 300 marks. What percentage is this?

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*(b)*   A runner completed a run of 14 miles in 2 hour 20 minutes. Calculate, in miles per hour, the average speed of the runner.

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**8.** The table shows the pairs of scores obtained by 8 pupils on Paper 1 and Paper 2 of a mathematics examination.

| Pupil | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| Paper 1 | 18 | 36 | 88 | 66 | 98 | 46 | 52 | 84 |
| Paper 2 | 34 | 30 | 86 | 68 | 80 | 54 | 52 | 68 |

A scatter diagram for these results is shown below.
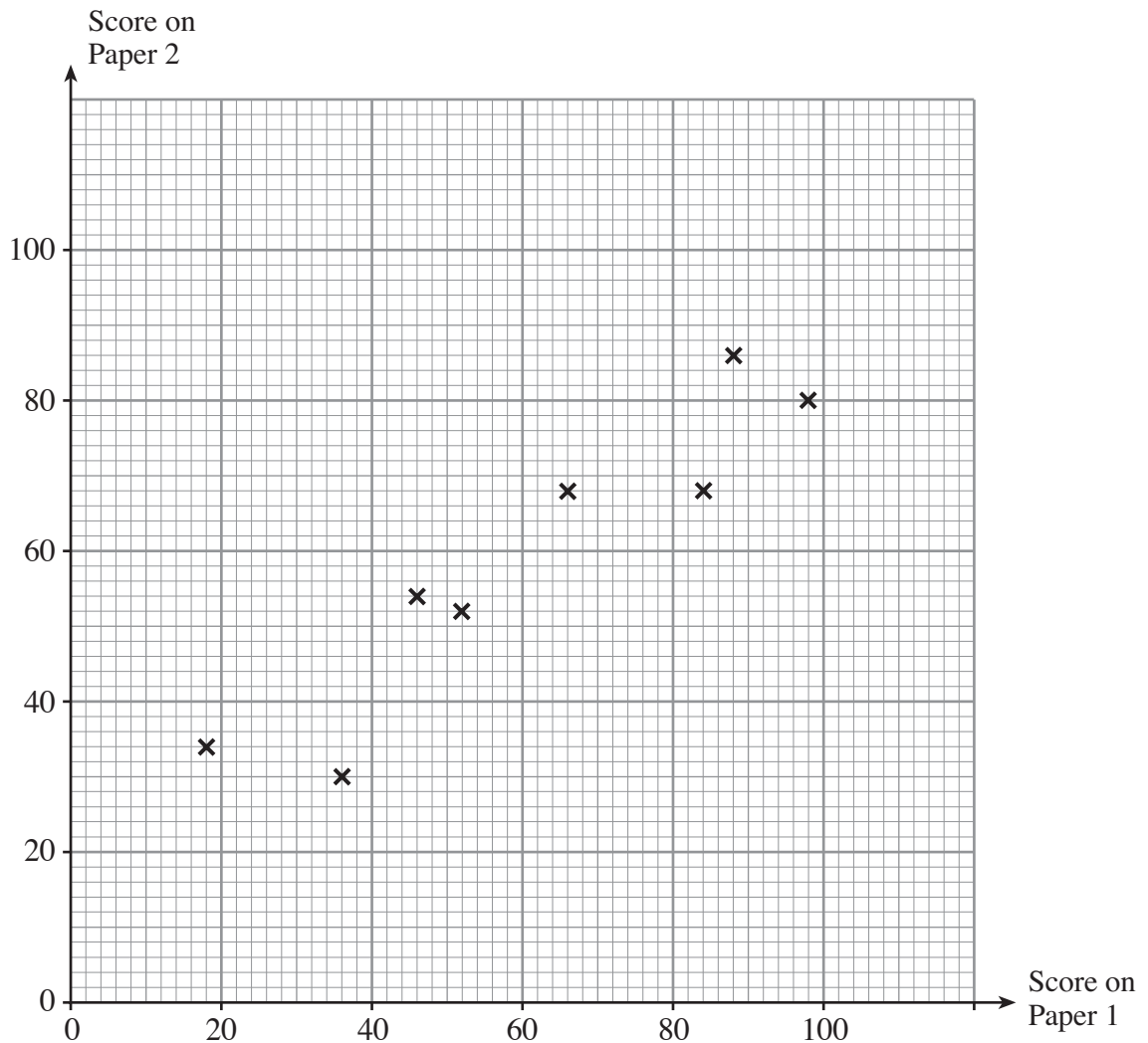


*(a)* The mean mark for the pupils on Paper 1 is 61 and the mean mark on Paper 2 is 59.
Draw a line of best fit on your scatter diagram.

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*(b)* Another pupil sat Paper 1 and was given a mark of 78, but was absent for Paper 2. Use your line of best fit to estimate the mark on Paper 2 for this pupil.

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**9.** *(a)* Express 1323 as a product of prime numbers in index form.

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*(b)* Write down the least whole number by which 1323 should be multiplied to make the result a perfect square.

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**10.** The table shows the values of $y = 2x^2 + x - 3$ for values of $x$ from $-3$ to $3$.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y = 2x^2 + x - 3$ | 12 | 3 | $-2$ | $-3$ | 0 | 7 | 18 |

*(a)* On the graph paper opposite, draw the graph of $y = 2x^2 + x - 3$ for values of $x$ between $-3$ and $3$.

[2]

*(b)* Draw the line $y = 6$ on your graph paper and write down the $x$-values of the points where your two graphs intersect.

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**For use with question 10.**

**Turn over.**
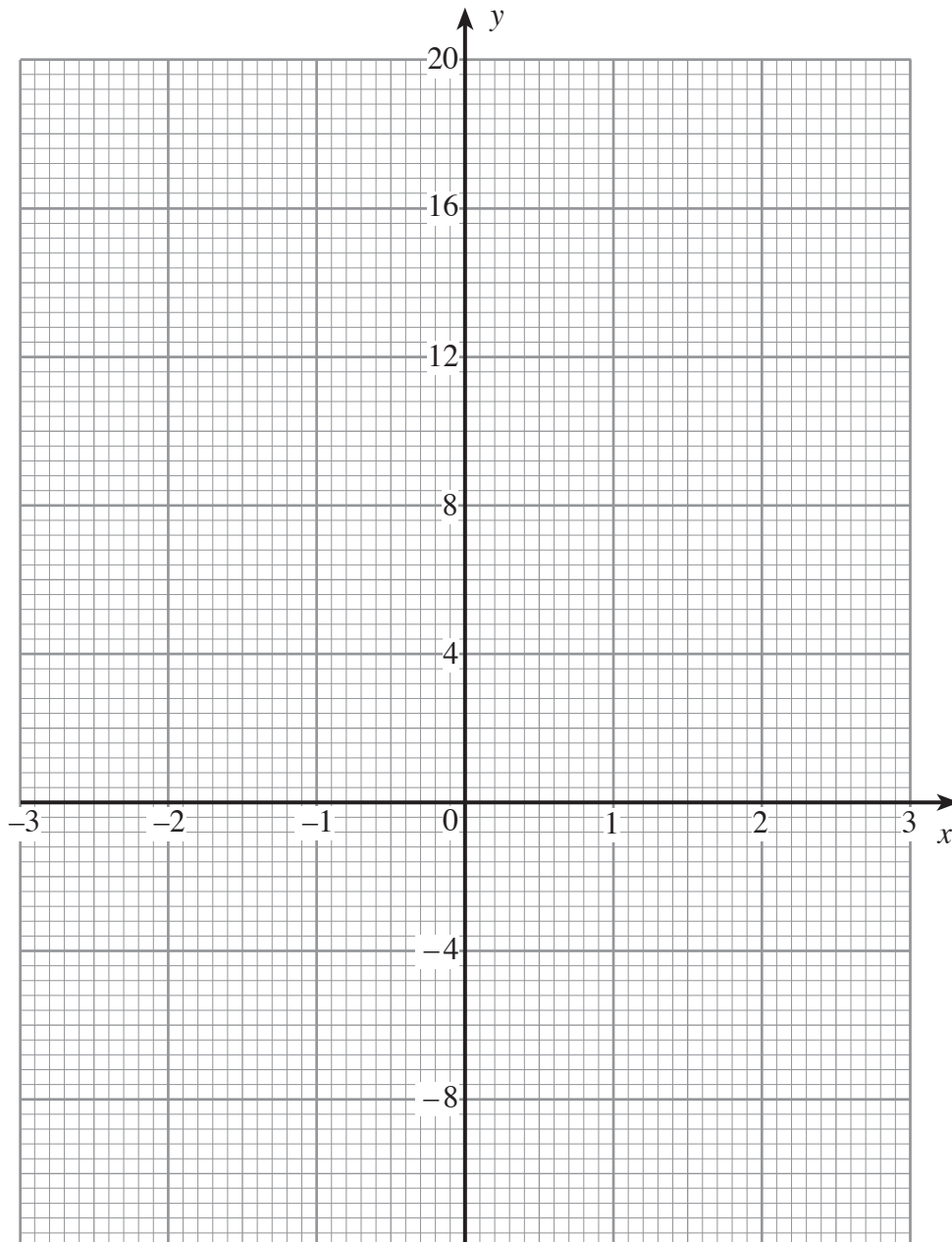
**11.** Find and shade the region of points inside the trapezium *ABCD* that satisfy **both** of the following conditions.

(i)    The points are nearer to *AD* than to *DC*.

(ii)    The points are further than 3 cm from the line *AB*.

[3]
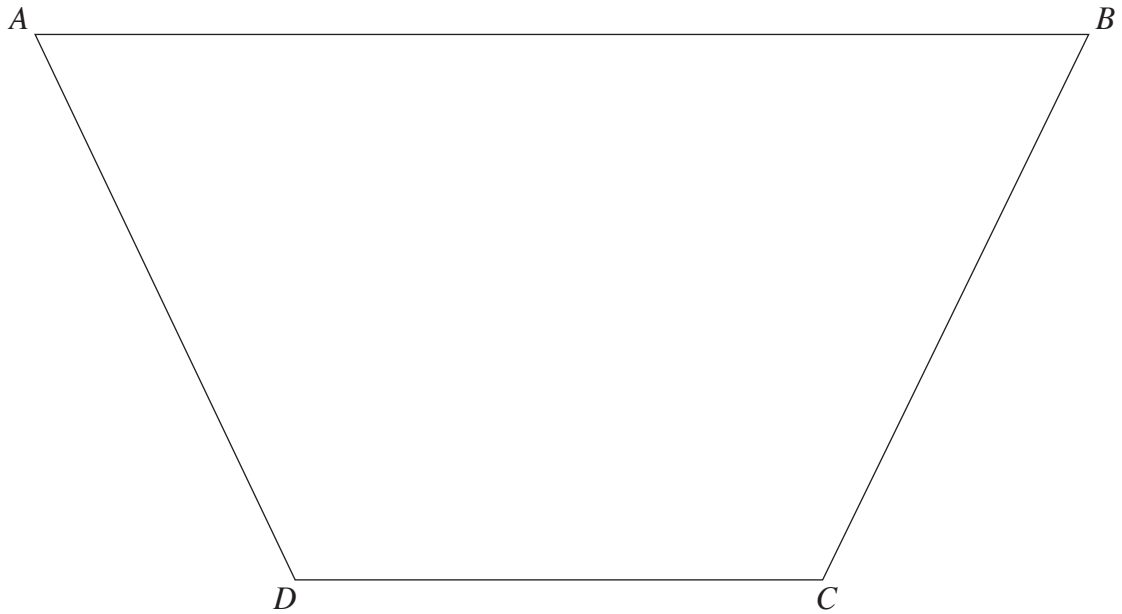
**12.** *(a)* Draw the image of the triangle **A** after a reflection in the line $y = -x$.
Label the image **B**.

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**Turn over.**
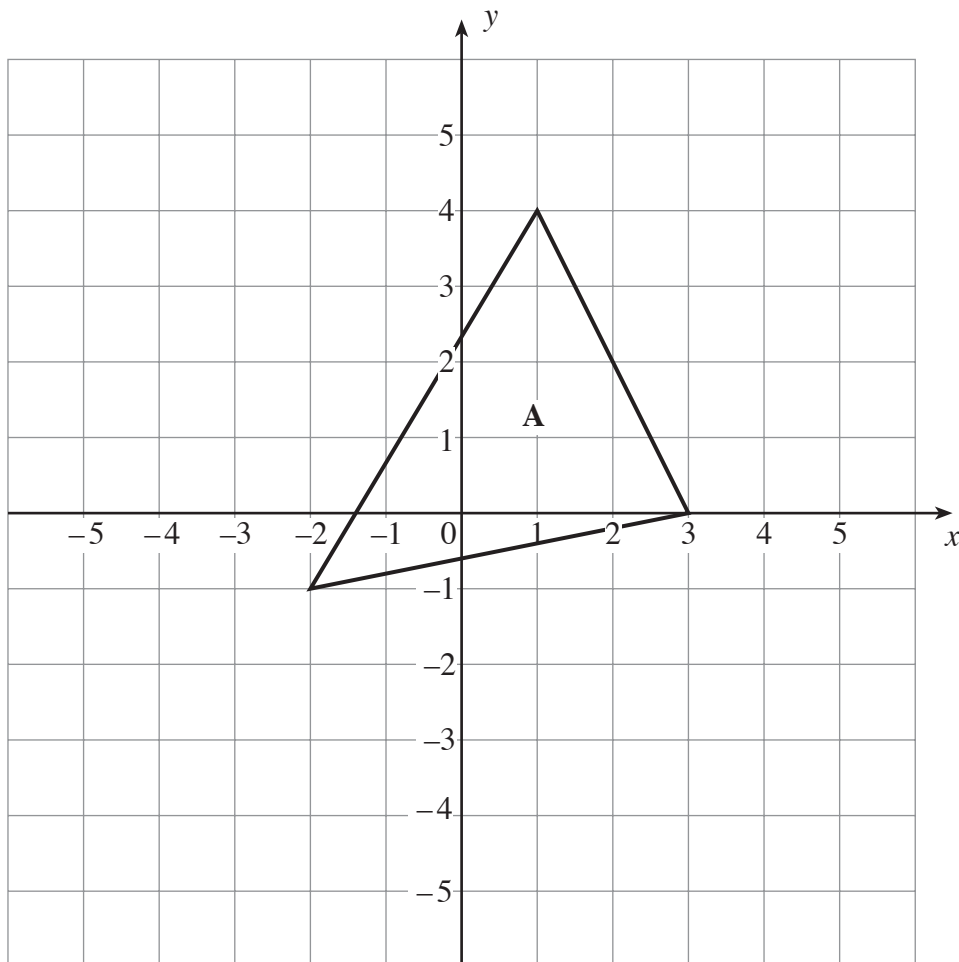
*(b)* Rotate the triangle **C** through 90° clockwise about the point $(-2, -1)$.
Label the image **D**.

[2]
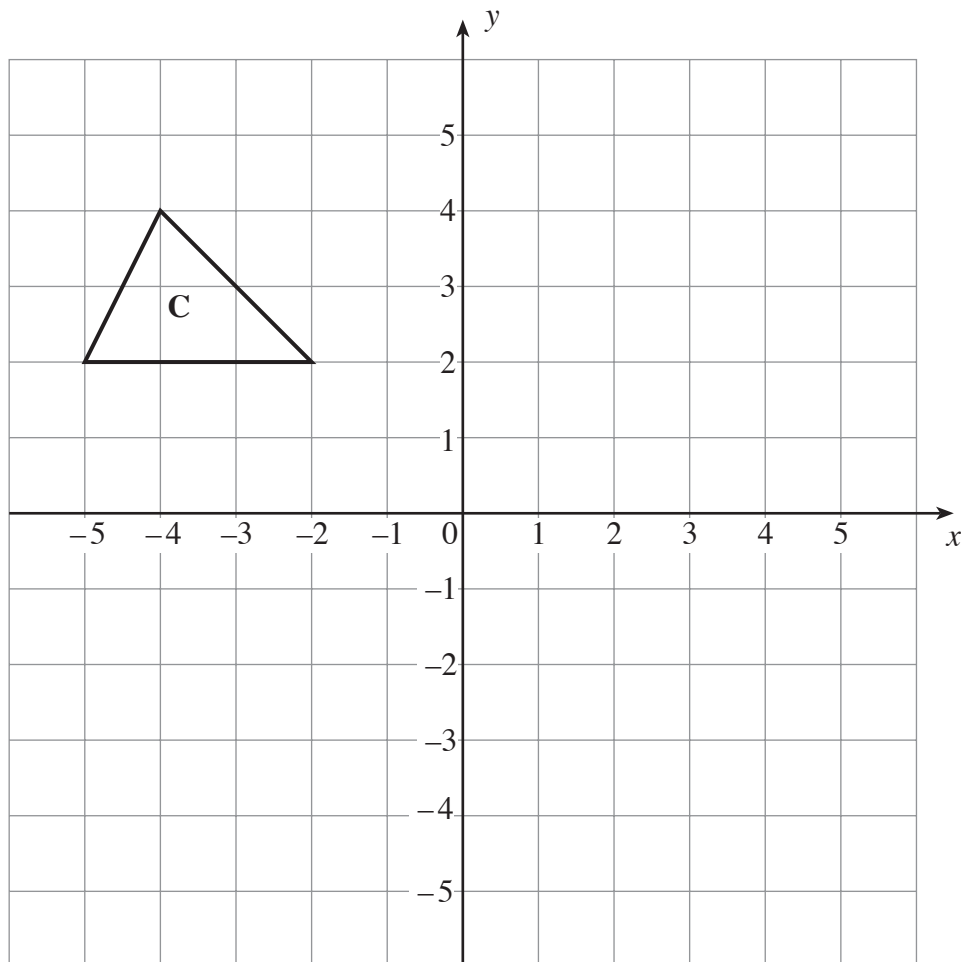
**13.** A jug has a volume of 1000 cm$^3$, measured to the nearest 50 cm$^3$.

(a) Write down the least possible value of the volume of the jug and the greatest possible value of the volume of the jug.

Least possible volume ................................. cm$^3$    Greatest possible volume ................................. cm$^3$

[2]

Water is poured from the jug into a tank of volume 52 litres measured to the nearest litre.

(b) Explain, showing all your calculations, why it is always possible to pour water from 50 full jugs into the tank without overflowing.

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**14.** (a) Simplify  $5c^6d^4 \times 4c^3d$.

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(b) Factorise  $6ab - 2a^2$.

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**15.** Solve the following equation.

$$\frac{4x-1}{3} - \frac{2x+7}{6} = \frac{5}{2}$$

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**16.** *(a)* Explain clearly why triangles *ABC* and *XYZ* are similar.
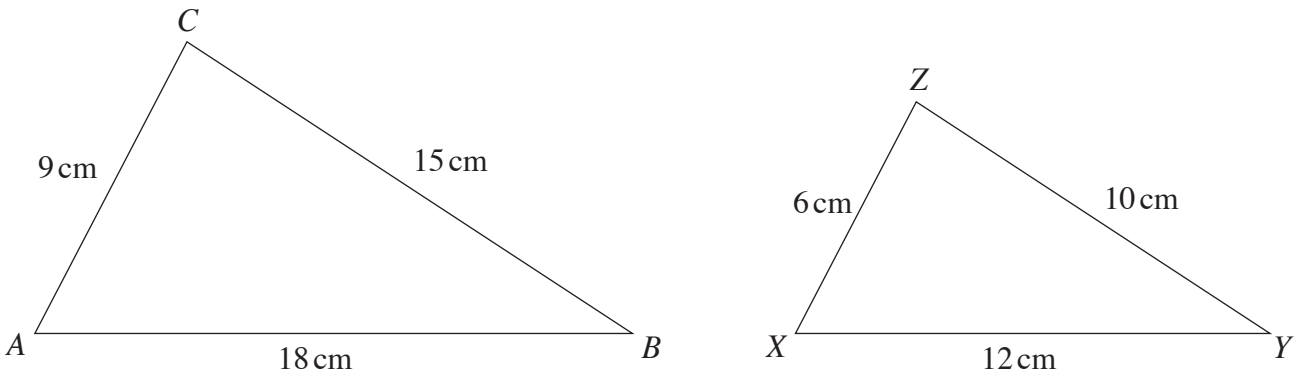
*Diagrams not drawn to scale.*

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*(b)* Triangle *PQR*, in which *PQ* = 15 cm, is similar to both triangles *ABC* and *XYZ*. Calculate the length of *QR*.

*Diagram not drawn to scale.*

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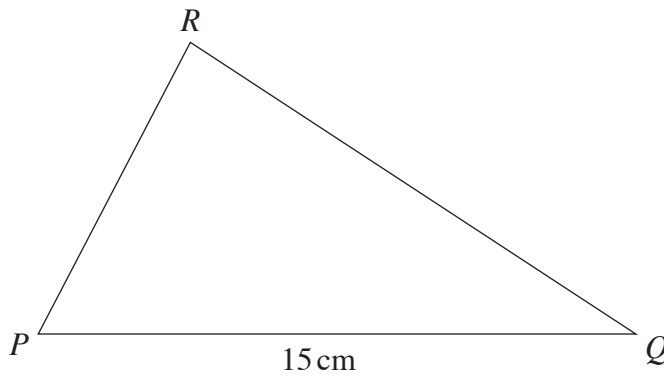
**17.** A box and a bag contain coloured balls identical except for their colour. When a ball is drawn at random from the box the probability that the ball is blue is $\frac{3}{5}$ . When a ball is drawn at random from the bag the probability that the ball is blue is $\frac{2}{9}$ .

Hywel draws one ball at random from the box and one ball at random from the bag.

*(a)* Complete the following tree diagram.

**Box**

**Bag**

Blue
ball

Blue
ball

Not a blue
ball

$\frac{2}{9}$

Blue
ball

Not a blue
ball

Not a blue
ball
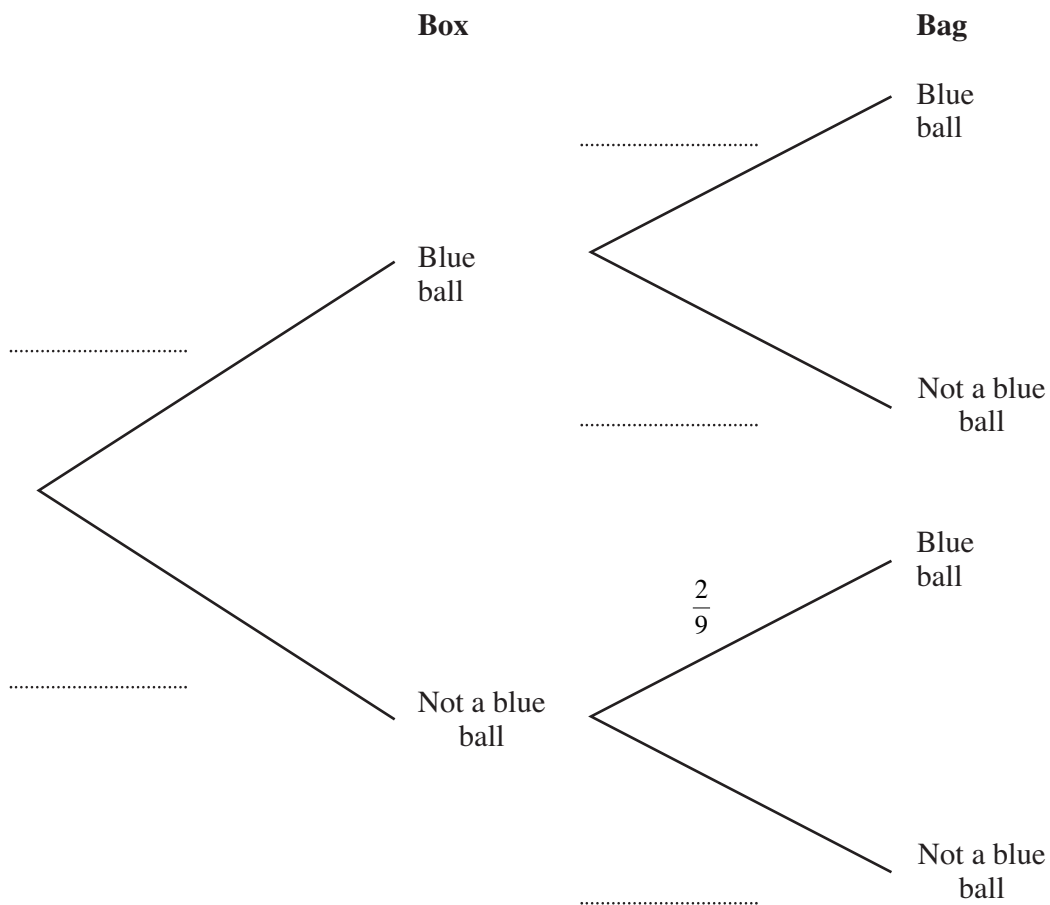
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*(b)* Calculate the probability that neither of the balls drawn is blue.

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**18.** *(a)*  Factorise $21x^2 + 4x - 1$. Hence solve $21x^2 + 4x - 1 = 0$.

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*(b)*  (i)  Factorise $49x^2 - 64$.

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(ii)  Hence simplify  $\dfrac{49x^2 - 64}{7x - 8}$ .

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**Turn over.**

**19.** The diagram shows two similar cylinders.
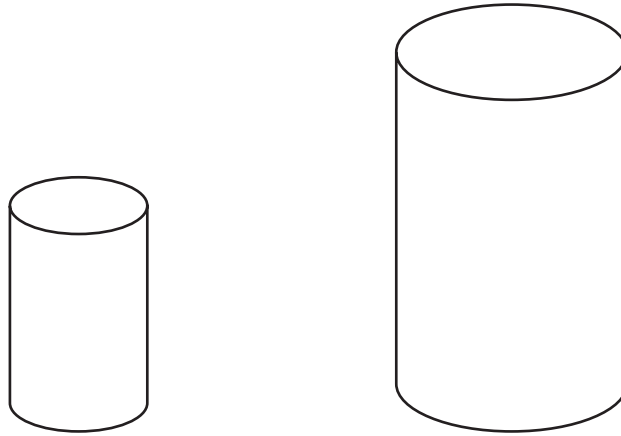


*Diagrams not drawn to scale.*

The areas of the ends of the smaller and larger cylinders are $16\,cm^2$ and $100\,cm^2$ respectively. Given that the height of the larger cylinder is $12{\cdot}5\,cm$, find the height of the smaller cylinder.

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**20.**
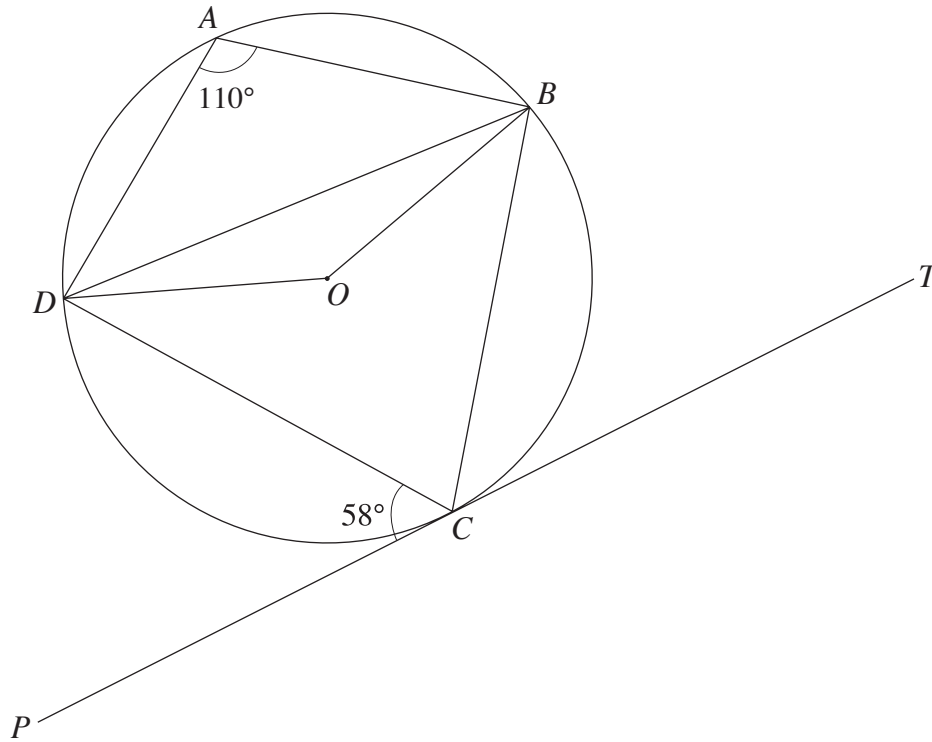


*Diagram not drawn to scale.*

Four points *A, B, C* and *D* lie on the circumference of the circle with centre *O*.
The tangent *TP* touches the circle at *C*. Given that $D\hat{C}P = 58°$ and $D\hat{A}B = 110°$, find **each** of the following angles, giving reasons for your answers.

*(a)* Reflex $D\hat{O}B$

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*(b)* $B\hat{D}C$

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**21.** Make *e* the subject of the following formula.

$$10b + 5be = 3e + 7c$$

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**22.** A sketch of $y = 9 + 2x^2 - x^3$ is shown in the diagram below for values of $x$ from $x = 0$ to $x = 3$.
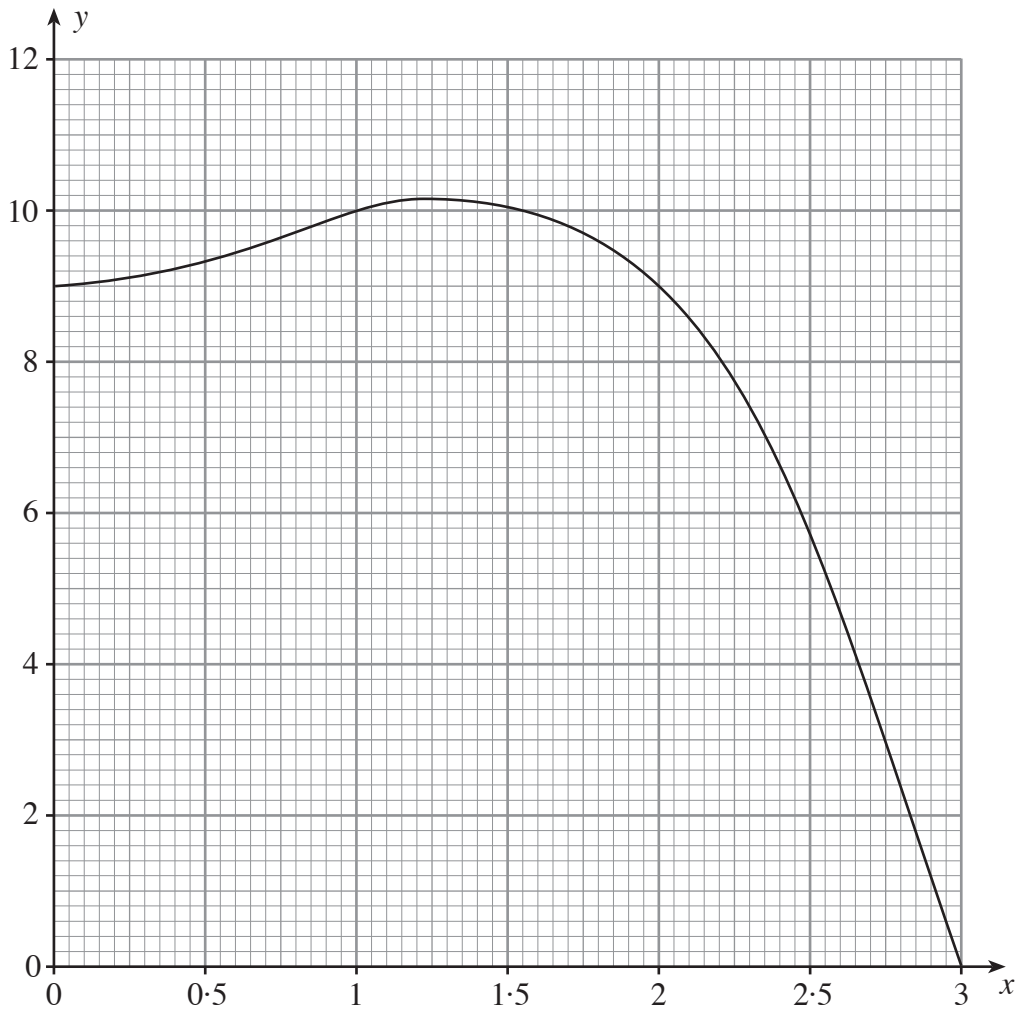


The following table of values of $y = 9 + 2x^2 - x^3$ for $x = 0, 1, 2$ and $3$.

| $x$ | 0 | 1 | 2 | 3 |
|-----|---|---|---|---|
| $y$ | 9 | 10 | 9 | 0 |

Use the values from the table and the trapezium rule with three strips to calculate an estimate for the area of the region bounded by the curve, the $x$-axis and the $y$-axis.

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**Turn over.**

**23.** Vectors **OM**, **ON** and **LN** are shown in the diagram below.



M

9**a** + 18**b**

O

−4**a** − 16**b**

N

4**a** − 8**b**

P

L
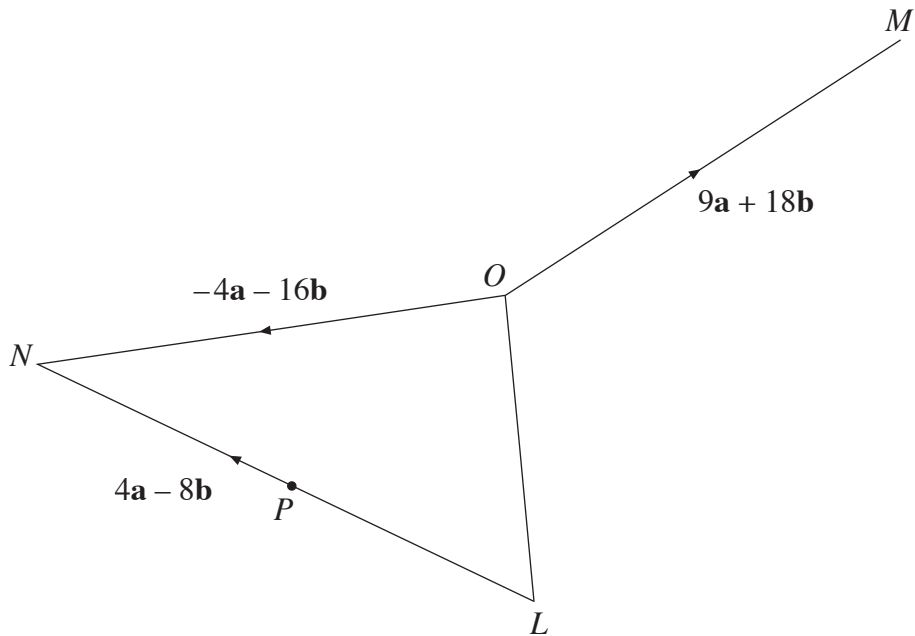
*Diagram not drawn to scale*.

Given that **OM** = 9**a** + 18**b**, **ON** = −4**a** − 16**b** and **LN** = 4**a** − 8**b** and point *P* is the mid-point of *LN*,

*(a)*   find **PO** in terms of **a** and **b** in its simplest form.

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*(b)*   Show that **PO** = *k***OM** where *k* is a constant.

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*(c)* State **two** geometrical relationships between *PO* and *OM*.

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**24.** Express $\dfrac{n}{n-3} - \dfrac{n}{n+2}$ as a single fraction in its simplest terms.

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**25.** Given that $f = \sqrt{2}, g = \sqrt{3}$ and $h = \sqrt{6}$, find in the simplest form,

*(a)* $\dfrac{fh}{g}$,

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*(b)* $fg + 2h$.

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**26.** Simplify the following expression.

$$\frac{21a^{\frac{7}{2}}(a+1)^{-\frac{3}{2}}}{7a^{-\frac{3}{2}}(a+1)^{\frac{5}{2}}}$$

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