4. Projectile Motion

Projectiles

From Unit 2:

A football of mass 500g is kicked so that it sets off horizontally across a flat field with a velocity $10ms^{-1}$.

Describe what happens:

Slows down until coming to rest

What other measures could we find?

deceleration, time of journey, distance travelled

What assumptions are we making in order to model mathematically?

acceleration is uniform
ball is a particle
no spin, bounce, etc.

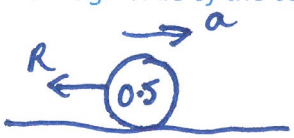
The ball comes to rest 25 seconds later.

How far did the ball travel?

$u=10, t=25, v=0, s=?$

$s=\frac{(u+v)}{2}t = 5\times 25 = 125$ metres

What is the magnitude of the constant resistive force acting on the ball?



N2L $F=ma$

$-R=0.5a$ —(1)

Need a. $v=u+at$

$0=10+25a$

$a=-\frac{10}{25}=-0.4\ ms^{-2}$

in (1) $-R=0.5\times -0.4$

$R=0.2$ Newtons.

What if the ball was kicked so that it sets off vertically from a point 1m above the ground with a velocity of $10ms^{-1}$.

Describe what happens:

decelerates uniformly under gravity until coming instantaneously to rest at its maximum height.

It then accelerates uniformly as it falls to the ground, impacting 1m below the point of projection

What other measures could we find?

MAX height, time to max height, time of flight, total dist travelled, speed of impact.

What assumptions are we making in order to model mathematically?

No air resistance, wind, etc.

How long was the ball in the air for? (~~arrow notation~~) ⇒ make all arrows same direction before substitution. Flip an arrow, change sign.

$u = 10\uparrow$, $a = -9.8\downarrow$, $s = -1m\downarrow$, $t = ?$

~~$u = -10\downarrow$~~

$s = ut + \frac{1}{2}at^2$

$-1 = 10t - 4.9t^2$

$1 = -10t + 4.9t^2 \Rightarrow 4.9t^2 - 10t - 1 = 0$

$t = 2.1$ sec (context allows ignore $t = -0.1$)

What was the maximum height of the ball?

at max height $v = 0$, $u = 10\uparrow$, $a = 9.8\downarrow$, $s = ?\uparrow$

$a = -9.8\uparrow$

$v^2 = u^2 + 2as$

$0^2 = 10^2 + 2 \times -9.8 \times s$

$19.6s = 100$

$s = \frac{100}{19.6} = 5.1$ m above point of projection

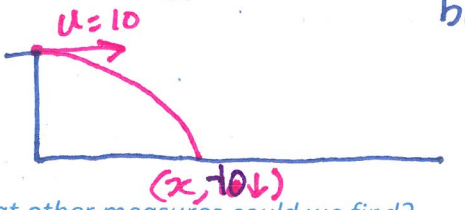
$+1$ m

6.1 m above ground

Unit 4:

Now the ball is kicked so that it sets off horizontally from a point 1m above the ground with a velocity $10ms^{-1}$.

Describe what happens:



accelerates towards

ball ~~falls to~~ ground, impacting x metres from the point of projection and 1m below.

What other measures could we find?

x, speed of impact, time of impact

What assumptions are we making in order to model mathematically?

No air resistance, etc.

In order to investigate the motion of a projectile, the horizontal and vertical motions should be considered separately.

The horizontal velocity of a projectile is constant since there is never any acceleration in the horizontal direction.

The vertical velocity on the other hand is subject to the acceleration due to gravity, and the equations of motion can be applied.

These problems are often represented using **i** and **j**, the unit vectors in a horizontal and vertical direction.

eg1 A particle is projected horizontally at 10 ms^{-1} from a point 1m above a horizontal surface. Find the time taken by the particle to reach the surface and the horizontal distance travelled.

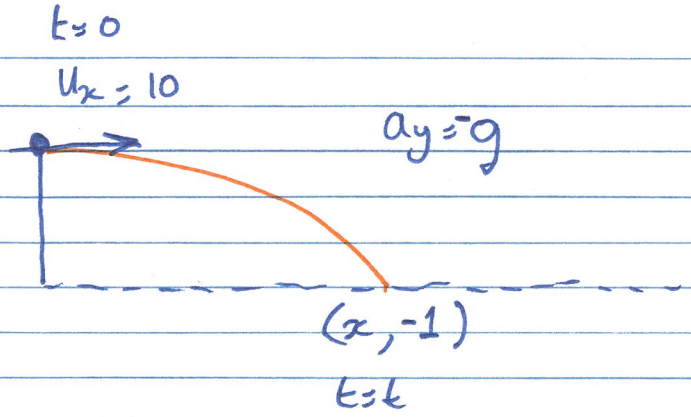
eg2 A particle is projected horizontally with a speed of 14.7 ms^{-1}. Find the horizontal and vertical displacements of the particle from the point of projection, after 2 seconds. Find also how far the particle then is from the point of projection.

eg3 A particle is projected horizontally from a point 44.1 m above a horizontal plane. The particle hits the plane at a point which is, horizontally, 39 m from the point of projection. Find the initial speed of the particle.

eg4 A particle is projected horizontally with a velocity of 39.2 ms^{-1}. Find the horizontal and vertical components of the velocity of the particle 3 seconds after projection. Find also, the speed and direction of motion of the particle at this time.

Exercise 4.1 Evens (except Q8 & 14)

Eg 1



x: $s_x = x$, $a_x = 0$, $v_x = u_x$, $t = t$, $u_x = 10$

y: $s_y = -1$, $a_y = -9.8$, $v_y = ?$, $t = t$, $u_y = 0$

Use $s = ut + \frac{1}{2}at^2$

$$\begin{pmatrix} x \\ -1 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2$$

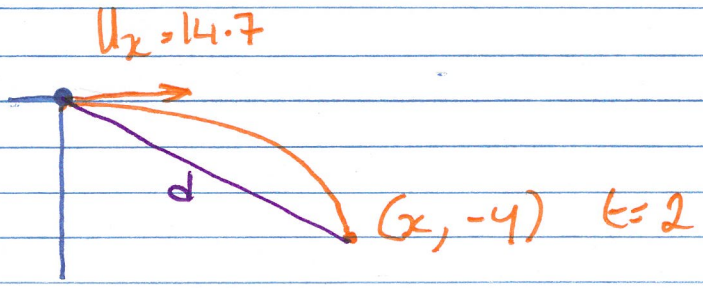
x: $x = 10t$ — ①

y: $-1 = -4.9t^2$ — ②

From ② $t = \sqrt{\frac{-1}{-4.9}} = +0.4517\ldots = 0.45$ sec {2 s.f.}

in ① $x = 10(0.4517\ldots) = 4.5$ metres

Eg 2



x: $s_x = x$, $a_x = 0$, $u_x = 14.7$, $t = 2$

y: $s_y = -y$, $a_y = -9.8$, $u_y = 0$, $t = 2$

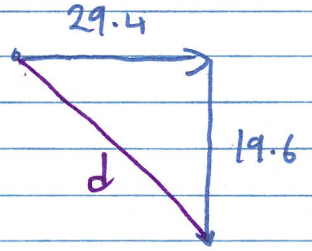
Use $s = ut + \frac{1}{2}at^2$

$$\begin{pmatrix} x \\ -4 \end{pmatrix} = \begin{pmatrix} 14.7 \\ 0 \end{pmatrix} 2 + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \cdot 2^2$$

x: $x = 14.7 \times 2 = 29.4\text{m}$

y: $-y = -4.9 \times 4 = 19.6\text{m}$

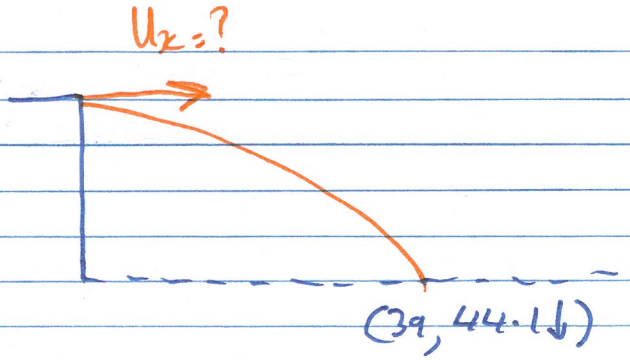
$\therefore$ particle is 29.4m horizontally 20 metres {2 s.f} vertically below



$d = \sqrt{29.4^2 + 19.6^2} = 35.3344\ldots$

$= 35\text{m}$ to 2 s.f.

(eg)



$(\rightarrow)$ $s_x = 39$, $u_x = ?$, $a_x = 0$, $t = ?$

$(\uparrow)$ $s_y = -44.1$, $u_y = 0$, $a_y = -9.8$, $t = ?$

$$\begin{pmatrix} 39 \\ -44.1 \end{pmatrix} = \begin{pmatrix} u_x \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2$$

$(\rightarrow)$ $39 = u_x t$ — ①

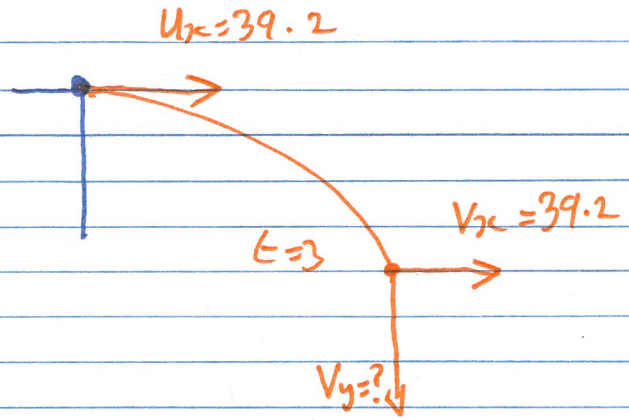
$(\uparrow)$ $-44.1 = 0 - 4.9t^2$

$$t = \sqrt{\frac{-44.1}{-4.9}} = 3 \text{ sec}$$

in ① $39 = u_x \times 3$

$u_x = 13 \text{ ms}^{-1}$

Eg 4

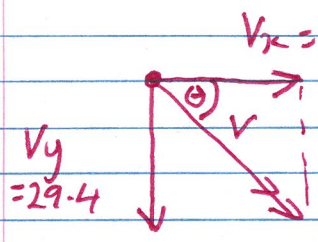


$$v = u + at$$

$$\begin{pmatrix} 39.2 \\ -v_y \end{pmatrix} = \begin{pmatrix} 39.2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 3$$

$(\downarrow)$ $\quad -v_y = -9.8 \times 3$

$$v_y = 29.4 = 29 \text{ ms}^{-1} \quad (2 \text{ s.f.})$$



$$v = \sqrt{29.4^2 + 39.2^2} = 49 \text{ ms}^{-1}$$

$$\theta = \tan^{-1}\left(\frac{29.4}{39.2}\right) = 36.869\ldots$$

$$= 37^\circ$$

Exercise 4.1

1 A particle is projected horizontally at 15 m s^{-1} from a point 60 m above a horizontal surface. Find how long it takes before the particle strikes the surface and the horizontal distance travelled in that time.

2 A particle is projected horizontally at 20 m s^{-1}. It strikes a horizontal surface after travelling 50 m horizontally. Find the height of the point of projection above the surface.

3 A stone is thrown horizontally from a window 3.6 m above horizontal ground. It hits the ground after travelling 10 m horizontally. Find the speed of projection.

4 A stone is thrown horizontally at 25 m s^{-1} from the edge of a vertical cliff which is 50 m high. By modelling the stone as a particle calculate its distance from the foot of the cliff when it enters the water.

5 A bird flying horizontally at 30 m s^{-1} at a height of 35 m drops a stone. Model the stone as a particle and hence determine how long it takes the stone to reach the ground. What is the horizontal distance travelled by the stone during this time?

6 A stone is thrown horizontally and hits the ground 7 m from the thrower 0.5 s later. Find the speed and height of projection.

7 At time $t = 0$ a particle is projected with a velocity $4\mathbf{i}$ m s^{-1} from a point with position vector $24\mathbf{j}$ m where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the horizontal and upward vertical directions respectively. Find the position vector of the particle after 1.5 s.

8 At time $t = 0$ a particle is projected horizontally from a point with position vector $(x\mathbf{i} + y\mathbf{j})$ m. Two seconds later it passes through the point with position vector $(8\mathbf{i} + 2\mathbf{j})$ m. If the speed of projection is 4 m s^{-1} find the values of x and y.

9 A child throws a ball horizontally at a wall 4 m away. The ball strikes the wall 0.4 m below the level of projection. Calculate the speed with which it was projected.

10 A particle slides along a horizontal table and over the edge. Given that the table is 1 m high and the speed of the particle on the table is 14 m s^{-1} calculate (a) the vertical velocity with which the particle hits the floor (assumed horizontal) (b) the speed and direction of motion of the particle when it hits the floor.

11 A boy standing on a garage roof throws a ball horizontally with a speed of 15 m s^{-1}. The ball just clears a wall 1.5 m high 10 m away. Model the ball as a particle and hence calculate the height of projection.

12 A particle is projected horizontally at 25 m s^{-1}. Find the horizontal and vertical components of the particle's velocity 1.5s later. Find the speed of the particle and its direction of motion at that time.

13 At time $t = 0$ a particle is projected with a velocity $4\mathbf{i}$ m s^{-1} from a point with position vector $(6\mathbf{i} + 2\mathbf{j})$ m. Find its position vector after 2 seconds.

Numerical Answers

1 3.50 s, 52.5 m **2** 80.6 m
3 11.7 m s^{-1} **4** 79.9 m
5 2.67 s, 80.2 m **6** 14 m s^{-1}, 1.23 m
7 $(6\mathbf{i} + 13.0\mathbf{j})$ m **8** 0, 21.6
9 14 m s^{-1}
10 (a) 4.43 m s^{-1} (b) 14.7 m s^{-1}, 17.6° below horizontal
11 3.68 m
12 25 m s^{-1}, 14.7 m s^{-1}, 29 m s^{-1}, 30.5° below horizontal
13 $(14\mathbf{i} - 17.6\mathbf{j})$ m
14 6 m s^{-1}, 2 s **15** 12.3 m **16** 9.49 m
17 16.8 m s^{-1}, 4.97 m **18** Yes, 0.475 m

14 At time $t = 0$ a particle is projected horizontally from a point A with position vector $(5\mathbf{i} + 30\mathbf{j})$ m. It passes through point B with position vector $(17\mathbf{i} + 10.4\mathbf{j})$ m. Find its speed of projection and the time taken to travel from A to B.

15 An aeroplane is flying horizontally at 400 m s^{-1}. A package is released and travels a distance of 2000 m horizontally before hitting the ground. Model the package as a particle and hence find the height of the aeroplane above the ground.

16 A batsman hits a ball horizontally with a speed of 21 m s^{-1} at 1 m above the ground. Find the distance travelled horizontally by the ball before it reaches the ground.

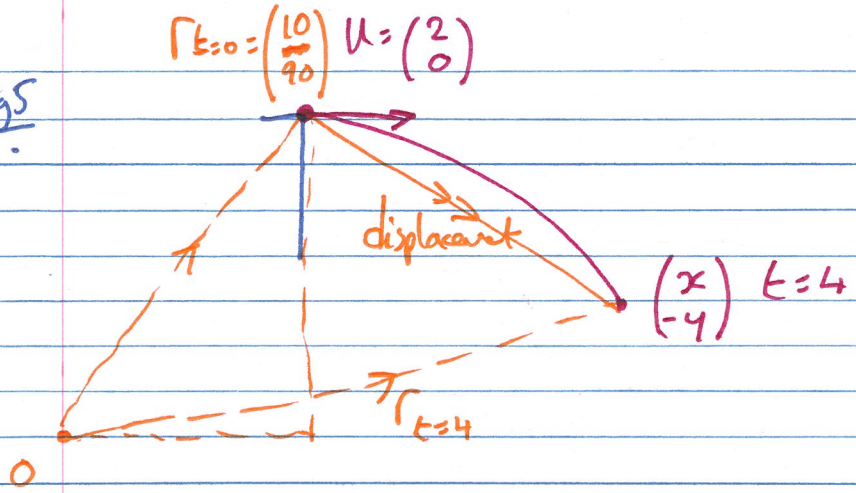
17 A child throws a ball horizontally from a window 5 m above horizontal ground. The ball just clears a vertical wall 2.5 m high and 12 m from the house. By modelling the ball as a particle calculate the speed of projection. The ball hits the ground at Q. Calculate the distance of Q from the wall.

18 A tennis ball is served horizontally at a speed of 24 m s^{-1} from a height of 2.7 m. The net is 1 m high and 12 m horizontally from the server. Model the ball as a particle and hence determine whether the ball clears the net and if so by what distance.

eg5 At time t = 0 a particle is projected with a velocity of $2\mathbf{i}$ ms^{-1} from a point with position vector $(10\mathbf{i} + 90\mathbf{j})$ m. Find the position vector of the particle when t = 4s.

Exercise 4.1 Q's 7, 8, 13, 14

Eg 5



$$r_{t=4} = r_{t=0} + \text{displacement}$$

$$= \begin{pmatrix} 10 \\ 90 \end{pmatrix} + \left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \right]$$

$$= \begin{pmatrix} 10 \\ 90 \end{pmatrix} + \left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} 4 + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} 4^2 \right]$$

$$= \begin{pmatrix} 10 \\ 90 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -78.4 \end{pmatrix}$$

$$r_{t=4} = \begin{pmatrix} 18 \\ 11.6 \end{pmatrix}$$

$\therefore r_{t=4} = 18\underline{i} + 12\underline{j}$ {2 s.f.}

Ex 4A 4.1

① $u_x = 15$

$u_x = 15$ $\quad a_x = 0$ $\quad s_x = x$ $\quad t = t$

$u_y = 0$ $\quad a_y = 9.8\downarrow$ $\quad s_y = 60\downarrow$

$$\begin{pmatrix} x \\ 60 \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

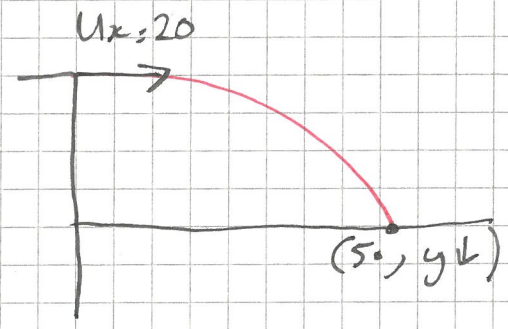
horizontally $\quad x = 15t \quad -①$

Vertically $\quad 60 = 4.9t^2$

$$t = \sqrt{\frac{60}{4.9}} = 3.5 \text{ sec}$$

in ① $\quad x = 15 \times 3.5 = 52.5 \text{ m}$

② $u_x = 20$



$u_x = 20$ $\quad a_x = 0$ $\quad s_x = 50$ $\quad t = t$

$u_y = 0$ $\quad a_y = 9.8\downarrow$ $\quad s_y = y\downarrow$

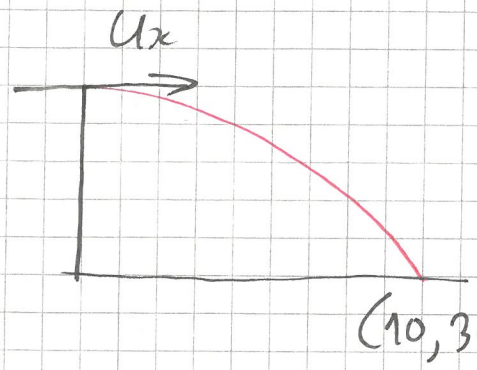
$$\begin{pmatrix} 50 \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

$$50 = 20t$$

$$t = 2.5 \text{ sec}$$

$$y = 4.9(2.5)^2 = 30.6 \text{ metres}$$

(3)



$u_x = ?$ $\quad a_x = 0$ $\quad s_x = 10$ $\quad t = t$

$u_y = 0$ $\quad a_y = 9.8\downarrow$ $\quad s_y = 3.6\downarrow$

$$\begin{pmatrix} 10 \\ 3.6 \end{pmatrix} = \begin{pmatrix} u_x \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

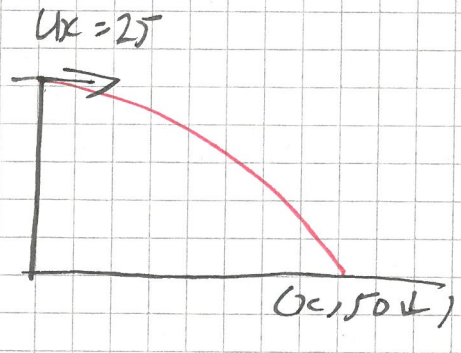
$10 = u_x t \quad -(1)$

$3.6 = 4.9 t^2$

$t = \sqrt{\frac{3.6}{4.9}} = 0.86 \text{ sec}$

$u(1) \quad u_x = \frac{10}{0.86} = 11.7 \text{ ms}^{-1}$

(4)



$$\begin{pmatrix} x \\ 50 \end{pmatrix} = \begin{pmatrix} 25 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

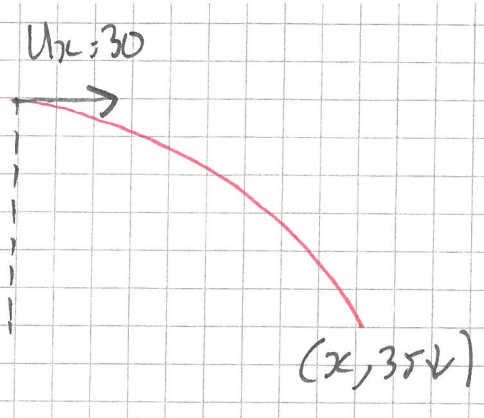
$x = 25t \quad -(1)$

$50 = 4.9 t^2$

$t = \sqrt{\frac{50}{4.9}} = 3.19 \text{ sec}$

$u(1) \quad x = 25 \times 3.19 = 79.9 \text{ m}$

(5)



$$\begin{pmatrix} x \\ 35 \end{pmatrix} = \begin{pmatrix} 30 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

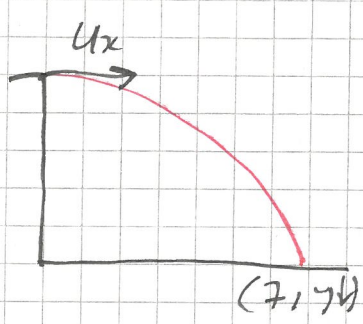
$x = 30t$ — ①

$35 = 4.9t^2$

$t = \sqrt{\frac{35}{4.9}} = 2.67 \text{ sec}$

sub ① $x = 30 \times 2.67 = 80.2 \text{m}$

(6)



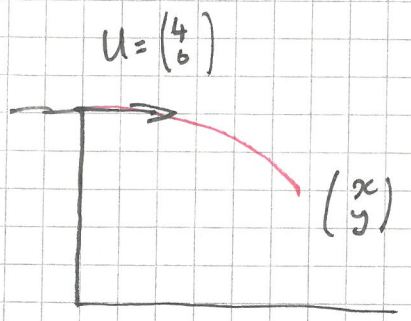
$$\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} u_x \\ 0 \end{pmatrix} 0.5 + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} 0.5^2$$

$7 = 0.5 u_x + 0$

$u_x = \frac{7}{0.5} = 14 \text{ ms}^{-1}$

$y = 4.9 \times 0.25 = 1.225 \text{m}$

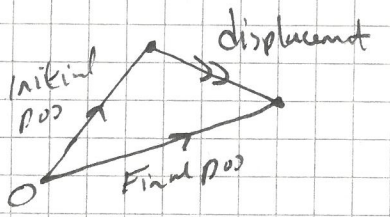
(7)



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} 1.5 + \frac{1}{2}\begin{pmatrix} 0 \\ 9.8 \end{pmatrix} 1.5^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -11.025 \end{pmatrix} = \begin{pmatrix} 6 \\ -11.025 \end{pmatrix}$$ below point of projection

Now relative to origin

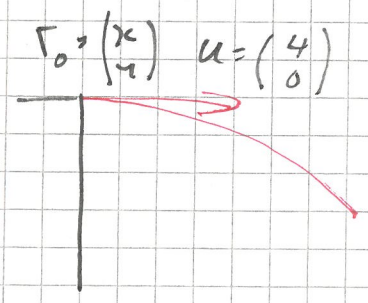


Final position = Initial pos + displacement

$$= \begin{pmatrix} 0 \\ 24 \end{pmatrix} + \begin{pmatrix} 6 \\ -11.025 \end{pmatrix} = \begin{pmatrix} 6 \\ 13.0 \end{pmatrix}$$

ie $6\underline{i} + 13.0\underline{j}$

(8).



when $t = 2$ $$\begin{pmatrix} 8 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \left[\begin{pmatrix} 4 \\ 0 \end{pmatrix} 2 + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 2^2 \right]$$

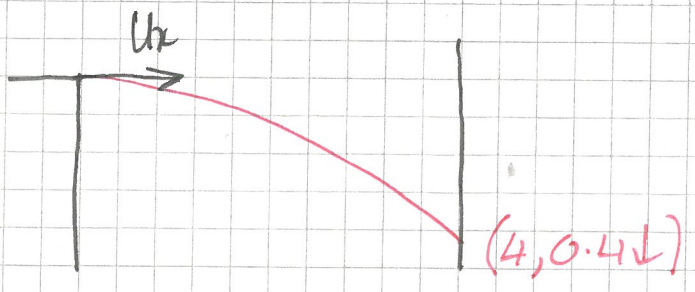
Final Pos = Initial pos + displacement

$$\begin{pmatrix} 8 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -19.6 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -19.6 \end{pmatrix} = \begin{pmatrix} 0 \\ 21.6 \end{pmatrix}$$

$x = 0$ $y = 21.6$

(a)



$$\begin{pmatrix}4\\0.4\end{pmatrix} = \begin{pmatrix}u_x\\0\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}0\\9.8\end{pmatrix}t^2$$

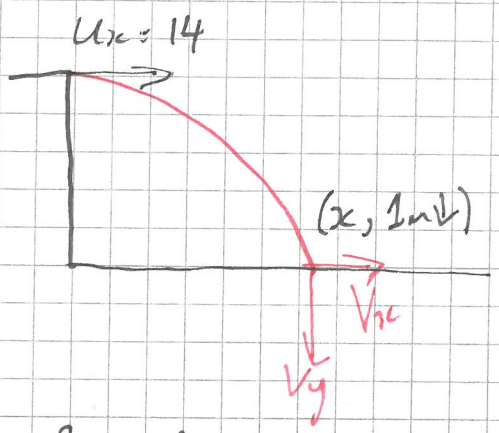
$$4 = u_x t \quad -(1)$$

$$0.4 = 4.9t^2$$

$$t = \sqrt{\frac{0.4}{4.9}} = 0.29 \text{ sec}$$

sub (1) $u_x = \frac{4}{0.29} = \underline{14 \text{ ms}^{-1}}$

(b)



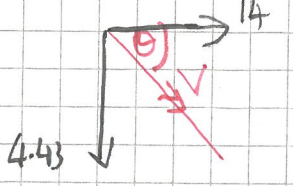
$$V^2 = u^2 + 2as$$

$$\begin{pmatrix}V_x\\V_y\end{pmatrix}^2 = \begin{pmatrix}14\\0\end{pmatrix}^2 + 2\begin{pmatrix}0\\9.8\end{pmatrix}\begin{pmatrix}x\\1\end{pmatrix}$$

(a) Vertical compont $\quad V_y^2 = 0^2 + 19.6$

$$V_y = \sqrt{19.6} = 4.43 \text{ ms}^{-1}$$

(b) No horizontal accel $\therefore V_x = u_x = 14$

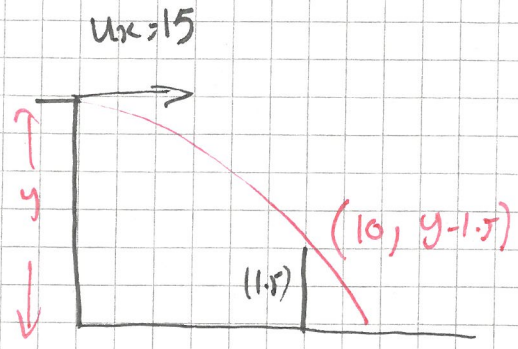


$$V^2 = \sqrt{14^2 + 4.43^2} = 14.7 \text{ ms}^{-1}$$

$$\Theta = \tan^{-1}\left(\frac{4.43}{14}\right) = 17.6°$$

ie particle travelling @ 14.7ms at an angle 17.6° below horizontal on impact.

(11)



$$\begin{pmatrix} 10 \\ y-1.5 \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

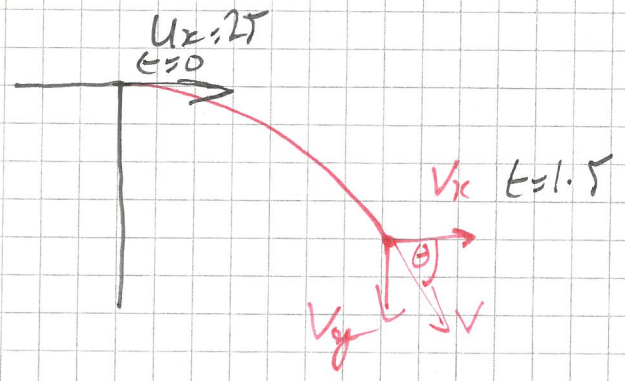
$10 = 15t$

$t = \frac{2}{3}$ sec

$$y - 1.5 = 4.9\left(\frac{2}{3}\right)^2$$

$$y = 4.9 \times \frac{4}{9} + 1.5 = 3.68\text{M}$$

(12)



$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} 25 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} 1.5$$

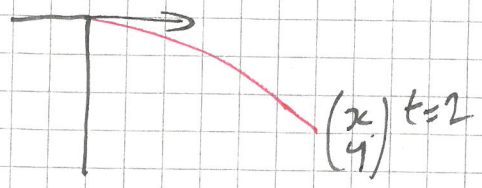
$V_x = 25$

$V_y = 14.7$

$$V = \sqrt{25^2 + 14.7^2} = 29 \text{ ms}^{-1}$$

$$\theta = \tan^{-1}\left(\frac{14.7}{25}\right) = 30.5^\circ \text{ below horizontal}$$

(13) $r_0 = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $u = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$



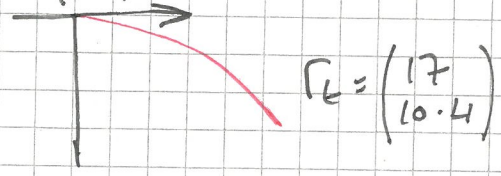
Final pos = initial pos + displacement

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \left(\begin{pmatrix} 4 \\ 0 \end{pmatrix} 2 + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 2^2 \right)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -19.6 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ -17.6 \end{pmatrix} \quad r_2 = 14\underline{i} - 17.6\underline{j}$$

(14) $r_0 = \begin{pmatrix} 5 \\ 30 \end{pmatrix}$ $u = \begin{pmatrix} u_x \\ 0 \end{pmatrix}$



$r_t = r_0 + \text{disp}$

$$\begin{pmatrix} 17 \\ 10.4 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \end{pmatrix} + \left[\begin{pmatrix} u_x \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2 \right]$$

$$\begin{pmatrix} 17 \\ 10.4 \end{pmatrix} - \begin{pmatrix} 5 \\ 30 \end{pmatrix} = \begin{pmatrix} u_x t \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -4.9t^2 \end{pmatrix}$$

$$12 = u_x t \quad -(1)$$

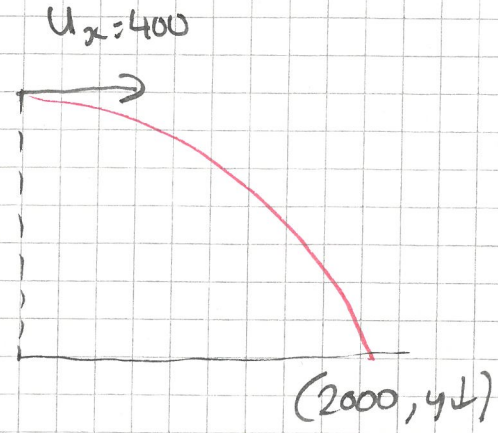
$$-19.6 = -4.9t^2$$

$$t = \sqrt{\frac{19.6}{4.9}} = 2 \text{ sec}$$

in (1) $12 = u_x \times 2$

$u_x = 6 \text{ ms}^{-1}$

(15)



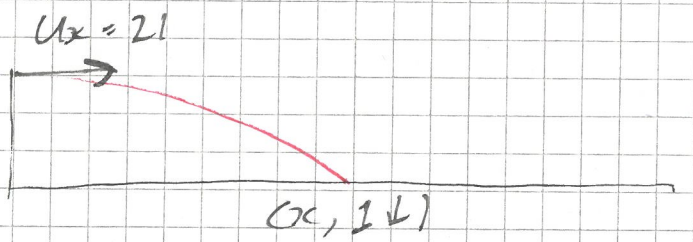
$$\begin{pmatrix} 2000 \\ 4 \end{pmatrix} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

$2000 = 400t$ — (1)

$t = 5$ sec

$y = 4.9(5)^2 = 122.5$ meters

(16)



$$\begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} 21 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$$

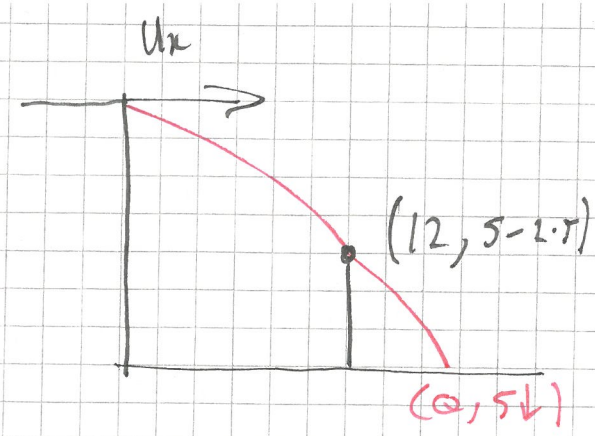
$x = 21t$ — (1)

$1 = 4.9t^2$

$t = \sqrt{\frac{1}{4.9}} = 0.45$ sec

in (1) $x = 21 \times 0.45 = 9.5$ meters

(17)



@ Wall: $\begin{pmatrix} 12 \\ 2.5 \end{pmatrix} = \begin{pmatrix} u_x \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$

$12 = u_x t$ — (1)

$2.5 = 4.9 t^2$

$t = \sqrt{\frac{2.5}{4.9}} = 0.71 \text{ sec}$

in (1) $u_x = \frac{12}{0.71} = 16.8 \text{ ms}^{-1}$

@ Q: $\begin{pmatrix} Q \\ 5 \end{pmatrix} = \begin{pmatrix} 16.8 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} t^2$

$Q = 16.8 t$ — (2)

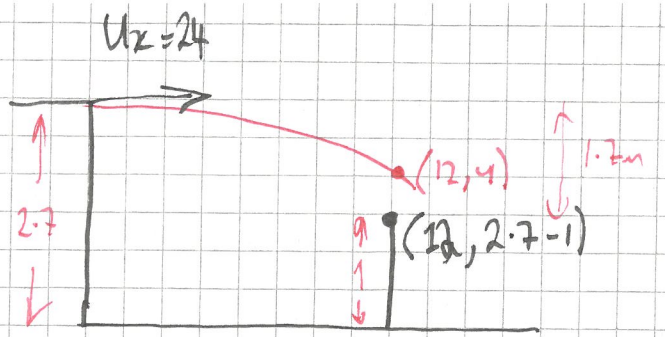
$5 = 4.9 t^2$

$t = \sqrt{\frac{5}{4.9}}$

in (2) $Q = 16.8 \times \sqrt{\frac{5}{4.9}} = \underline{17.0 \text{M}}$

* back of book answer is incorrect.

(18)



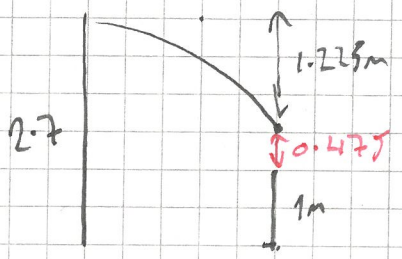
$$\begin{pmatrix} 12 \\ 4 \end{pmatrix} = \begin{pmatrix} 24 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ 9·8 \end{pmatrix} t^2$$

$$12 = 24t$$

$$t = 0·5 \text{ sec}$$

$$y = 4·9(0·5)^2 = 1·225$$

So when horizontally 12m from point of projection ball has fallen 1·225m thus will clear net. ✱



$$2·7 - 1 - 1·225 = 0·475 \text{ metres}$$