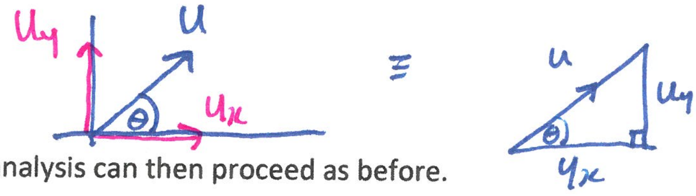
**Particles projected at an angle to the horizontal**

Suppose a particle is projected at an angle θ° above the horizontal with a velocity $Ums^{-1}$.


As before, in order to model the trajectory of the particle, we must consider the horizontal and vertical motion of the particle separately.

This requires us to split the initial velocity into its horizontal and vertical components:




Using trig $\frac{U_x}{U} = \cos\theta$ $\quad U_x = U\cos\theta$

$\frac{U_y}{U} = \sin\theta$ $\quad U_y = U\sin\theta$

The analysis can then proceed as before.

Eg6 A particle P is projected from a point O on a horizontal plane with speed $28ms^{-1}$ and with angle of elevation 30°. After projection, the particle moves freely under gravity until it strikes the plane at a point A. Find
a. the greatest height above the plane reached by P
b. the time of flight of P
c. the distance OA

Eg7 A particle is projected from a point O with speed V $ms^{-1}$ and at an angle of elevation θ, where $\tan\theta = \frac{4}{3}$. The point O is 42.5m above a horizontal plane. The particle strikes the plane, at a point A, 5 seconds after it is projected.
a. Show that V = 20
b. Find the distance OA

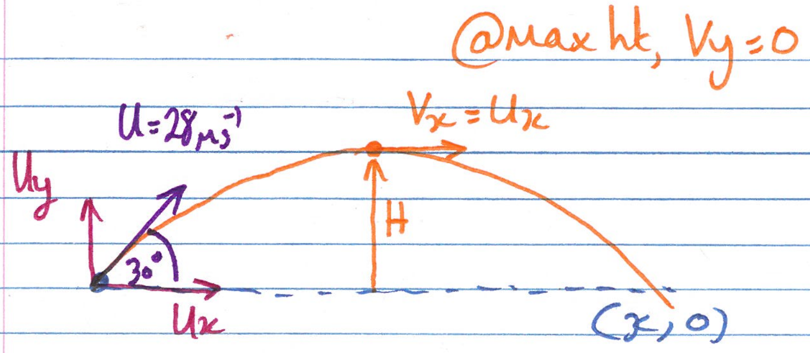
Ex4.2 2,4,6,8.

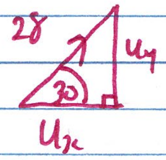
Eg8 A particle is projected from a point O with speed $35ms^{-1}$ at an angle of elevation of 30°. The particle moves freely under gravity.
Find the length of time for which the particle is 15m or more above O.

Eg9 A particle is projected from a point with speed u at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.
a. Show that $y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + tan^2\alpha)$
A particle is projected from a point A on a horizontal plane, with speed $28ms^{-1}$ at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32m from A and at a height of 8m above the plane.
b. Find the two possible values of α, giving your answers correct to the nearest degree.

Eg10 A ball is struck by a racket at a point A which is 2m above horizontal ground. Immediately after being struck, the ball has velocity $(\mathbf{5i} + \mathbf{8j})ms^{-1}$. After being struck, the ball travels freely under gravity until it strikes the ground at a point B. Find
a. the greatest height above the ground reached by the ball
b. the speed of the ball as it reaches B
c. the angle the velocity of the ball makes with the ground as the ball reaches B.
d.

Eg6




$\frac{U_x}{28} = \cos 30$, $U_x = 28\cos 30 = 14\sqrt{3}$

$\frac{U_y}{28} = \sin 30$, $U_y = 28\sin 30 = 14$

(a) @ Max ht, $S_y = H$, $a_y = -9.8$, $U_y = 14$, $t = ?$, $V_y = 0$

$V = u + at$

$0 = 14 - 9.8t$

$t = \frac{14}{9.8} = \frac{10}{7}$ sec

Now $S = ut + \frac{1}{2}at^2$

$H = 14\left(\frac{10}{7}\right) + \frac{1}{2} - 9.8\left(\frac{10}{7}\right)^2$

$H = 10$ metres.

(b) over whole flight, vertical displacement $S_y = 0$

$0 = 14t + \frac{1}{2} - 9.8t^2$

$4.9t^2 - 14t = 0$

$t(4.9t - 14) = 0$

either $t = 0$ or $4.9t = 14$

$t = \frac{14}{4.9} = 2.8571\ldots = 2.9$ sec {2 s.f.)

(c) Consider horizontal motion

$S_x = x$, $a_x = 0$, $u_x = 14\sqrt{3}$, $t = \frac{20}{7}$

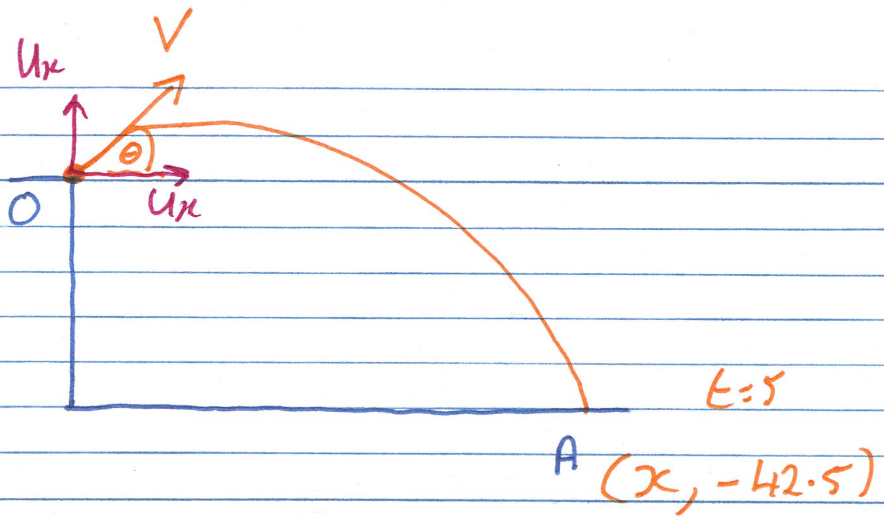
Using $s = ut + \frac{1}{2}at^2$

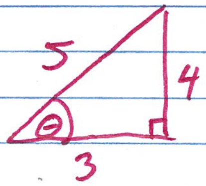
$x = 14\sqrt{3} \cdot \frac{20}{7} = 40\sqrt{3}$ metres.

$= 69.282\ldots$

$= 70$ metres. $\{2\text{ s.f.}\}$

Eg7




$\tan\theta = \frac{4}{3}$ , $\cos\theta = \frac{3}{5}$ , $\sin\theta = \frac{4}{5}$

(a) $(\rightarrow)$ : $S_x = x$ , $U_x = U\cos\theta = \frac{3V}{5}$ , $a_x = 0$ , $t = 5$

$(\uparrow)$ : $S_y = -42.5$ , $U_y = U\sin\theta = \frac{4V}{5}$ , $a_y = -9.8$ , $t = 5$

$$\begin{pmatrix} x \\ -42.5 \end{pmatrix} = \begin{pmatrix} 3/5\,V \\ 4/5\,V \end{pmatrix} 5 + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 5^2$$

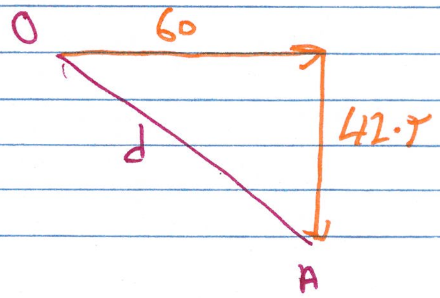
$(\uparrow)$ $-42.5 = 4V - 4.9 \times 25$

$4V = 122.5 - 42.5$

$V = 20\ ms^{-1}$ As required.

(b) $(\rightarrow)$ $x = 3V$

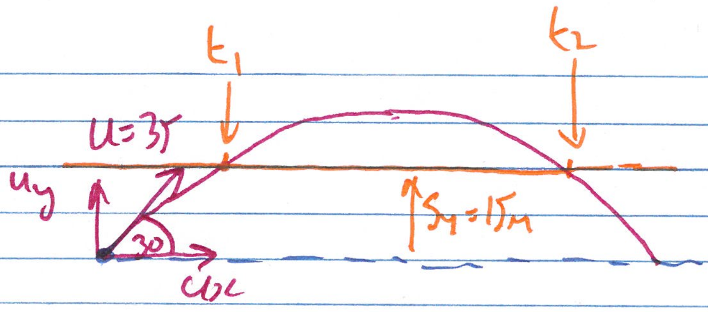
$x = 3(20) = 60$ metres




$d = \sqrt{60^2 + 42.5^2} = 73.527\ldots$

$\therefore$ OA $= 74$m {2 s.f.}

Eg8




$S_y = 15$, $u_y = 35\sin 30 = \frac{35}{2}$, $a_y = -9.8$, $t = ?$

$15 = \frac{35}{2}t + \frac{1}{2} \times -9.8t^2$

$15 = 17.5t - 4.9t^2$

$4.9t^2 - 17.5t + 15 = 0$

$t_2 = 2.142\ldots$

$t_1 =$ ~~1.485...~~ $1.4285\ldots$

$\therefore$ time elapsed above 15m $= 2.142\ldots -$ ~~1.485...~~ $1.4285$

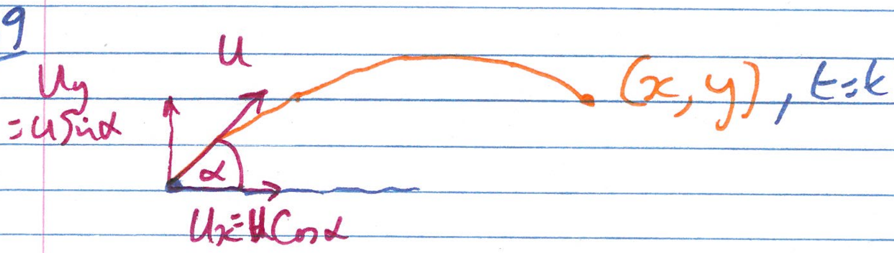
~~= 0.657~~

~~= 0.66 sec~~ {2 sig fig

$= 0.7135$

$= 0.71$ {2 sig fig}

Eg 9




Using $s = ut + \frac{1}{2}at^2$

@ $t = t$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u\cos\alpha \\ u\sin\alpha \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2$

Need to eliminate $t$:

$(\rightarrow) \quad x = u\cos\alpha\, t \quad -①$

$(\uparrow) \quad y = u\sin\alpha\, t - \frac{g}{2}t^2 \quad -②$

from ① $\quad t = \dfrac{x}{u\cos\alpha}$

in ② $\quad y = \cancel{u}\sin\alpha\left(\dfrac{x}{\cancel{u}\cos\alpha}\right) - \dfrac{g}{2}\left(\dfrac{x}{u\cos\alpha}\right)^2$

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\left(\frac{1}{\cos^2\alpha}\right)$$

Now $\quad \dfrac{1}{\cos\alpha} = \sec\alpha$

So $\quad \dfrac{1}{\cos^2\alpha} = \sec^2\alpha$

also, $\sec^2\alpha = 1 + \tan^2\alpha$

$\therefore \quad y = x\tan\alpha - \dfrac{gx^2}{2u^2}(1 + \tan^2\alpha)$ As required.

(b) $$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

$x = 32,\ u = 28,\ y = 8$

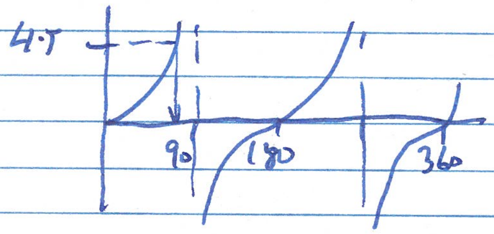
hence $$8 = 32\tan\alpha - \frac{9.8(32^2)}{2(28)^2}(1+\tan^2\alpha)$$

$$8 = 32\tan\alpha - 6.4(1+\tan^2\alpha)$$

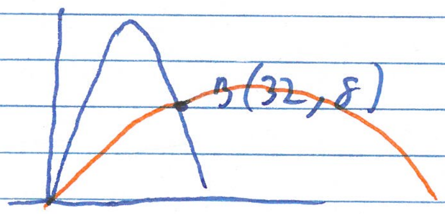
$$8 = 32\tan\alpha - 6.4 - 6.4\tan^2\alpha$$

$$6.4\tan^2\alpha - 32\tan\alpha + 14.4 = 0$$

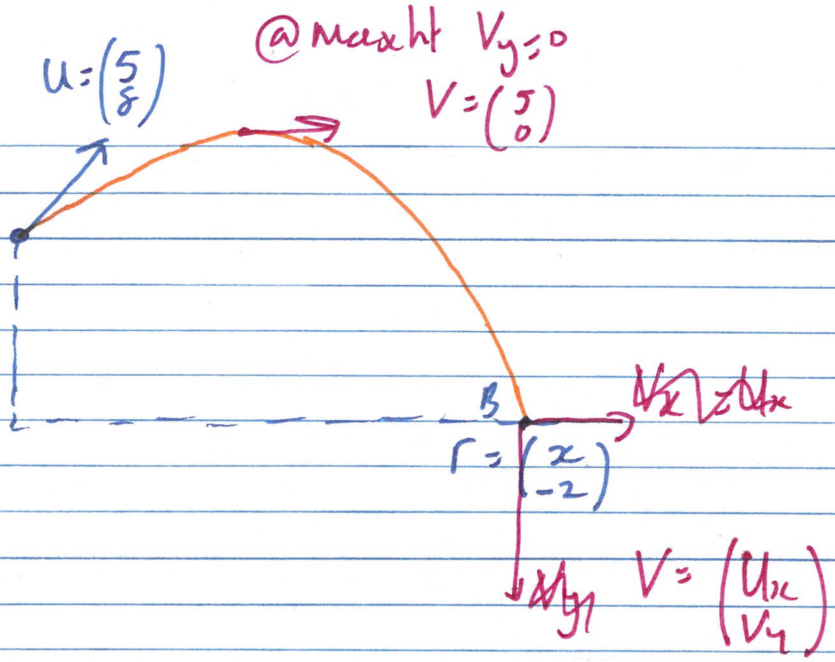
either $\tan\alpha = 4.5$ or $\tan\alpha = 0.5$




$\alpha = 77°$ or $\alpha = 27°$



Eg10




(a) @ Max ht $\quad V_y = U_y + a_y t$

$$0 = 8 - 9.8t$$

$$t = \frac{8}{9.8} = \frac{40}{49}$$

Now $\quad S_y = U_y t + \frac{1}{2} a_y t^2$

$$S_y = 8\left(\frac{40}{49}\right) - 4.9\left(\frac{40}{49}\right)^2$$

$$= 3.265\ldots$$

$$= 3.3\text{m} \ \{2.s.f\} \text{ above point of projection}$$

$\therefore \ 3.3 + 2 = 5.3\text{m}$ above ground

(b) Speed @ B. $\quad V_x = U_x = 5 \text{ ms}^{-1}$

Using $\quad V^2 = u^2 + 2as$

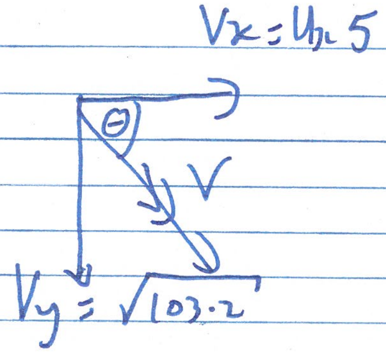
$$V_y^2 = 8^2 + 2 \times -9.8 \times -2$$

$$V_y^2 = 64 + 39.2$$

$$V_y^2 = 103.2$$

$$V_y = \sqrt{103.2} = 10.15\ldots$$

Now for speed




$$V = \sqrt{(\sqrt{103.2})^2 + 5^2} = 11.322 = 11 \text{ ms}^{-1} \ \{\text{to } 2.\text{s.f}\}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{103.2}}{5}\right) = 63.79\ldots = 64^\circ \ \{2 \text{ s.f.}\}$$

## Exercise 4.2

Whenever a numerical value of $g$ is required, take $g = 9.8\text{ m s}^{-2}$.

**1** A particle is projected with speed $35\text{ m s}^{-1}$ at an angle of elevation of 60°. Find the time the particle takes to reach its greatest height.

**2** A ball is projected from a point 5 m above horizontal ground with speed $18\text{ m s}^{-1}$ at an angle of elevation of 40°. Find the height of the ball above the ground 2 s after projection.

**3** A stone is projected horizontally from a point above horizontal ground with speed $32\text{ m s}^{-1}$. The stone takes 2.5 s to reach the ground. Find

**a** the height of the point of projection above the ground,

**b** the distance from the point on the ground vertically below the point of projection to the point where the stone reached the ground.

**4** A projectile is launched from a point on horizontal ground with speed $150\text{ m s}^{-1}$ at an angle of 10° to the horizontal. Find

**a** the time the projectile takes to reach its highest point above the ground,

**b** the range of the projectile.

**5** A particle is projected from a point $O$ on a horizontal plane with speed $20\text{ m s}^{-1}$ at an angle of elevation of 45°. The particle moves freely under gravity until it strikes the ground at a point $X$. Find

**a** the greatest height above the plane reached by the particle,

**b** the distance $OX$.

**6** A ball is projected from a point $A$ on level ground with speed $24\text{ m s}^{-1}$. The ball is projected at an angle $\theta$ to the horizontal where $\sin\theta = \frac{4}{5}$. The ball moves freely under gravity until it strikes the ground at a point $B$. Find

**a** the time of flight of the ball,

**b** the distance from $A$ to $B$.

**7** A particle is projected with speed $21\text{ m s}^{-1}$ at an angle of elevation $\alpha$. Given that the greatest height reached above the point of projection is 15 m, find the value of $\alpha$, giving your answer to the nearest degree.

**8** A particle is projected horizontally from a point $A$ which is 16 m above horizontal ground. The projectile strikes the ground at a point $B$ which is at a horizontal distance of 140 m from $A$. Find the speed of projection of the particle.

**9** A particle $P$ is projected from the origin with velocity $(12\mathbf{i} + 24\mathbf{j})\text{ m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find

**a** the position vector of $P$ after 3 s,

**b** the speed of $P$ after 3 s.

**10** A stone is thrown with speed $30\,\text{m s}^{-1}$ from a window which is 20 m above horizontal ground. The stone hits the ground 3.5 s later. Find

**a** the angle of projection of the stone,

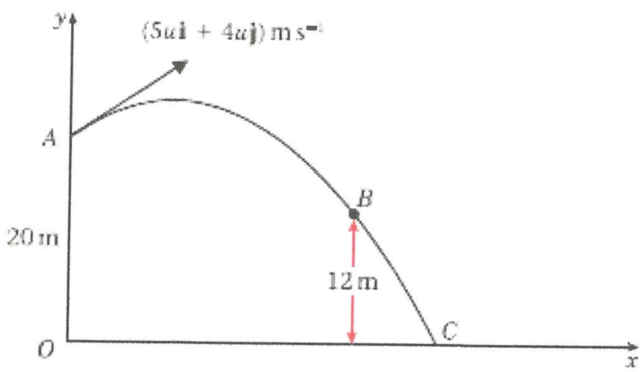
**b** the horizontal distance from the window to the point where the stone hits the ground.

**11** A ball is thrown from a point $O$ on horizontal ground with speed $u\,\text{m s}^{-1}$ at an angle of elevation of $\theta$, where $\tan\theta = \frac{3}{4}$. The ball strikes a vertical wall which is 20 m from $O$ at a point which is 3 m above the ground. Find

**a** the value of $u$,

**b** the time from the instant the ball is thrown to the instant that it strikes the wall.

**12**




*[In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.]*

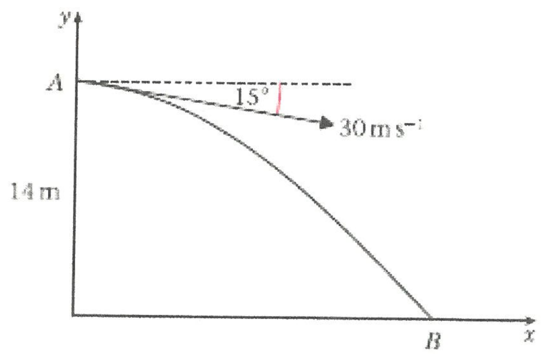
A particle $P$ is projected from a point $A$ with position vector $20\mathbf{j}$ m with respect to a fixed origin $O$. The velocity of projection is $(5u\mathbf{i} + 4u\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity, passing through a point $B$, which has position vector $(k\mathbf{i} + 12\mathbf{j})$ m, where $k$ is a constant, before reaching the point $C$ on the $x$-axis, as shown in the figure above.
The particle takes 4 s to move from $A$ to $B$. Find

**a** the value of $u$,

**b** the value of $k$,

**c** the angle the velocity of $P$ makes with the $x$-axis as it reaches $C$.

**13**




A stone is thrown from a point $A$ with speed $30\,\text{m s}^{-1}$ at an angle of 15° below the horizontal. The point $A$ is 14 m above horizontal ground. The stone strikes the ground at the point $B$, as shown in the figure above. Find

**a** the time the stone takes to travel from $A$ to $B$,

**b** the distance $AB$.

**14** A particle is projected from a point with speed $21\text{ m s}^{-1}$ at an angle of elevation $\alpha$ and moves freely under gravity. When the particle has moved a horizontal distance $x$ m, its height above the point of projection is $y$ m.

**a** Show that $y = x\tan\alpha - \dfrac{x^2}{90\cos^2\alpha}$.

**b** Given that $y = 8.1$ when $x = 36$, find the value of $\tan\alpha$.

**15** A projectile is launched from a point on a horizontal plane with initial speed $u\text{ m s}^{-1}$ at an angle of elevation $\alpha$. The particle moves freely under gravity until it strikes the plane. The range of the projectile is $R$ m.

**a** Show that the time of flight of the particle is $\dfrac{2u\sin\alpha}{g}$ seconds.

**b** Show that $R = \dfrac{u^2\sin 2\alpha}{g}$.

**c** Deduce that, for a fixed $u$, the greatest possible range is when $\alpha = 45°$.

**d** Given that $R = \dfrac{2u^2}{5g}$, find the two possible values of the angle of elevation at which the projectile could have been launched.

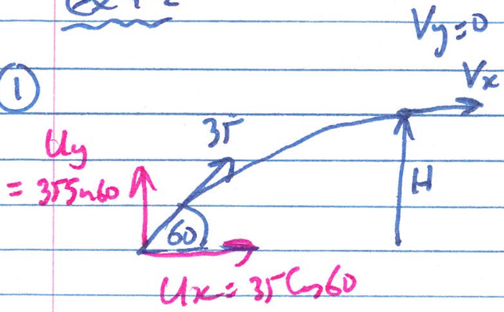
**16** A particle is projected from a point on level ground with speed $u\text{ m s}^{-1}$ and angle of elevation $\alpha$. The maximum height reached by the particle is 42 m above the ground and the particle hits the ground 196 m from its point of projection.

Find the value of $\alpha$ and the value of $u$.

**Numerical Answers**

1 3.1 (2 s.f.)
2 8.5 m (2 s.f.)
3 **a** 31 m (2 s.f.) **b** 80 m
4 **a** 2.7 s (2 s.f.) **b** 790 m (2 s.f.)
5 **a** 10 m (2 s.f.) **b** 41 m (2 s.f.)
6 **a** 3.9 s (2 s.f.) **b** 56 m (2 s.f.)
7 55° (nearest degree)
8 $77\text{ m s}^{-1}$ (2 s.f.)
9 **a** $(36\mathbf{i} + 27.9\mathbf{j})$ m **b** $13\text{ m s}^{-1}$ (2 s.f.)
10 **a** 22° (2 s.f.) **b** 97 m (2 s.f.)
11 **a** 16 (2 s.f.) **b** 1.6 s (2 s.f.)
12 **a** 4.4 **b** 88 **c** 50° (2 s.f.)
13 **a** 1.1 s (2 s.f.)
**b** 34 m (2 s.f.)
14 **b** $\tan\alpha = \frac{3}{4}$
15 **d** 12° and 78° (nearest degree)
16 $\alpha = 40.6°$ (nearest 0.1°)
$u = 44$ (2 s.f.)

Ex 4.2

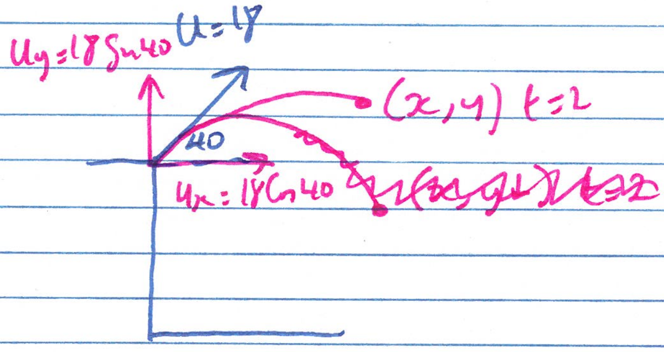
① 


@ Max height, $V_y = 0$, $U_y = 35\sin 60 \uparrow$, $a_y = 9.8 \downarrow$, $t = ?$
$= -9.8 \uparrow$

$$V_y = U_y + a_y t$$

$$0 = \frac{35\sqrt{3}}{2} - 9.8t$$

$$t = 3.1 \text{ sec}$$

② 


$S_y = y$, $a_y = -9.8$, $U_y = 18\sin 40$, $t = 2$

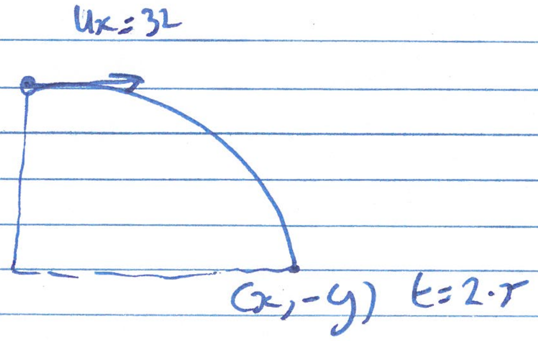
$$y = +18\sin 40\,(2) - \frac{1}{2}\,9.8\,2^2$$

$$y = -19.6 + 36\sin 40 = +3.54$$

ie 3.5 m ~~below~~ above point of projection

which will be $5 + 3.5 = 8.5$m above ground

(3)




$$\begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} 32 \\ 0 \end{pmatrix} 2.5 + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 2.5^2$$

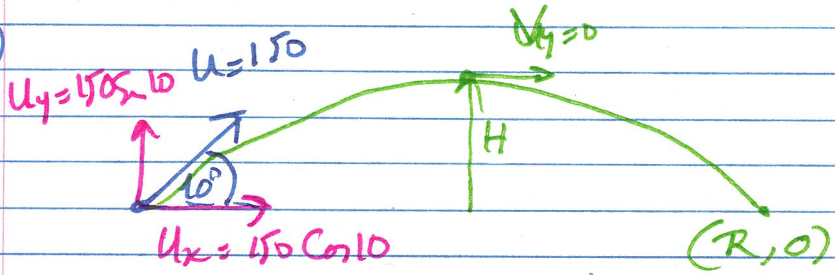
$x = 32 \times 2.5 = 80$

$-y = -\cancel{\ldots}\ 30.625$

$y = \cancel{\ldots}\ 31$ meters

(a) 31 meters (b) 80 meters

(4)




(a) @ Max ht, $V_y = 0$, $u_y = 150 \sin 10$, $a_y = -9.8$, $t = ?$

$$V_y = u_y + a_y t$$

$$0 = 150 \sin 10 - 9.8t$$

$$t = \frac{150 \sin 10}{9.8} = 2.65788\ldots = 2.7 \text{ sec}$$

(b) $$\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} 150 \cos 10 \\ 150 \sin 10 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \quad \begin{matrix} -(1) \\ -(2) \end{matrix}$$

From (2) $0 = 150 \sin 10 . t - 4.9t^2$

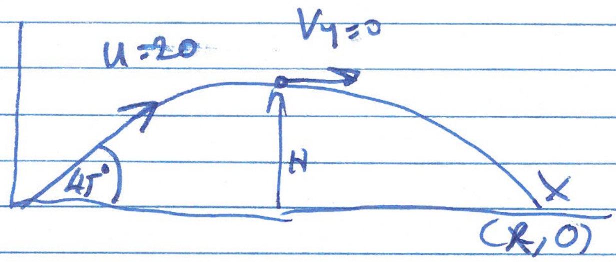
(4)(b) $t(4.9t - 150\sin 10) = 0$

either $t = 0$ or $t = \frac{150\sin 10}{4.9}$

in (1) $R = 150\cos 10\left(\frac{150\sin 10}{4.9}\right) = 785.25\ldots$

$= 790$ m (2.s.f).

(5)




(a) @ Max ht, $V_y = 0$, $a_y = -9.8$, $U_y = 20\sin 45$, $s = H$

$V^2 = u^2 + 2as$

$0 = (20\sin 45)^2 - 19.6s$

$s = \frac{(20\sin 45)^2}{19.6} = 10.204\ldots$

$S = 10$m (2 sf)

(b) at point of $\begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} 20\cos 45 \\ 20\sin 45 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2$ — (1) — (2)

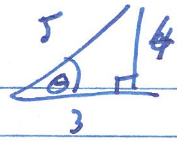
from (2) $0 = 20\sin 45\, t - 4.9t^2$

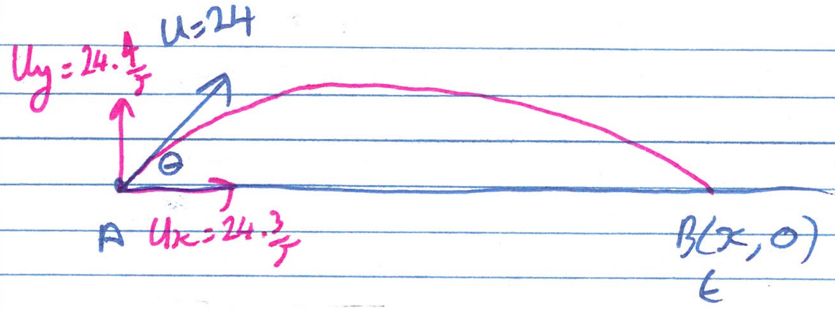
$t(4.9t - 20\sin 45) = 0$

$t = \frac{20\sin 45}{4.9}$

in (1) $x = 20\cos 45\left(\frac{20\sin 45}{4.9}\right) = 40.816\ldots = 41$m (2sf)

6)




$$\begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} 14.4 \\ 19.2 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \quad \begin{matrix} -(1) \\ -(2) \end{matrix}$$

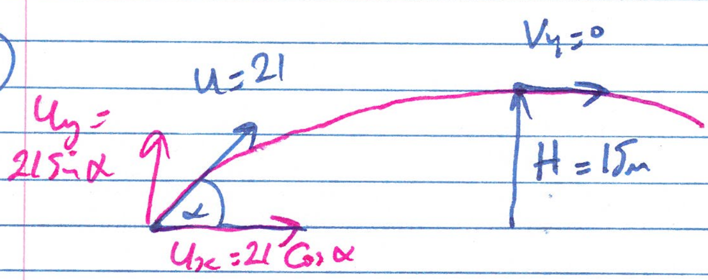
From (2) $\quad 0 = 19.2t - 4.9t^2$

$$t(4.9t - 19.2) = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{19.2}{4.9} = 3.9 \text{ sec}$$

In (1) $\quad x = 14.4(3.9) = 56\text{ m}.$

7)




@ Max Ht, $V_y = 0$

$$V_y = U_y - gt$$

$$0 = 21\sin\alpha - gt$$

$$t = \frac{21\sin\alpha}{g}$$

Now $\quad 15 = 21\sin\alpha\, t - \frac{1}{2}gt^2$

$$15 = 21\sin\alpha\left(\frac{21\sin\alpha}{g}\right) - 4.9\left(\frac{21\sin\alpha}{g}\right)^2$$

(7) contd

$$15 = \frac{21^2}{9.8}\sin^2\alpha - \frac{4.9(21)^2\sin^2\alpha}{9^2}$$

$$15 = \sin^2\alpha\left[\frac{21^2}{9.8} - \frac{4.9(21)^2}{9.8^2}\right]$$

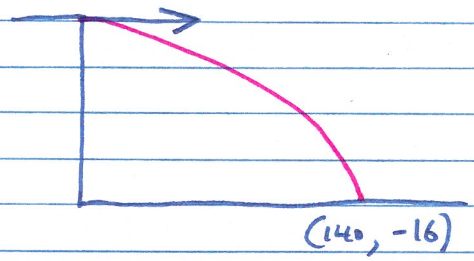
$$15 = 22.5\sin^2\alpha$$

$$\sin\alpha = \sqrt{\frac{15}{22.5}}$$

$$\sin\alpha = \sqrt{\frac{2}{3}}$$

$$\alpha = 54.73\ldots = 55^\circ$$

(8) $u_x = ?$




$$\begin{pmatrix}140\\-16\end{pmatrix} = \begin{pmatrix}u_x\\0\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}0\\-9.8\end{pmatrix}t^2 \quad \begin{matrix}—(1)\\—(2)\end{matrix}$$

From (2) $-16 = -4.9t^2$

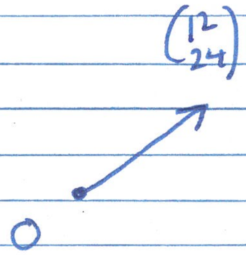
$$t = \sqrt{\frac{16}{4.9}}$$

in (1) $140 = u_x\sqrt{\frac{16}{4.9}}$

$$u_x = 77.47\ldots$$

$$= 77\ \mathrm{ms}^{-1}$$

(9)



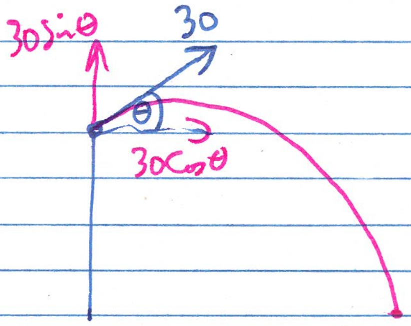

@ $t=3$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 24 \end{pmatrix} 3 + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 3^2$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 36 \\ 72 \end{pmatrix} + \begin{pmatrix} 0 \\ -44.1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 36 \\ 27.9 \end{pmatrix}$$

$\therefore \quad r_{t=3} = 36\underline{i} + 27.9\underline{j}$

(10)




$(x, -20)$, $t = 3.5$

@ impact, $\begin{pmatrix} x \\ -20 \end{pmatrix} = \begin{pmatrix} 30\cos\theta \\ 30\sin\theta \end{pmatrix} 3.5 + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} 3.5^2$ — (1) — (2)

From (2) $\quad -20 = 105\sin\theta - 60.025$

$$105\sin\theta = 40.025$$

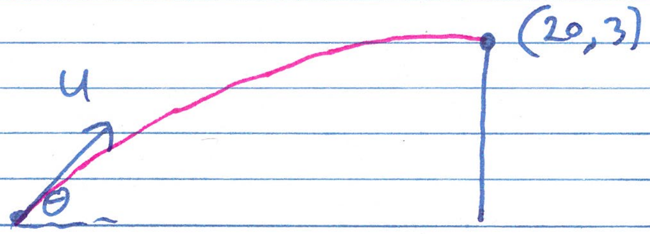
$$\sin\theta = \frac{40.025}{105}$$

$$\theta = 22.407\ldots = 22^\circ$$

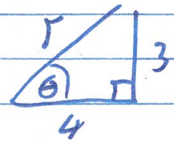
(10) cont

in (1) $\quad x = 105\cos(22.407) = 97\,m.$

(11)




$\tan\theta = \frac{3}{4}$




$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

$u_x = \cancel{u\cos}\,\frac{4u}{5}, \quad u_y = \frac{3}{5}u$

$$\begin{pmatrix} 20 \\ 3 \end{pmatrix} = \begin{pmatrix} 0.8u \\ 0.6u \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \quad \begin{matrix} -(1) \\ -(2) \end{matrix}$$

from (1) $\quad 20 = 0.8ut \quad -(3)$

from (2) $\quad 3 = 0.6ut - 4.9t^2 \quad -(4)$

from (3) $\quad u = \frac{25}{t} \quad -(5)$

in (4) $\quad 3 = 0.6 \cdot \frac{25}{\cancel{t}} \cdot \cancel{t} - 4.9t^2$

$3 = 15 - 4.9t^2$

$4.9t^2 = 12$

$t = \sqrt{\frac{12}{4.9}} = 1.564\ldots = 1.6$ sec

in (5) $\quad u = \frac{25}{1.564\ldots} = 15.975\ldots = 16\,ms^{-1}$

from point of projection

(12) @ B. $\begin{pmatrix} k \\ -8 \end{pmatrix} = \begin{pmatrix} 5u \\ 4u \end{pmatrix} 4 + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} 4^2$ — (1) — (2)

(a) from (2) $-8 = 16u - 78.4$

$u = \frac{70.4}{16} = 4.4\,ms^{-1}$

(b) in (1) $k = 20(4.4) = 88$ metres

(c) @ C. $\cancel{V_y^2 = u_y^2 + 2a_y s}$

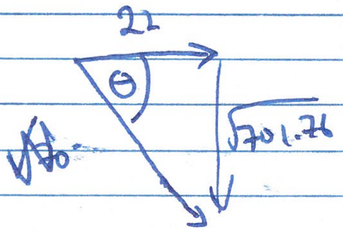
$\cancel{V_y^2 =}$

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix}^2 = \begin{pmatrix} u_x \\ u_y \end{pmatrix}^2 + 2\begin{pmatrix} a_x \\ a_y \end{pmatrix}\begin{pmatrix} x \\ -20 \end{pmatrix}$$

$V_x = u_x = 5 \times 4.4 = 22\,ms^{-1}$

$V_y^2 = 17.6^2 + 2 \times -9.8 \times -20$

$V_y = \sqrt{701.76}$




$\theta = \tan^{-1}\left(\frac{\sqrt{701.76}}{22}\right) = 50.29\ldots$

$= 50°$

(13) $U_x = 30\cos 15$

$U_y = -30\sin 15$

$$\begin{pmatrix} x \\ -14 \end{pmatrix} = \begin{pmatrix} 30\cos 15 \\ -30\sin 15 \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2 \qquad \begin{matrix} -(1) \\ -(2) \end{matrix}$$

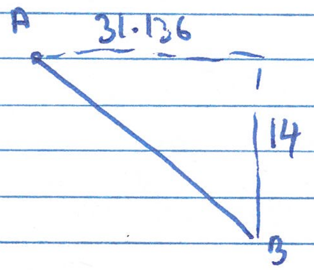
From (2) $\quad -14 = -30\sin 15 t - 4.9t^2$

$4.9t^2 + 30\sin 15 t - 14 = 0$

$t = 1.0744\ldots$

$t = 1.1$ sec

In (1) $\quad x = 30\cos 15(1.0744\ldots) = 31.136\ldots = 31\text{ m}$




$AB = \sqrt{31.136^2 + 14^2} = 34.138\ldots$

$= 34$ metres

(14)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 21\cos\alpha \\ 21\sin\alpha \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2$$

$$x = 21\cos\alpha\, t$$

$$t = \frac{x}{21\cos\alpha}$$

$$y = \cancel{21}\sin\alpha\left(\frac{x}{\cancel{21}\cos\alpha}\right) - 4.9\left(\frac{x}{21\cos\alpha}\right)^2$$

$$y = x\tan\alpha - \frac{4.9}{21^2}\frac{x^2}{\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{x^2}{90\cos^2\alpha}$$

@ $(36, 8.1)$

$$8.1 = 36\tan\alpha - \frac{36^2}{90\cos^2\alpha}$$

$$8.1 = 36\tan\alpha - \frac{14.4}{\cos^2\alpha}$$

$$8.1 = 36\tan\alpha - 14.4\sec^2\alpha$$

$$8.1 = 36\tan\alpha - 14.4(1 + \tan^2\alpha)$$

$$8.1 = 36\tan\alpha - 14.4 - 14.4\tan^2\alpha$$

$$14.4\tan^2\alpha - 36\tan\alpha + 22.5 = 0$$

$$\tan\alpha = 1.25$$

(15) $\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} u\cos\alpha \\ u\sin\alpha \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$ — (1)
— (2)

(a) From (2) $0 = ut\sin\alpha - \frac{1}{2}gt^2$

$$\frac{1}{2}gt^2 - ut\sin\alpha = 0$$

$$t\left(\frac{1}{2}gt - u\sin\alpha\right) = 0$$

$$\frac{1}{2}gt = u\sin\alpha$$

$$t = \frac{2u\sin\alpha}{g}$$ as required

(b) in (1) $R = u\cos\alpha\left(\frac{2u\sin\alpha}{g}\right)$

$$R = \frac{u^2}{g} . 2\sin\alpha\cos\alpha$$

$$R = \frac{u^2}{g}\sin(2\alpha)$$ as required. — (3)

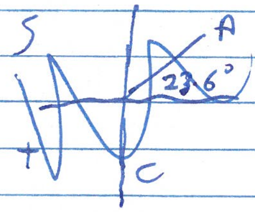
(c) $\sin 2\alpha$ takes values $0 \leq \sin 2\alpha \leq 1$

So for max R, $\sin 2\alpha = 1$

$$2\alpha = 90$$
$$\alpha = 45^\circ$$

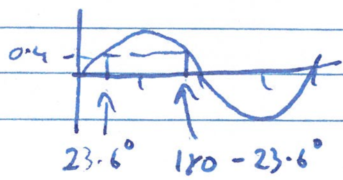
(d) if $R = \frac{2u^2}{5g}$ then (3) $\frac{2\cancel{u^2}}{5\cancel{g}} = \frac{\cancel{u^2}}{\cancel{g}}\sin(2\alpha)$

$$\sin(2\alpha) = 0.4$$




$$2\alpha = 23.6^\circ, 180 - 23.6^\circ$$

$$\alpha = \frac{23.6}{2}, \frac{156.4}{2}$$




$$\alpha = 12^\circ, 78^\circ$$

(16) @ Max ht, $V_y = 0$

$$0 = u\sin\alpha - 9.8t$$

$$t = \frac{u\sin\alpha}{9.8}$$

position $$42 = u\sin\alpha\, t - 4.9t^2$$

$$42 = u\sin\alpha\left(\frac{u\sin\alpha}{9.8}\right) - 4.9\left(\frac{u\sin\alpha}{9.8}\right)^2$$

$$42 = \frac{(u\sin\alpha)^2}{9.8} - \frac{4.9}{(9.8)^2}(u\sin\alpha)^2$$

$\times 9.8^2$

$$42(9.8)^2 = 9.8(u\sin\alpha)^2 - 4.9(u\sin\alpha)^2$$

$$42(9.8)^2 = 4.9(u\sin\alpha)^2$$

$$(u\sin\alpha)^2 = 823.2 \quad — \text{(A)}$$

@ Max Range

$$\begin{pmatrix}196\\0\end{pmatrix} = \begin{pmatrix}u\cos\alpha\\u\sin\alpha\end{pmatrix}t + \begin{pmatrix}0\\-4.9\end{pmatrix}t^2 \quad \begin{matrix}—(1)\\—(2)\end{matrix}$$

from (1) $$196 = u\cos\alpha\, t$$

$$t = \frac{196}{u\cos\alpha}$$

in (2) $$0 = \cancel{u}\sin\alpha\left(\frac{196}{\cancel{u}\cos\alpha}\right) - 4.9\left(\frac{196}{u\cos\alpha}\right)^2$$

$$0 = 196\tan\alpha - \frac{4.9\,(196^2)}{(u\cos\alpha)^2}$$

$\div 4.9$

$$0 = 40\tan\alpha - \frac{196^2}{(u\cos\alpha)^2}$$

(16) Contd

$$\frac{196^2}{(u\cos\alpha)^2} = 40\tan\alpha$$

$$(u\cos\alpha)^2 = \frac{196^2}{40\tan\alpha} \qquad \text{—— (B)}$$

(A) $\div$ (B) $\quad \tan^2\alpha = 823.2 \div \frac{196^2}{40\tan\alpha}$

$$\tan^2\alpha = 823.2 \times \frac{40\tan\alpha}{196^2}$$

$$\tan^2\alpha = \frac{6}{7}\tan\alpha$$

$$\tan\alpha\left[\tan\alpha - \frac{6}{7}\right] = 0$$

either $\tan\alpha = 0$ $\quad$ or $\quad \tan\alpha = \frac{6}{7}$

$\alpha = 0$ $\qquad\qquad \alpha = 40.6^\circ$

in (A) $\quad u = \sqrt{\frac{823.2}{(\sin 40.6)^2}} = 44.087\ldots = 44 \text{ ms}^{-1}$