**Classwork** **10/04/08**

## Graphs of Quadratic Expressions & Other Curves (Grade B/A)

We are now becoming used to recognising a quadratic expression such as

$$x^2 - x - 12$$

We are able to factorise it $(x-4)(x+3)$

and we could go on to find the two solutions of the quadratic equation

$$x^2 - x - 12 = 0$$

either $x-4=0$ or $x+3=0$

$x=4$ $x=-3$

We could even solve the above equation using the quadratic equation formula if it was appropriate (which it ***isn't***!!).

Now we are going to see what the graph of such expressions look like.

In order to plot a graph we need co-ordinates, so the quadratic expression is written as

$$y = x^2 - x - 12$$

We then substitute into the expression different values of x and calculate corresponding values of y in order to produce a set of co-ordinates.

*In an exam you are usually given the x co-ordinates and most of the corresponding y co-ordinates. You then just have to find the missing values. It is usually worthwhile starting with an x value which you have been given the y co-ordinate for so that you can be happy that you are correctly substituting and simplifying.*

| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|
| y | 8 | 0 | -6 | -10 | -12 | -12 | -10 | -6 | 0 | 8 |

Use x = -4 to check: $y = (-4)^2 - (-4) - 12 = 16 + 4 - 12 = 8$ ✓

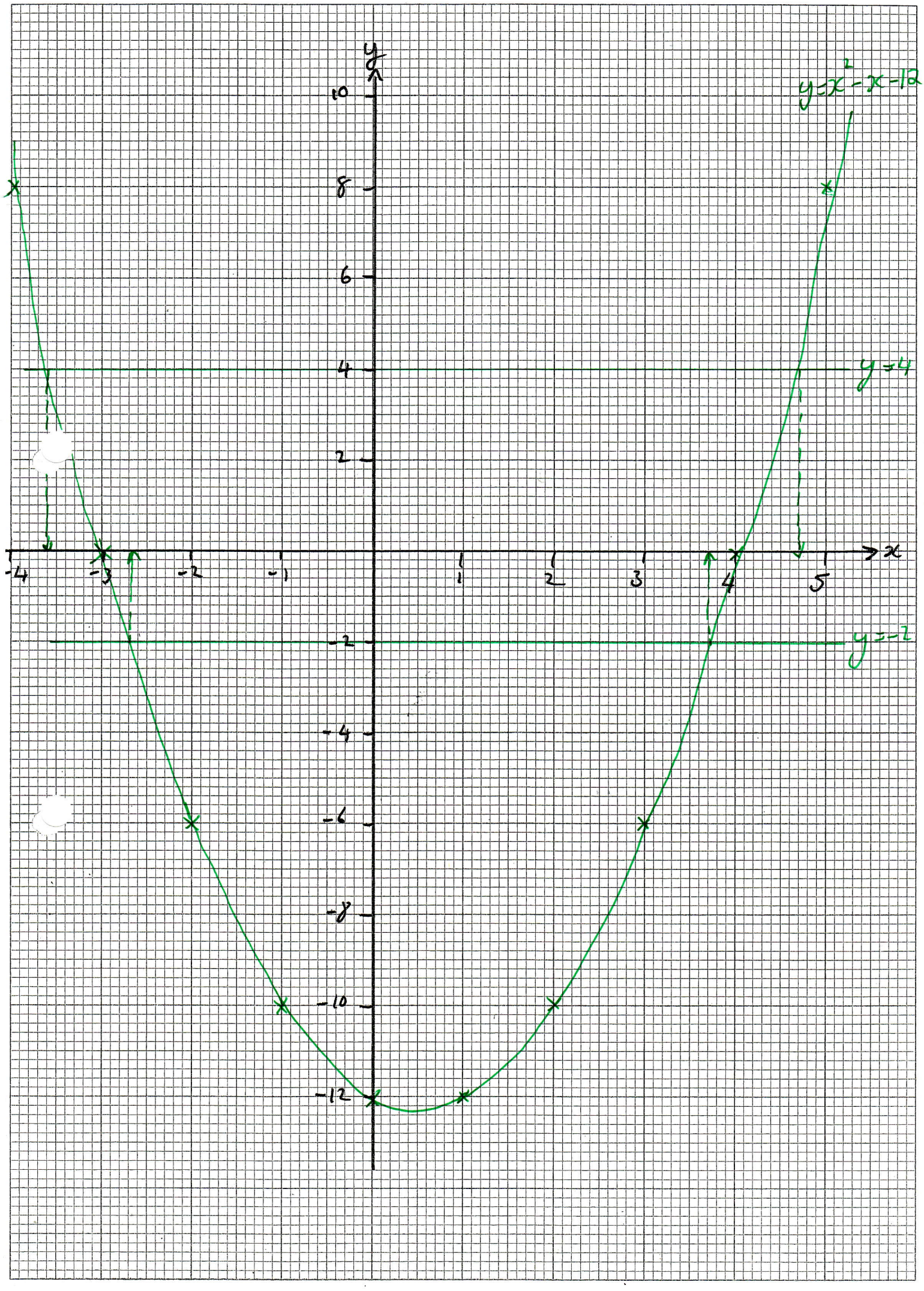
Now do x = -2: $y = (-2)^2 - (-2) - 12 = 4 + 2 - 12 = -6$ ✓

Use x = 3 to check: $y = (3)^2 - (3) - 12 = 9 - 3 - 12 = -6$ ✓

Now do x = 2: $y = (2)^2 - 2 - 12 = 4 - 2 - 12 = -10$ ✓

When we were drawing straight line graphs, we only needed to plot two points in order to draw the line. For curved graphs we need to plot enough points to produce the shape.

Plot the points on the graph paper overleaf and join with a smooth curve (freehand not ruler.


y
x
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-4
-3
-2
-1
1
2
3
4
5
y = x² - x - 12
y = 4
y = -2

A graph of a quadratic expression should always be U shaped (could be upside down), so if any point produces an extra bump in the graph you have probably miscalculated or misplotted a point.

Now do the PPQ's questions 1 and 2.

The same method applies to graphs of other expressions – try PPQ's questions 3 and 8 (calculator OK in 8)

**Using the Graphs to Solve Equations**

If we return to our graph $y = x^2 - x - 12$, we can use the graph to find the solutions to the graph $x^2 - x - 12 = 0$ by finding the intersection between the graph $y = x^2 - x - 12$ and $y = 0$.

So from our graph, the solutions to the equation $x^2 - x - 12 = 0$ are $x = -3, x = 4$

By drawing other lines onto the graph and reading of the x co-ordinates where the graphs intersect we can solve other quadratic equations:

Draw the line $y = 4$ and write down the x co-ordinates where the graphs intersect:

$x = 4.7$ or $x = -3.6$

What quadratic equation does are these the solutions to?

$x^2 - x - 12 = 4$

$x^2 - x - 16 = 0$

***Grade A Question:*** ***How can we use the graph $y = x^2 - x - 12$ to solve the equation $x^2 - x - 10 = 0$?***

$x^2 - x - 10 = 0$

$x^2 - x - 12 + 2 = 0$

$x^2 - x - 12 = -2$

$x = -2.7$ or $x = 3.7$.

PPQ's 4, 5, 6 & 7 (calculator OK on 7)

# Quadratic & Other Curves

Remember the questions are usually on non-calculator papers

Examiner only *Arholwr yn unig*

**1** The table shows some of the values of $y = x^2 + x$ for values of $x$ from –3 to 2.

*(a)* Complete the table by finding the value of $y$ for $x = -1$.

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $y = x^2 + x$ | 6 | 2 | 0 | 0 | 2 | 6 |

remember to choose a test point (especially if $x$ is a negative number)

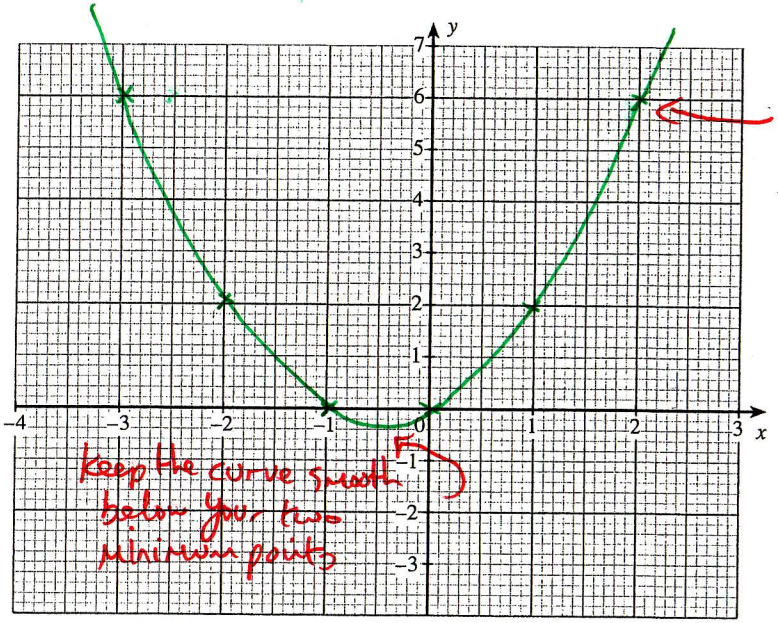
test using $x=-2$: $y = (-2)^2 + (-2) = 4 - 2 = 2$ ✓

now $x = -1$ : $y = (-1)^2 + (-1) = 1 - 1 = 0$

[1]

*(b)* On the graph paper below, draw the graph of $y = x^2 + x$ for values of $x$ from –3 to 2.

[2]




Make sure that the points you plot can be seen clearly beneath your curve.

Keep the curve smooth below your two minimum points



**Turn over.**

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test $x=-2$ $y = 5-(-2)^2$ $= 5-(4) = 5-4 = 1$ ✓

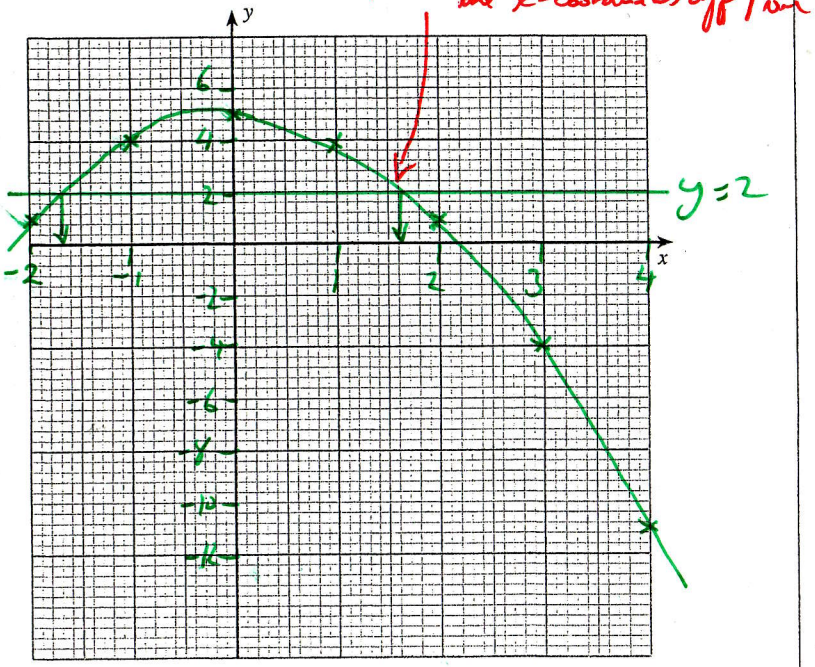
**2** *(a)* Complete the following table which gives values of $y = 5 - x^2$ for values of $x$ from –2 to 4. [1]

$y$ $x=-1$ $y = 5-(-1)^2 = 5-1 = 4$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y = 5 - x^2$ | 1 | 4 | 5 | 4 | 1 | –4 | –11 |

*(b)* Using suitable scales draw the graph of $y = 5 - x^2$ for values of $x$ from –2 to 4 on the graph paper below. [3]

Mark where you are reading the $x$-coordinates off from




*(c)* Draw the line $y = 2$ on the graph paper and write down the $x$-values of the points where the two graphs intersect.

$x = -1.7$ and $x = 1.6$

[2]

WARM UP 1

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(a) The cost of a stand season ticket last year was £200. This year it has increased to £250. Find the percentage increase in the cost of the stand season ticket.

increase = £50

% increase = $\frac{\text{increase}}{\text{original}} \times 100 = \frac{50}{200} \times 100 = 25\%$

[3]

(b) Two friends, Nigel and Paul, decide to share the cost of a £100 field season ticket in the ratio 4:1.

(i) How much **each** should each of Nigel and Paul pay towards the cost of the ticket?

4+1 = 5 equal parts  100 ÷ 5 = £20 per part

Nigel Pays 4 x 20 = £80

Paul Pays 1 x 20 = £20

Nigel pays ..... Paul pays .....

[2]

(ii) In the season there are 45 matches to attend. Nigel suggests that they take it in turns to attend every other match. Would this be a fair suggestion? You must explain your answer giving an alternative suggestion if you decide that this would not be a fair method.

Not fair, Nigel paid much more.

Nigel attends $\frac{4}{5} \times 45 = 36$ matches

Paul attends $\frac{1}{5} \times 45 = 9$ matches

[2]



cubed (times itself 3 times)

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3 The table shows some of the values of $y = x^3 + 3$ for values of $x$ from –3 to 3.

(a) Complete the table by finding the value of $y$ for $x = -1$ and $x = 2$.

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = x^3 + 3$ | –24 | –5 | 2 | 3 | 4 | 11 | 30 |

test $x=-2$ $y = (-2)^3 + 3 = -8 + 3 = -5$ ✓

$x=-1$ $y = (-1)^3 + 3 = -1 + 3 = 2$

$x=2$ $y = (2)^3 + 3 = 8 + 3 = 11$

[2]

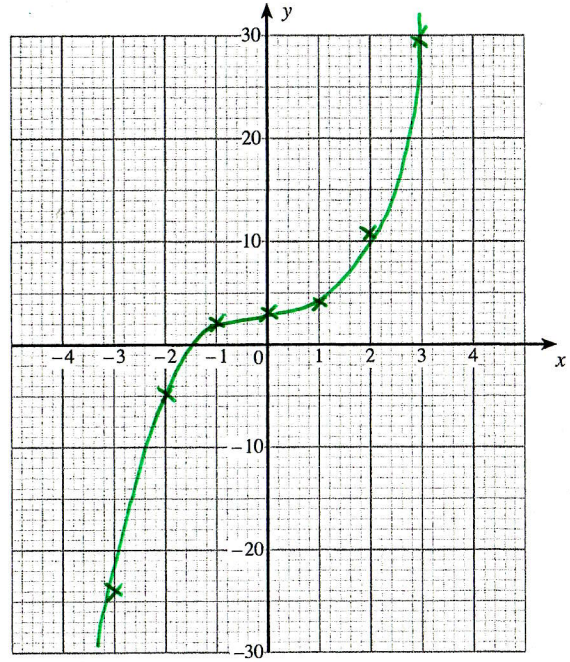
$(-2)^3$

$= -2 \times -2 \times -2$

$= -8$

(b) On the graph paper below, draw the graph of $y = x^3 + 3$ for values of $x$ from –3 to 3.

[2]



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**4.** The table shows the values of $y = 3x^2 - 2x - 5$ for values of $x$ from –2 to 4.

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y = 3x^2 - 2x - 5$ | 11 | 0 | –5 | –4 | 3 | 16 | 35 |

*(a)* On the graph paper opposite, draw the graph of $y = 3x^2 - 2x - 5$ for values of $x$ between –2 and 4.

[2]

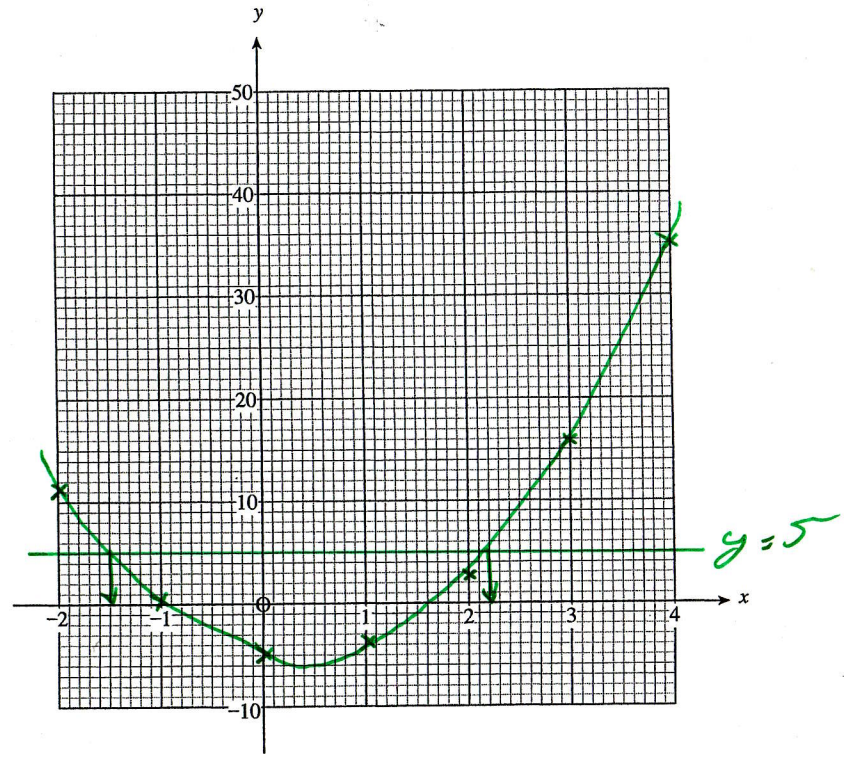
*(b)* Draw the line $y = 5$ on your graph paper and write down the $x$-values of the points where your two graphs intersect.

$x = -1.5$ and $x = 2.2$

[2]



**For use with Question 4**






**Turn over.**

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5 The table shows some of the values of $y = 2x^2 - 5x - 8$ for values of $x$ from –2 to 4.

(a) Complete the table by finding the value of $y$ for $x = 3$.

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y = 2x^2 - 5x - 8$ | 10 | –1 | –8 | –11 | –10 | –5 | 4 |

for $x=2$ $y = 2(2)^2 - 5(2) - 8$
$= 2\times4 - 10 - 8 = 8 - 10 - 8 = -10$ ✓

Now $x=3$ $y = 2(3)^2 - 5(3) - 8$
$= 2\times9 - 15 - 8 = 18 - 15 - 8 = -5$ [1]

(b) On the graph paper opposite, draw the graph of $y = 2x^2 - 5x - 8$ for values of $x$ between –2 and 4. [2]

(c) Draw the line $y = 3$ on your graph paper and write down the $x$-values of the points of intersection of your line with $y = 2x^2 - 5x - 8$.

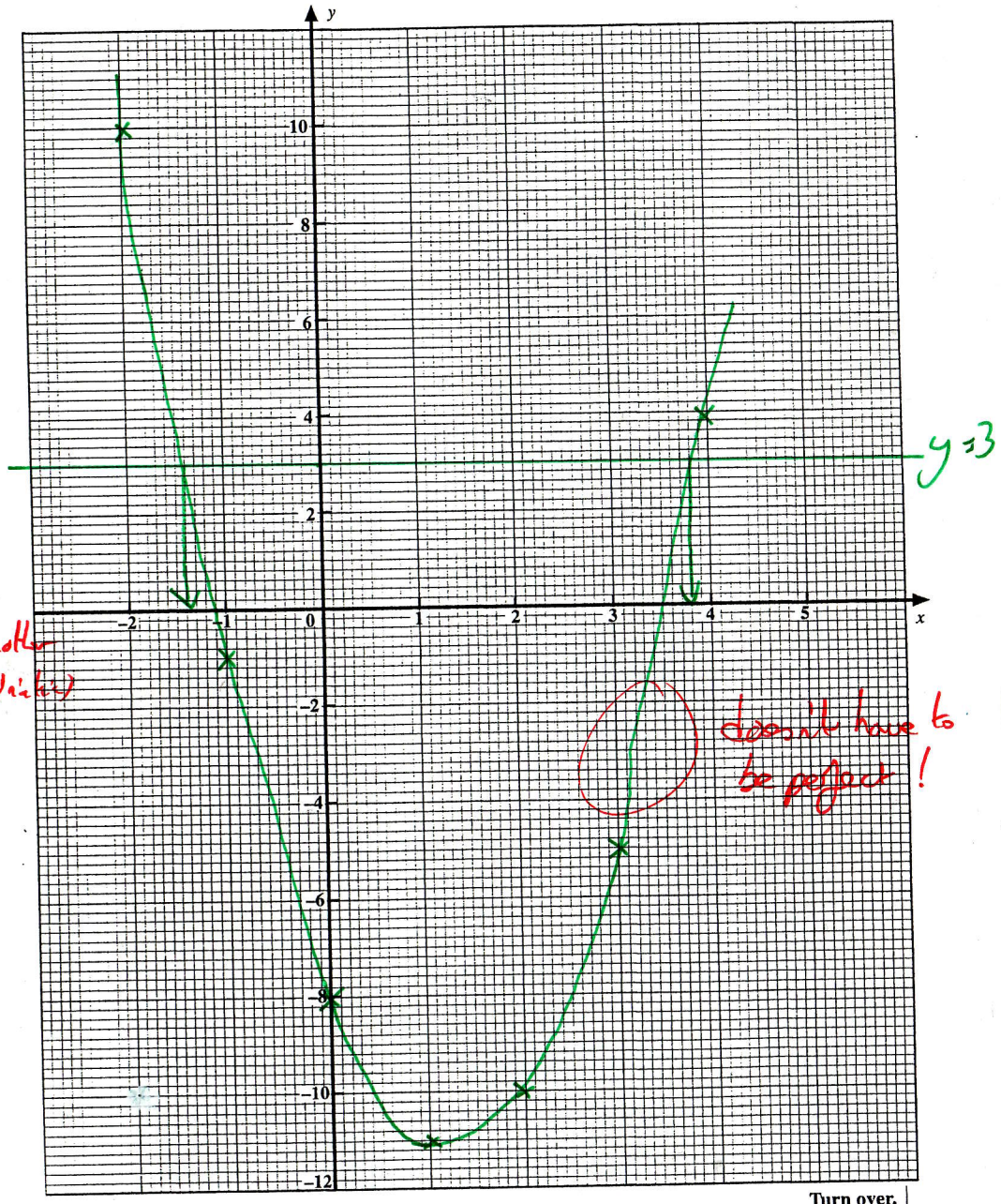
$x = -1.4$ and $x = 3.5$

[2]

(d) Write down and simplify the equation in $x$ whose solutions you found in (c).

$2x^2 - 5x - 8 = 3$ → put the two equations equal to each other
$2x^2 - 5x - 8 - 3 = 0$ → rearrange to make = 0 (for a quadratic)
$2x^2 - 5x - 11 = 0$ → simplify.

[1]




doesn't have to be perfect!

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**6** The table shows some of the values of $y = 2x^2 - 5x - 3$ for values of $x$ from –2 to 4.

*(a)* Complete the table by finding the value of $y$ for $x = -1$.

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y = 2x^2 - 5x - 3$ | 15 | 4 | –3 | –6 | –5 | 0 | 9 |

first $x=-2$ $y = 2(-2)^2 - 5(-2) - 3 = 2\times4 + 10 - 3 = 8 + 10 - 3 = 15$ ✓

now $x=-1$ $y = 2(-1)^2 - 5(-1) - 3 = 2\times1 + 5 - 3 = 2 + 5 - 3 = 4$ [1]

*(b)* On the graph paper opposite, draw the graph of $y = 2x^2 - 5x - 3$ for values of $x$ between –2 and 4.

[3]

*(c)* Draw the line $y = 3$ on the graph paper and write down the $x$-values of the points where your two graphs intersect.

$x = -0.9$ and $x = 3.4$

Don't know what all this space is for!

[2]

*(d)* Write down the equation in $x$ whose solutions are the $x$-values you found in *(c)*.

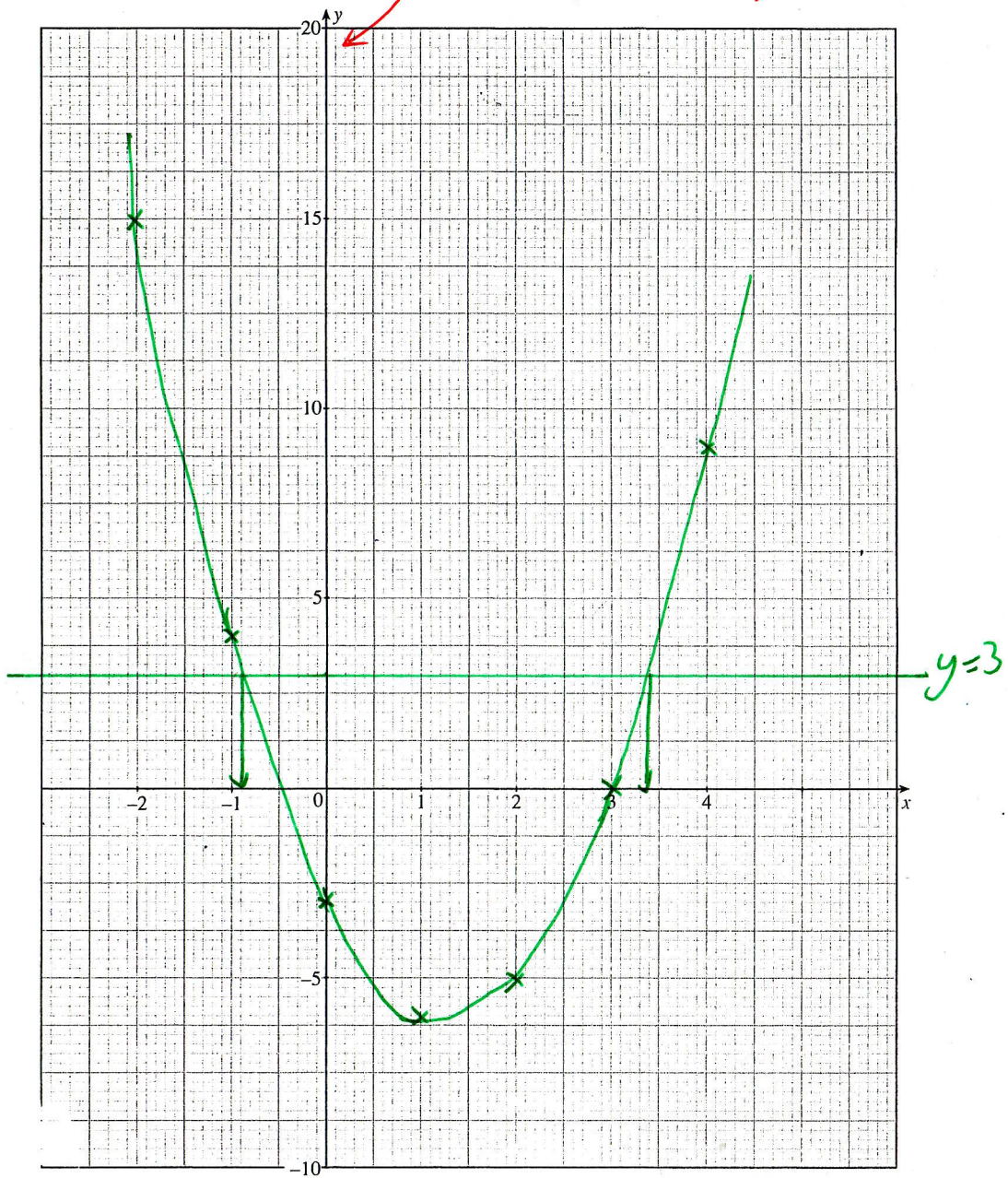
$2x^2 - 5x - 3 = 3$

$2x^2 - 5x - 3 - 3 = 0$

$2x^2 - 5x - 6 = 0$

[1]

horrible scale... 1 little square = 0.25
or 4 little squares = 1



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The table shows some of the values of $y = 3x^2 + x - 5$ for values of $x$ from –3 to 3.

*(a)* Complete the table by finding the value of $y$ for $x = -2$.

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y = 3x^2 + x - 5$ | 19 | 5 | –3 | –5 | –1 | 9 | 25 |

check over when you are allowed to use a calculator

check $x=-1$ $y = 3(-1)^2 + (-1) - 5 = 3\times1 - 1 - 5$
$= 3 - 1 - 5 = 3 - 6 = -3$ ✓

now $x=-2$ $y = 3(-2)^2 + (-2) - 5 = 3\times4 - 2 - 5$
$= 12 - 2 - 5 = 5$

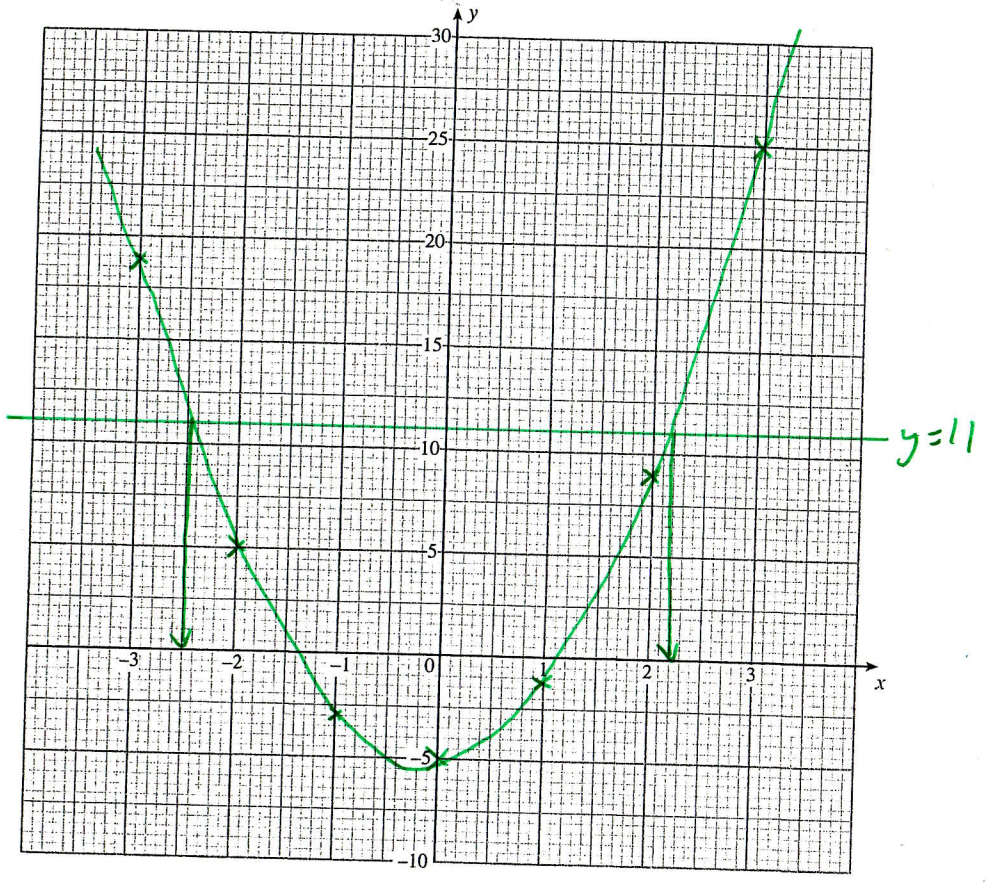
[1]

*(b)* On the graph paper opposite, draw the graph of $y = 3x^2 + x - 5$ for values of $x$ between –3 and 3.
[3]

*(c)* Draw the line $y = 11$ on your graph paper and write down the $x$–values of the points where your two graphs intersect.

$x = -2.5$ and $x = 2.2$ (Remember that your answers are off your graph and can differ slightly)

[2]



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8

The table shows some of the values of $y = 3x + \frac{10}{x}$ for values of $x$ from 1 to 5.

*(a)* Complete the table by finding the value of $y$ when $x = 1$ and $x = 2$.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 13 | 11 | 12·33 | 14·5 | 17 |

check $y = 3(3) + \frac{10}{3} = 9 + (\frac{10}{3}) = 12.33$

Now $y = 3(1) + \frac{10}{1} = 3 + 10 = 13$

And $y = 3(2) + \frac{10}{2} = 6 + 5 = 11$

[1]

*(b)* On the graph paper opposite, draw the graph of $y = 3x + \frac{10}{x}$ for values of $x$ between 1 and 5.
[1]

*(c)* Find the coordinates of the point of intersection of $y = 3x + \frac{10}{x}$ and $y = 27 - 4x$ which lies between $x = 1$ and $x = 5$.

bit more work here – need two points etc

$y = 27 - 4x$ when $x = 0$ $y = 27 - 4 \times 0 = 27$ (0, 27)

$x = 1$ $y = 27 - 4 \times 1 = 23$

$x = 3.4$

[2]