Solutions

Relative Frequency PPQs

1. An experiment was carried out to investigate the probability of obtaining a head when a biased coin is thrown.
The number of times the coin landed and showed a head in 4 sets of ten throws is shown in the table below.

Number of throws	Number of times a head is recorded
1st ten throws	2
2nd ten throws	4
3rd ten throws	3
4th ten throws	1

(a) Complete the table below to show the relative frequency of obtaining a head after throwing the coin a total of 10 times, 20 times, 30 times and 40 times. [2]

Number of times the coin is thrown altogether		10	20	30	40
Relative frequency of obtaining a head	Fraction	$\frac{2}{10}$	$\frac{6}{20}$	$\frac{9}{30}$	$\frac{10}{40}$
	Decimal	0·2	0·3	0·3	0·25

B1 B1

(b) Using the above results, write down the best estimate for the probability of obtaining a head when this biased coin is thrown.
Give a reason for your answer. [2]

0·25 B1

Most reliable as measured from the most throws E1

2.

An experiment was carried out to investigate the probability of obtaining an even number when a biased dice is thrown.
The number of even numbers obtained in each of 5 sets of 20 throws is shown in the table below.

	Number of times an even number is recorded
First set of 20 throws	14
Second set of 20 throws	8
Third set of 20 throws	14
Fourth set of 20 throws	16
Fifth set of 20 throws	10

(a) Complete the table below to show the relative frequency of an even number occurring after throwing the dice a total of 20 times, 40 times, 60 times, 80 times and 100 times.

Number of times the dice is thrown altogether		20	40	60	80	100
Relative frequency of obtaining an even number	Fraction	$\frac{14}{20}$	$\frac{22}{40}$	$\frac{36}{60}$	$\frac{52}{80}$	$\frac{62}{100}$
	Decimal	0·7	0·55	0·6	0·65	0·62

B3 AO4
B2 6,7,8 correct
B1 4 or 5 correct

.....

.....

.....

.....

[3]

(b) Using the above results, write down the best estimate for the probability of obtaining an even number when this biased dice is thrown. Give a reason for your answer.

0·62 B1

Most reliable after most throws E1

.....

[2]

(c) Explain what you think might happen to the relative frequency if the experiment was continued with more throws of the biased dice.

The relative frequency will stabilise E1

[1]

(d) What would be your best estimate of the probability of obtaining an odd number on this biased dice?

$1 - 0.62 = 0.38$ M1

A1

[2]

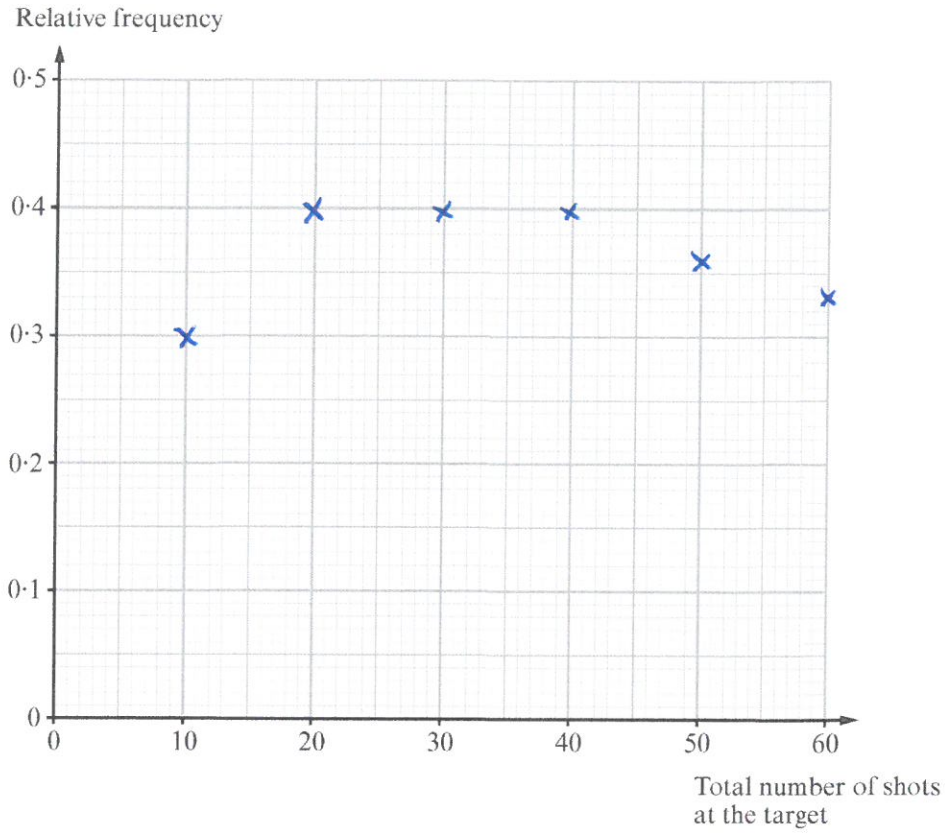
3. Yasmin carried out an experiment.
In the experiment, she shot 10 balls at a target and recorded the number of shots hitting the target.
She carried out this experiment 6 times.
The results are shown in the following table.

Experiment	1st	2nd	3rd	4th	5th	6th
Number of shots hitting the target	3	5	4	4	2	2

Yasmin decided to draw a graph showing the relative frequency of 'shots hitting the target' after 10 shots, 20 shots, 30 shots, 40 shots, 50 shots, 60 shots.

(a) Use the graph paper opposite to draw the graph of the relative frequencies.

Total Shots	10	20	30	40	50	60	
Total hits	3	8	12	16	18	20	B1
Rel Freq	3/10	8/20	12/30	16/40	18/50	20/60	M1
	0.3	0.4	0.4	0.4	0.36	0.33	A2



P1

[5]

(b) Do you consider that the experiment has been carried out enough times to give a good estimate for the probability of a shot hitting the target?
You must give a reason for your answer.

Yes – appears to be stabilizing between 0·3 & 0·4

or

No – doesn't appear stable yet

E1

[1]

4. Alan is a professional darts player. He claims that, with any throw, he can hit the bull's-eye (in the centre of the board) with a probability of 50%.

Ffion challenges him to prove this by throwing 5 sets of 10 darts.

Alan's results are given in the following table.

Number of throws	10	10	10	10	10
Number of throws hitting the bull's-eye	4	8	3	3	2

Ffion then creates a table to show the cumulative number of bull's-eyes and to calculate the relative frequencies.

Total number of throws	10	20	30	40	50
Total number of throws hitting the bull's-eye	4	12	15	18	20
Relative frequency of hitting a bull's-eye	$\frac{4}{10}$	$\frac{12}{20}$	$\frac{15}{30}$	$\frac{18}{40}$	$\frac{20}{50}$
	0·4	0·6	0·5	0·45	0·4

B1 B1 B1

(a) Complete the table above. [3]

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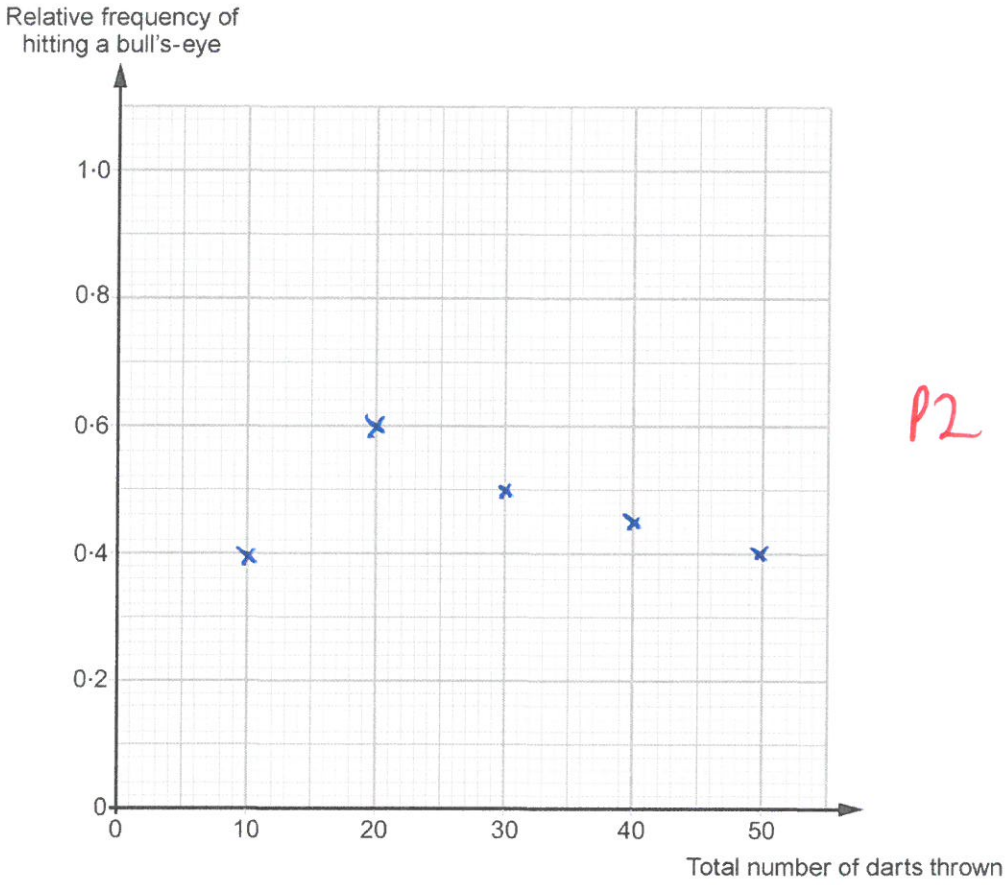
(b) Draw a graph to show the relative frequency of hitting a bull's-eye. [2]

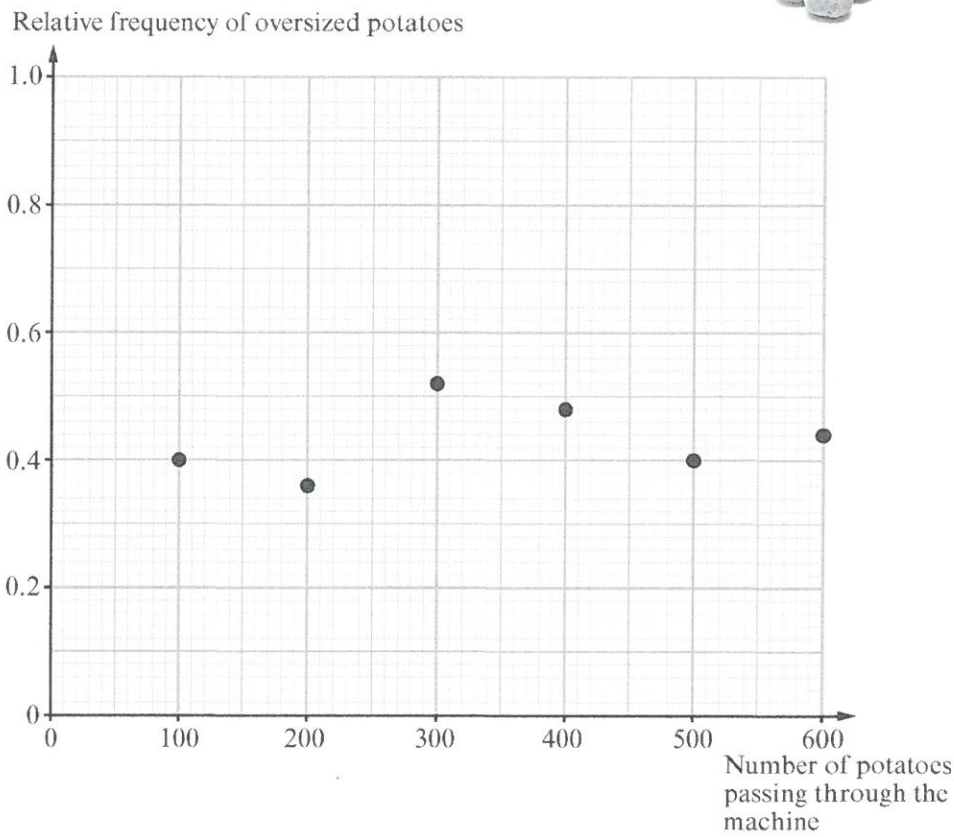


P2

(c) Is Alan correct to claim that he has a probability of 50% of hitting the bull's-eye? Explain your answer. [1]

No - because the proportion relative frequency suggests how 40% success rather than 50%. E1

5. A potato producer uses a machine to sort his potatoes.
The potato producer carried out a survey to investigate the probability of oversized potatoes passing through his sorting machine.
The relative frequency of oversized potatoes passing through the machine was calculated after a total of 100, 200, 300, 400, 500 and 600 potatoes.
The results are plotted on the graph below.



(a) Write down the best estimate for the probability that one of these potatoes, selected at random, will be oversized.
You must give a reason for your answer.

0.44 B1

Most reliable after most trials E1

[2]

(b) A trader offers to buy oversized potatoes at 15p each.
How much would the potato producer receive if he decided to sell, to the trader, all the oversized potatoes in the first 100 potatoes sorted by the machine?

0·4 × 100 = 40 oversized pots B1

So 40 × 0·15 = £6 M1 A1

[3]

(c) The potato producer decides not to sell his potatoes to the trader.
He sells 900 potatoes to a market stall holder.
The potato producer sells these potatoes for £4.50 per 100 potatoes.
He has agreed with the market stall holder that he will give a 2p refund per oversized potato discovered.
What would your best estimate be for the amount you would expect the potato producer to make from this transaction?

Sells for 9 × 4.50 = £40·50 M1 A1

estimated oversize = 0·44 × 900 = 396 B1

refund = 396 × 0·02 = £7·92 M1 A1

So makes 40·50 − 7·92 = £32·58 B1

[6]

6. A factory production line packs buttons into bags.
There are exactly 80 buttons packed into each bag.
There is a mixture of different coloured buttons in each bag.
A total of 600 bags of buttons were packed in a day.

The first 100 bags were checked and it was found that a total of 1200 red buttons had been used.
In the 600 bags of buttons, it was found that the relative frequency of red buttons packed was 40%.

Calculate the relative frequency of red buttons packed in the final 500 bags.

Total button packed = $80 \times 600 = 48000$ button M2

N° of red button $40\% \times 48000 = 19200$ red A1

N° of reds in Final 500 = $19200 - 1200 = 18000$ red B1

Total button in Final 500 = $80 \times 500 = 40000$ M2

So relative freq in Final 500 = $\frac{18000}{40000} = \frac{9}{20} = 0.45$ A1
or 45%

[7]

Marking Scheme

1.

12 (a) 9/30 and 0.3	B1	
10/40 and 0.25	B1	FT from 'their 9/30' If B0 awarded for part (a) award SC1 for 9/30 AND 10/40
(b) 0.25 or equivalent	B1	FT their final column entry in (a)
Reason eg 'most throws', 'last value', 'uses all the data'.	E1	Do not accept 'better estimate'.
	4	

2.

8.(a) (14/20) | 22/40 | 36/60 | 52/80 | 62/100 0.7 | 0.55 | 0.6 | 0.65 | 0.62	B3	B2 for 6, 7 or 8 correct entries, including FT values and calculations, or B1 for 4 or 5 correct entries, including FT values and calculations
(b) 0.62 or equivalent	B1	FT their final column entry in (a)
Reason, e.g. "last value", "most throws"	E1	Do not accept 'most accurate'. Mark independently of B1 If no estimate given, but statement that 100 throws as more results then award B0, E1
(c) Conclusion, e.g. "settle", "stable", "smooth out", "would get a more accurate answer"	E1	Do not accept implication that it stays at 0.62
(d) 1- 0.62 or 1 – 62/100	M1	FT 1 – (b), or 1 – 'their final result in the table in (a)'
0.38 or 38/100 (=19/50)	A1	Ignore incorrect cancelling

3.

11.(a) Sight of (3), 8, 12, 16, 18, 20 OR 10, 20, 30, 40, 50, 60	B1	Cumulative totals FT both their cumulative totals for shots on target
3/10, 8/20, 12/30, 16/40, 18/50, 20/60	M1	Expressed as fractions, from both cumulative
0.3, 0.4, 0.4, 0.4, 0.36, 0.33 (not 0.3)	A2	Conversion to decimals. A1 for any 4 correct conversions
All their 6 points plotted accurately	P1	FT their cumulative decimals Do not award if 'bars' are drawn
11.(b) 'Yes' with reason, e.g. 'all around the same' or ' between 0.3 and 0.4' or '0.3(3..)' stated as an estimate, **OR** 'No' with reason, e.g. 'still swing in results', 'results still changing'	E1	Must FT as an interpretation of stability from cumulative totals used to create the graph in (a)

4.
5.

8(a) The last reading (0.44)	M1	Not from an incorrect calculation
E.G. "more potatoes" checked	A1	
(b) Use of 0.4 or sight of 40	B1	
40 × 0.15 or equivalent	M1	Ignore incorrect place value
(£)6 or 600(p)	A1	Accept 600 without units but not with incorrect unit
(c) 0.44	B1	CAO. Not from an incorrect method FT 'their 0.44' or (a), provided within the range 0.36 to 0.52 inclusive, excluding 0.5
× 900 × (0.0)2	M1	Ignore incorrect place value
792(p) or (£)7.92	A1	FT from M1
9 × 4.5(0)	M1	
(£)40.5(0)	A1	
(£)32.58 or 3258(p)	B1	CAO
	11	

6.

8. $80 \times 600 \times 0.4(0)$	M2	M1 for product of any two seen. Or equivalent calculation
$= 19200$	A1	
$19200 - 1200$ OR 18000 red buttons	B1	
$\div 500$ AND $\div 80$ OR $\div 40\,000$	m2	FT 'their 18000' provided M2 awarded m1 for $\div 500$ or $\div 80$
0.45 or 45%	A1	Accept 36 buttons per bag as evidence for m1 CAO