Components of forces

We have seen that two forces can be combined into a single force which is called their *resultant*.

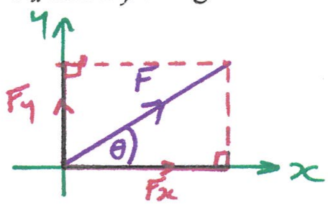
There is a reverse process which consists of expressing a single force in terms of its two *components*. These components are sometimes referred to as the *resolved parts* of the force.

It is particularly useful to find two mutually perpendicular components of a force.

The directions may, for example, be horizontal and vertical, or parallel and perpendicular to the surface of an inclined plane.

The component of the force F in any given direction is a measure of the effect of the force F in that direction.

Consider a force F acting at and angle θ to the x-axis as shown below. The components F_x and F_y being the horizontal and vertical components of F respectively.

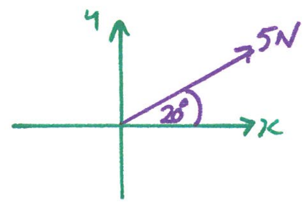


$$\frac{F_y}{F} = \frac{opp}{hyp} = \sin\theta \qquad \frac{F_x}{F} = \frac{adj}{hyp} = \cos\theta$$

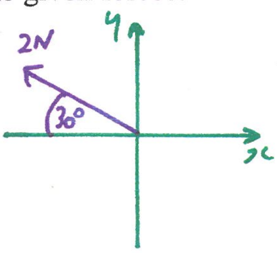
$$F_y = F\sin\theta \qquad F_x = F\cos\theta$$

Eg 1 Find the components F_x and F_y of the given forces:

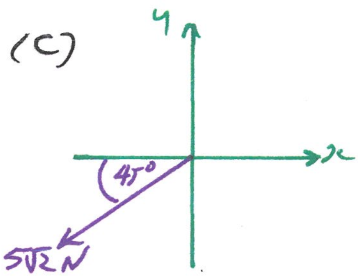
(a)



(b)



(c)

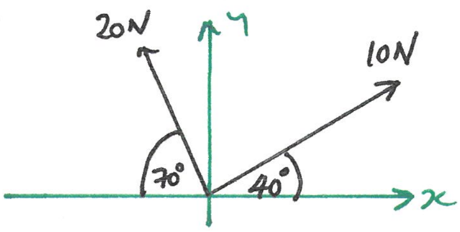


Exercise Q's 1 & 2

Eg 2 A body of mass 4kg rests on an incline of 35°. Find the component of the weight of the body parallel and perpendicular to the plane.

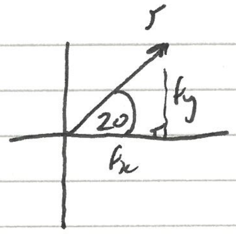
Exercise Q3

Eg 3 Find the sum of the components of the given forces in the direction of (i) x-axis (ii) y-axis.



Exercise Q's 4 & 5

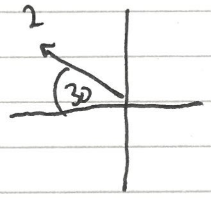
Eg1 ~~Eg13~~ (a)



$F_x = 5\cos 20 = 4.70\,N$

$F_y = 5\sin 20 = 1.71\,N$

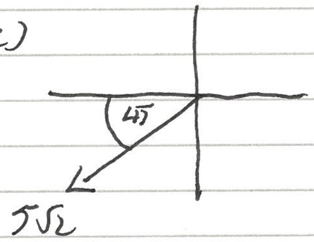
(b)



$F_x = -2\cos 30 = -2 \times \frac{\sqrt{3}}{2} = -\sqrt{3}\,N$

$F_y = +2\sin 30 = 2 \times \frac{1}{2} = 1\,N$

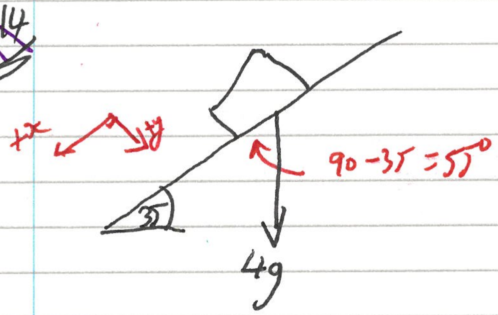
(c)



$F_x = -5\sqrt{2}\cos 45 = -5\sqrt{2} \times \frac{1}{\sqrt{2}} = -5\,N$

$F_y = -5\sqrt{2}\sin 45 = -5\sqrt{2} \times \frac{1}{\sqrt{2}} = -5\,N$

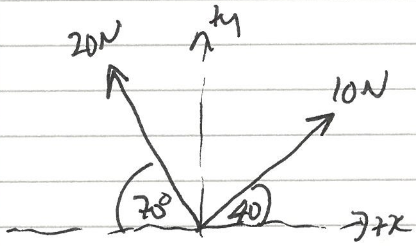
Eg2 ~~Eg14~~



$F_x = 4g\cos 55 = 22.5\,N$

$F_y = 4g\sin 55 = 32.1\,N$

Eg3 ~~Eg15~~

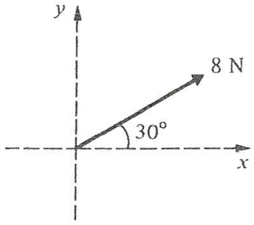


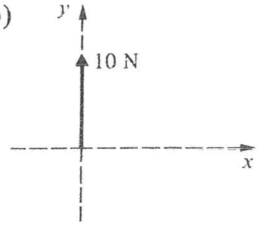
$\Sigma F_x = 10\cos 40 - 20\cos 70 = 0.82\,N$

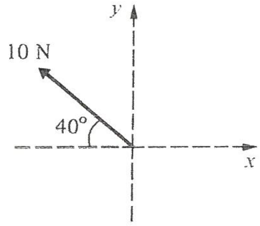
$\Sigma F_y = 10\sin 40 + 20\sin 70 = 25.2\,N$

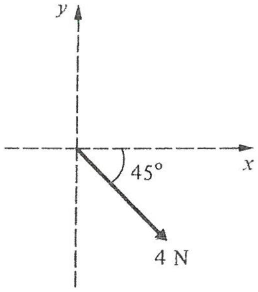
COMPONENTS of FORCES EXERCISE 6.1

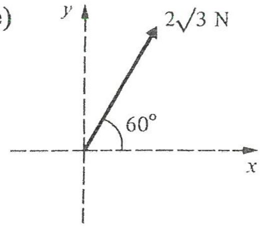
1. For each of the forces shown below, find the components in the direction of (i) the x-axis and (ii) the y-axis.

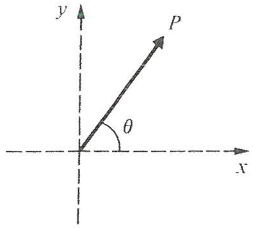
(a)


(b)


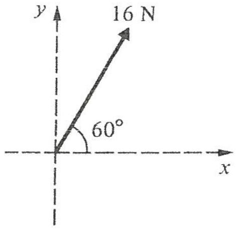
(c)


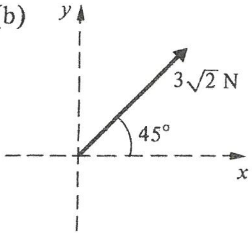
(d)


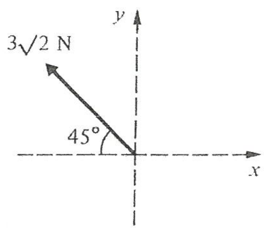
(e)


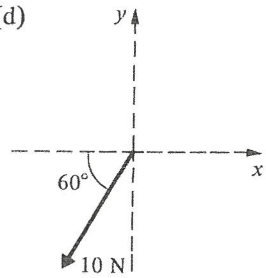
(f)


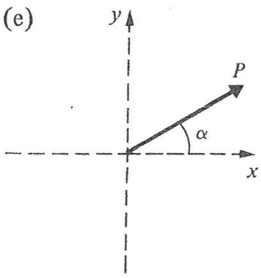
2. Express each of the following forces in the form $a\mathbf{i} + b\mathbf{j}$.

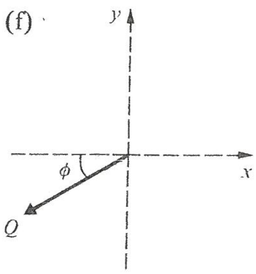
(a)


(b)


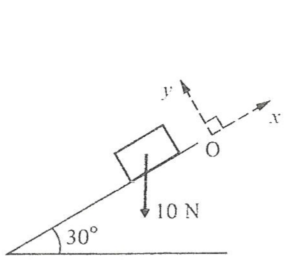
(c)


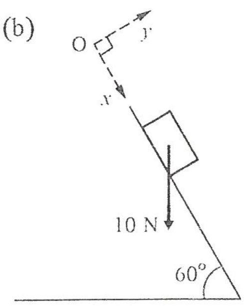
(d)


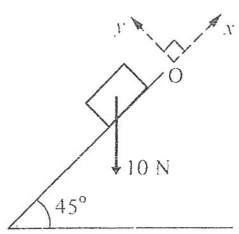
(e)


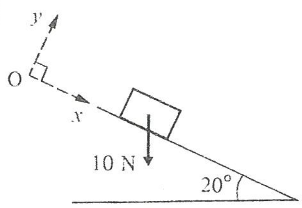
(f)


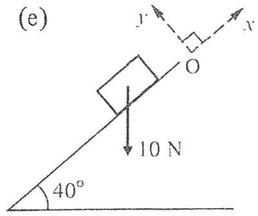
3. Each of the following diagrams shows a body of weight 10 N on an incline. In each case find the component of the weight of the body (i) in the Ox direction and (ii) in the Oy direction.

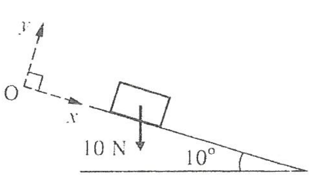
(a)


(b)


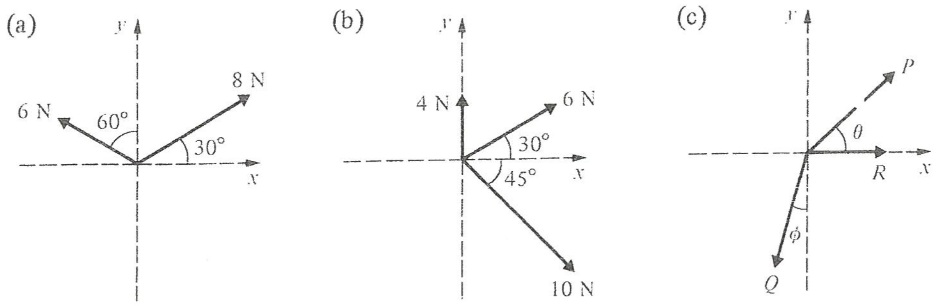
(c)


(d)


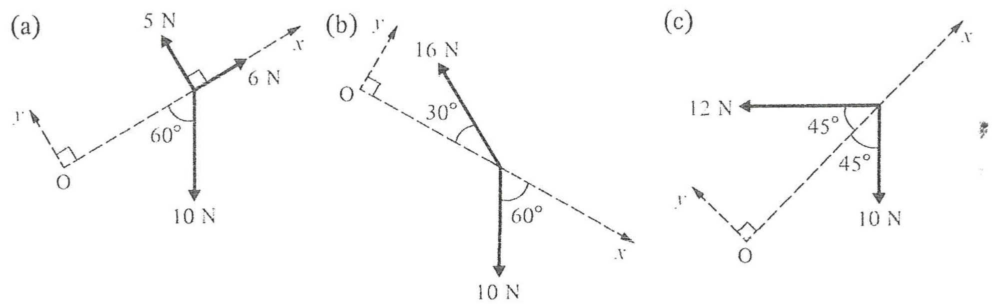
(e)


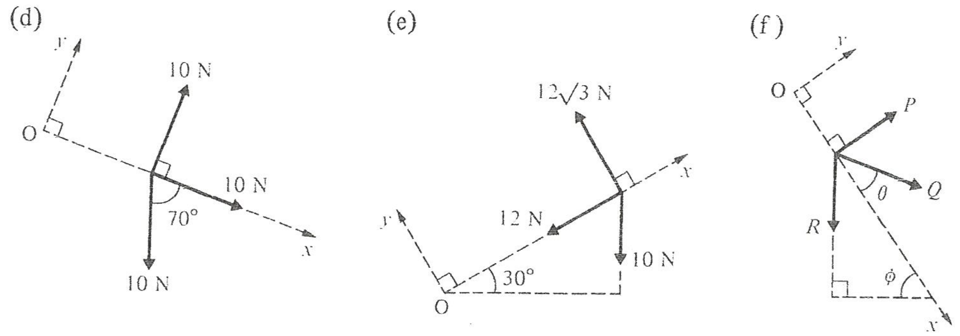
(f)


4. For each of the following systems of forces, find the sum of the components in the direction of (i) the x-axis and (ii) the y-axis.



5. For each of the following systems of forces, find the sum of the components (i) in the Ox direction and (ii) in the Oy direction.





ANSWERS

1. (a) (i) $4\sqrt{3}$ N (ii) 4 N (b) (i) 0 (ii) 10 N
(c) (i) $-7{\cdot}66$ N (ii) $6{\cdot}43$ N (d) (i) $2\sqrt{2}$ N (ii) $-2\sqrt{2}$ N
(e) (i) $\sqrt{3}$ N (ii) 3 N (f) (i) $P\cos\theta$ (ii) $P\sin\theta$

2. (a) $(8\mathbf{i} + 8\sqrt{3}\mathbf{j})$ N (b) $(3\mathbf{i} + 3\mathbf{j})$ N (c) $(-3\mathbf{i} + 3\mathbf{j})$ N (d) $(-5\mathbf{i} - 5\sqrt{3}\mathbf{j})$ N
(e) $P\cos\alpha\mathbf{i} + P\sin\alpha\mathbf{j}$ (f) $-Q\cos\phi\mathbf{i} - Q\sin\phi\mathbf{j}$

3. (a) (i) -5 N (ii) $-5\sqrt{3}$ N (b) (i) $5\sqrt{3}$ N (ii) -5 N
(c) (i) $-5\sqrt{2}$ N (ii) $-5\sqrt{2}$ N (d) (i) $3{\cdot}42$ N (ii) $-9{\cdot}40$ N
(e) (i) $-6{\cdot}43$ N (ii) $-7{\cdot}66$ N (f) (i) $1{\cdot}74$ N (ii) $-9{\cdot}85$ N

4. (a) (i) $\sqrt{3}$ N (ii) 7 N (b) (i) $12{\cdot}3$ N (ii) $-0{\cdot}071$ N
(c) (i) $P\cos\theta + R - Q\sin\phi$ (ii) $P\sin\theta - Q\cos\phi$

5. (a) (i) 1 N (ii) $-3{\cdot}66$ N (b) (i) $-8{\cdot}86$ N (ii) $-0{\cdot}66$ N
(c) (i) $-11\sqrt{2}$ N (ii) $\sqrt{2}$ N (d) (i) $13{\cdot}4$ N (ii) $0{\cdot}60$ N
(e) (i) -17 N (ii) $7\sqrt{3}$ N (f) (i) $R\sin\phi + Q\cos\theta$ (ii) $P + Q\sin\theta - R\cos\phi$

Further N2L Problems

Now that we can resolve forces, we can extend the complexity of the situations which require Newton's second law to solve.

Eg4 A body of mass 4kg has an acceleration *a* when it is acted upon by a force of $25\sqrt{2}N$ which is inclined at 45° to the smooth horizontal surface on which the body rests. By resolving the forces parallel and perpendicular to the plane, calculate the normal reaction between the body and the plane and the acceleration, *a*, of the body.

Exercise 6.2 Q's 1 to 5

Eg5 A body of mass $3\sqrt{3}kg$ on the surface of a smooth plane inclined at 60° is acted on by a horizontal force of 15g N. Calculate the normal reaction of the plane on the body, and the acceleration of the body up the surface of the smooth inclined plane.

Exercise 6.2 Q's 6 to 9

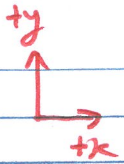
Rough Surfaces

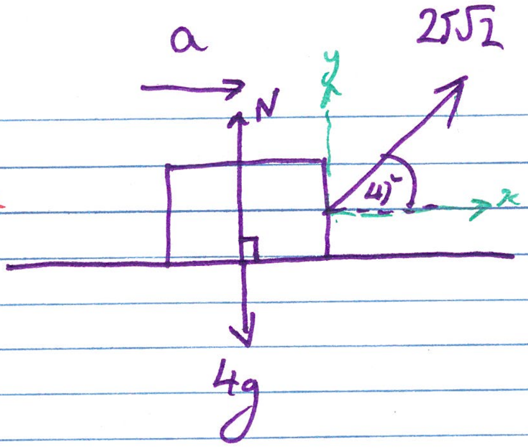
In the previous examples, motion has taken place on smooth surfaces. In practice this does not happen; all surfaces tend to impede motion. The resistance to motion is an external force acting upon the body, parallel to the surfaces in contact. It will be considered to be a constant force.

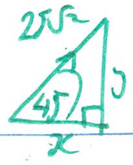
Eg6 A body of mass 5kg is released from rest on the surface of a rough plane which is inclined at $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If the body takes 3 seconds to reach a speed of 4.9ms^{-1} from rest, find the resistance to motion which the body must be experiencing.

Exercise 6.2 Q's 12 to 17

Eg 4







$x = 25\sqrt{2}\cos 45$

$y = 25\sqrt{2}\sin 45$

Σf_x: N2L $\Sigma f_x = ma$

Σf_y: no acceleration $\therefore$ equilibrium $\therefore$ $\Sigma f_y = 0$

Σf_x: $25\sqrt{2}\cos 45 = 4a$ — ①

Σf_y: $25\sqrt{2}\sin 45 + N - 4g = 0$ — ②

From ① $25\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} = 4a$

$4a = 25$

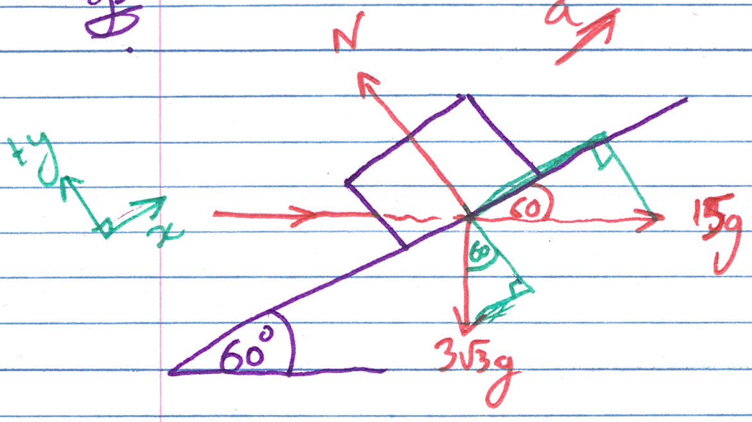
$a = \frac{25}{4} = 6.25\,ms^{-2}$ $\quad$ $6.3\,ms^{-1}$ to ~~1dp.~~ 2 s.f.

From ② $25\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} + N - 4g = 0$

$N = 4g - 25$

$N = 14.2\,N$ $\quad$ 14N sig fig

Eg5



① ↗ a

② ↙ a

③ → a

④ ← a

⑤ ↑ a

⑥ ↓ a

$\Sigma f_x = Ma$

$\Sigma f_y = 0$

$\Sigma f_x: \; 15g\cos 60 - 3\sqrt{3}g\sin 60 = 3\sqrt{3}\,a \quad -①$

$\Sigma f_y: \; N - 15g\sin 60 - 3\sqrt{3}g\cos 60 = 0 \quad -②$

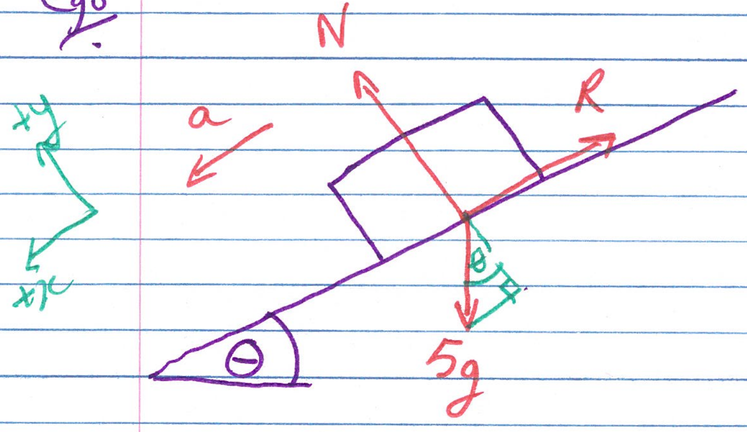
From ① $\quad a = 5.7$

From ② $\quad N = 15g\sin 60 + 3\sqrt{3}g\cos 60$

$= 152.8$

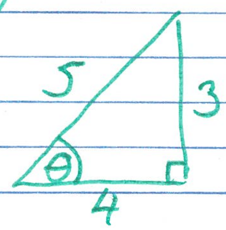
$= 150\,N \quad$ 2 s.f.

Eg6.



$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin\theta = \frac{3}{5}$ $\frac{opp}{hyp}$



$\Sigma F_x = Ma: \quad 5g\sin\theta - R = 5a$ — ①

$\Sigma F_y = 0 \quad : \quad N - 5g\cos\theta = 0$ — ②

Need to find a: $u=0,\ v=4.9,\ t=3,\ a=?$

$$v = u + at$$

$$4.9 = 0 + 3a$$

$$a = \frac{4.9}{3}$$

Sub in ① $\cancel{5}g\left(\frac{3}{\cancel{5}}\right) - R = 5\left(\frac{4.9}{3}\right)$

$$R = 3g - \frac{49}{6} = 21.2\,N.$$

Exercise 6.2

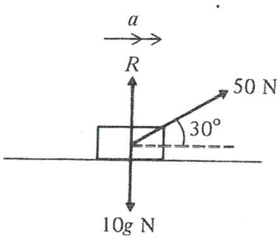
Each of the diagrams in questions **1** to **9** shows a body of mass 10 kg accelerating along a surface in the direction indicated. All of the forces acting are as shown. In each case

(a) obtain an equation by resolving perpendicular to the direction of motion,

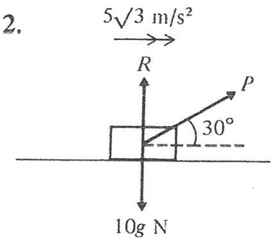
(b) obtain an equation by applying $F = ma$ parallel to the direction of motion,

(c) use your equations to (a) and (b) to find the unknown forces, accelerations and angles.

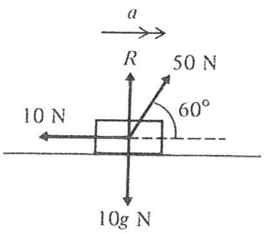
1.



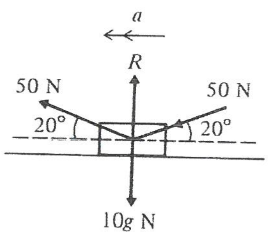
2.



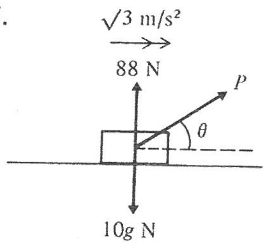
3.



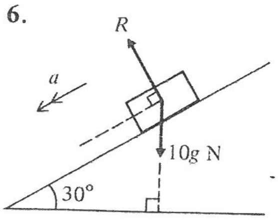
4.



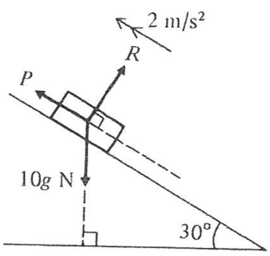
5.



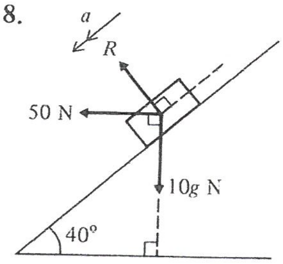
6.



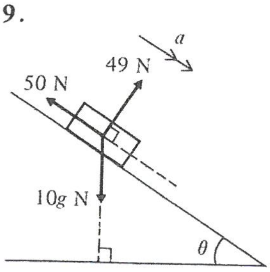
7.



8.



9.



12. A body of mass 10 kg is initially at rest on a rough horizontal surface. It is pulled along the surface by a constant force of 60 N inclined at 60° above the horizontal. If the resistance to motion totals 10 N, find the acceleration of the body and the distance travelled in the first 3 s.

13. A body of mass 5 kg, initially at rest on a smooth horizontal surface, is pulled along the surface by a constant force P inclined at 45° above the horizontal. In the first 5 seconds of motion the body moves a distance of 10 m along the surface. Find the acceleration of the body, the magnitude of P and the normal reaction between the body and the surface.

14. A mass of 5 kg is initially at rest at the bottom of a smooth slope which is inclined at $\sin^{-1}\frac{3}{5}$ to the horizontal. The mass is pushed up the slope by a horizontal force of 50 N. Find the normal reaction between the mass and the plane and the acceleration up the slope. How far up the slope will the mass travel in the first 4 s?

15. A body of mass 100 kg is released from rest at the top of a smooth plane which is inclined at 30° to the horizontal. Find the velocity of the body when it has travelled 20 m down the slope. What would your answer be if the mass had been 50 kg?

16. A body of mass 20 kg is released from rest at the top of a rough slope which is inclined at 30° to the horizontal. If the body accelerates down the slope at 3 m/s², find the resistance to motion experienced by the body. (Assume this resistance to be constant throughout).

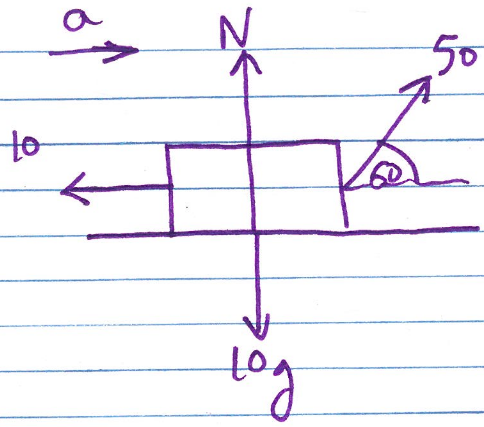
17. A body of mass 20 kg is released from rest at the top of a rough slope which is inclined at 30° to the horizontal. Six seconds later the body has a velocity of 21 m/s down the slope. Find the resistance to motion experienced by the body. (Assume this resistance to be constant throughout).

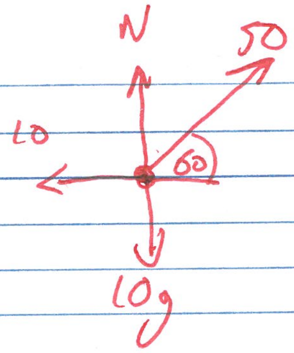
ANSWERS

1. (a) $R + 50\sin 30° = 10g$ (b) $50\cos 30° = 10a$ (c) $R = 73$ N, $a = \frac{5\sqrt{3}}{2}$ m/s²
2. (a) $R + P\sin 30° = 10g$ (b) $P\cos 30° = 50\sqrt{3}$ (c) $P = 100$ N, $R = 48$ N
3. (a) $R + 50\sin 60° = 10g$ (b) $50\cos 60° - 10 = 10a$ (c) $R = 54{\cdot}7$ N, $a = 1{\cdot}5$ m/s²
4. (a) $R + 50\sin 20° = 10g + 50\sin 20°$ (b) $50\cos 20° + 50\cos 20° = 10a$
   (c) $R = 98$ N, $a = 9{\cdot}40$ m/s²
5. (a) $88 + P\sin\theta = 10g$ (b) $P\cos\theta = 10\sqrt{3}$ (c) $P = 20$ N, $\theta = 30°$
6. (a) $R = 10g\cos 30°$ (b) $10g\sin 30° = 10a$ (c) $R = 84{\cdot}9$ N, $a = 4{\cdot}9$ m/s²
7. (a) $R = 10g\cos 30°$ (b) $P - 10g\sin 30° = 20$ (c) $R = 84{\cdot}9$ N, $P = 69$ N
8. (a) $R + 50\sin 40° = 10g\cos 40°$ (b) $50\cos 40° + 10g\sin 40° = 10a$
   (c) $R = 42{\cdot}9$ N, $a = 10{\cdot}1$ m/s²
9. (a) $49 = 10g\cos\theta$
   (b) $10g\sin\theta - 50 = 10a$ (c) $a = 3{\cdot}49$ m/s², $\theta = 60°$
10. 65·3 N, 1·63 m/s², 84·9 N 11. 34·6°, 80·7 N, 25·6 N 12. 2 m/s², 9 m
13. 0·8 m/s², $4\sqrt{2}$ N, 45 N 14. 69·2 N, 2·12 m/s², 16·96 m
15. 14 m/s, 14 m/s 16. 38 N 17. 28 N 18. 0·74 s
19. 3·8 m/s², 7·6 m 20. 31 N 21. 30° 22. 0·663 s

Ex 6.2

Q3





ΣF_x: N2L: $\quad 50\cos 60 - 10 = 10a \quad - (1)$

$\Sigma F_y = 0$: $\quad 50\sin 60 + N - 10g = 0 \quad - (2)$

From (1) $\quad 50 \times \frac{1}{2} - 10 = 10a$

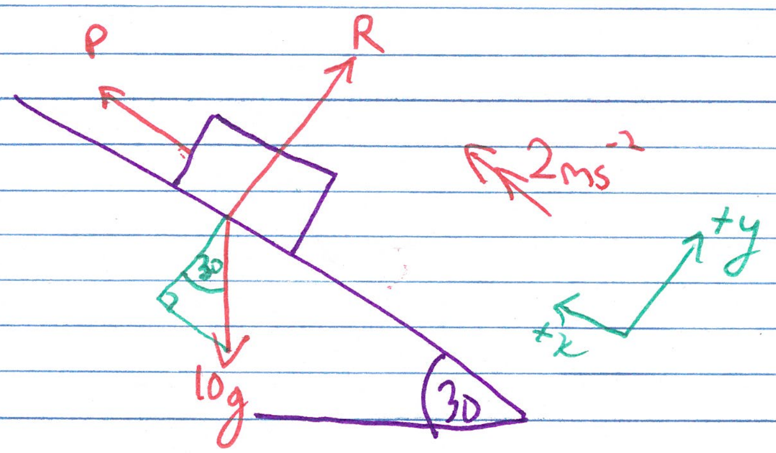
$25 - 10 = 10a$

$a = \frac{15}{10} = 1.5\,\text{ms}^{-2}$

From (2) $\quad N = 10g - \frac{50\sqrt{3}}{2}$

$= 54.7\text{ N}$

Q7



$\Sigma F_x = Ma \qquad P - 10g \sin 30 = 10 \times 2 \qquad — (1)$

$\Sigma F_y = 0 \qquad R - 10g \cos 30 = 0 \qquad — (2)$

From (1) $\quad P = 20 + 10g \cos 30$

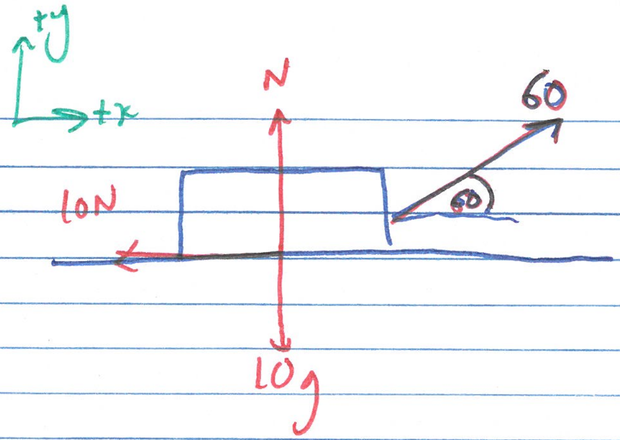
$P = 104.87\ldots$

$= 100$ to 2sf

From (2) $\quad R = 10g \sin 30$

$= 49N$

(12)



$\Sigma F_x = Ma$ $\quad 60\cos 60 - 10 = 10a$ — (1)

$\Sigma F_y = 0$ $\quad N + 60\sin 60 - 10g = 0$ — (2)

From (1) $\quad a = \dfrac{60\cos 60 - 10}{10} = 2 ms^{-2}$

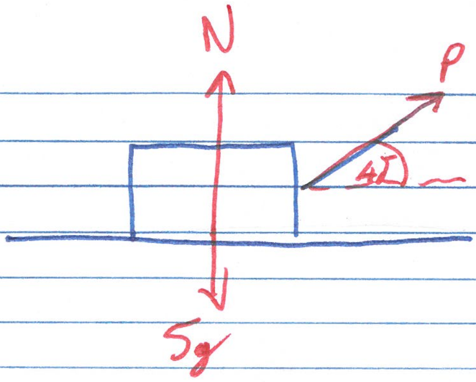
Using SUVAT: $u = 0$, $t = 3$, $a = 2$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2} \times 2 \times 3^2$

$s = 9$ meters.

(13)



$\Sigma F_x = Ma \qquad P\cos 45 = 5a \qquad$ — (1)

$\Sigma F_y = 0 \qquad N + P\sin 45 - 5g = 0 \qquad$ — (2)

Suvat: $u = 0,\ t = 5,\ s = 10,\ a = ?$

$$10 = 0 + \frac{1}{2} \times a \times 5^2$$

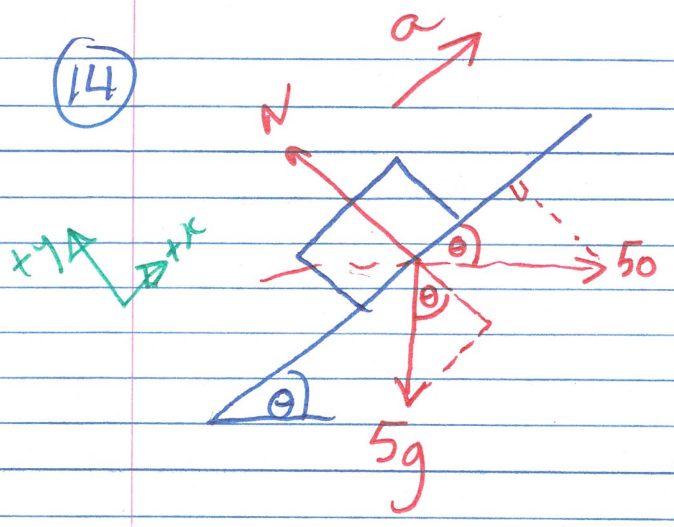
$$10 = \frac{25}{2} a$$

$$a = \frac{20}{25} = 0.8\ ms^{-2}$$

In (1) $\quad P\cos 45 = 5(0.8)$

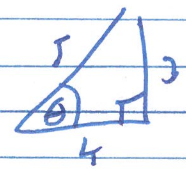
$$P = \frac{4}{\cos 45} = 4\sqrt{2}\ N$$

14



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



$\Sigma F_x = ma$: $50\cos\theta - 5g\sin\theta = 5a$ — (1)

$\Sigma F_y = 0$: $N - 5g\cos\theta - 50\sin\theta = 0$ — (2)

From (1) $50\left(\frac{4}{5}\right) - 5g\left(\frac{3}{5}\right) = 5a$

$a = 2.12\,ms^{2}$

From (2) $N = 5g\left(\frac{4}{5}\right) + 50\left(\frac{3}{5}\right) = 69.2N$

SUVAT: $u=0$, $a=2.12$, $t=4$, $s=?$

$s = 0 + \frac{1}{2} \times 2.12 \times 4^2$

$s = 16.96\,m$