Sine Rule, Cosine Rule & Area of $\triangle$

1. The diagram shows a circle with centre O and chord JK.

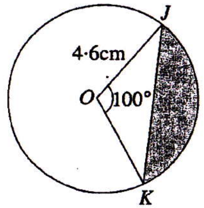


Diagram not drawn to scale.

The circle has a radius of 4·6 cm and $J\hat{O}K = 100°$.
Calculate the area of the shaded region.

Area of sector OJK $= \frac{100}{360} \times \pi \times 4.6^2 = 18.5$

Area of $\triangle$ OJK $= \frac{1}{2} \times 4.6 \times 4.6 \times \sin 100 = 10.4$

Area of shaded region $= 18.5 - 10.4 = 8.1\text{cm}^2$

[6]

Turn over.

2.

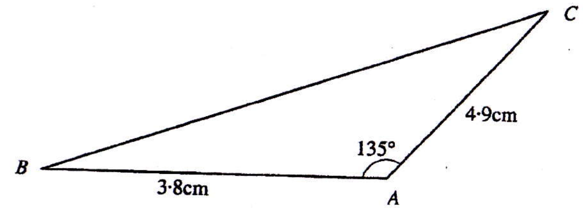


Diagram not drawn to scale.

In triangle ABC, $B\hat{A}C = 135°$ measured correct to the nearest degree.
$AC = 4·9$ cm and $AB = 3·8$ cm both measured correct to the nearest mm.

Find correct to three significant figures, the greatest possible area of the triangle ABC.

Max area when lengths and angles are Maximum

$A_{MAX} = \frac{1}{2} \times 3.85 \times 4.95 \times \sin 135.5$

$= 6.7 \text{ cm}^2$

[3]

(3) The diagram shows two triangles *ABC* and *ACD* with the common side *AC*.

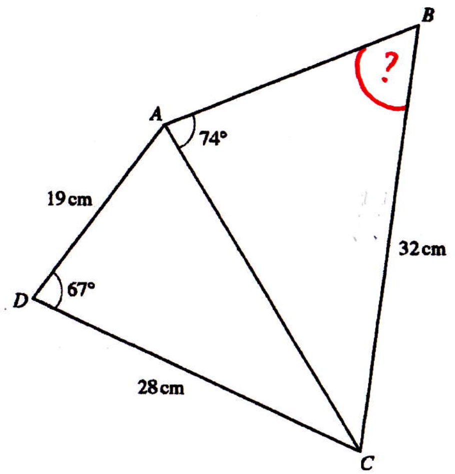
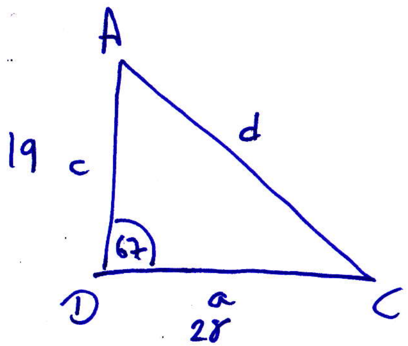
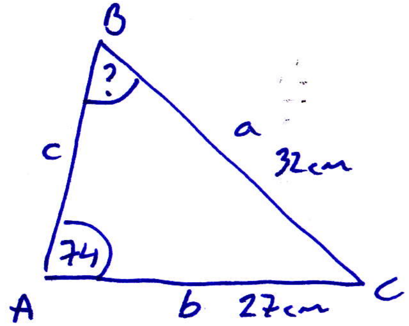


Diagram not drawn to scale.





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The triangles *ABC* and *ACD* are such that *BC* = 32 cm, *AD* = 19 cm, *CD* = 28 cm, $B\hat{A}C = 74°$ and $A\hat{D}C = 67°$.

Find the size of $A\hat{B}C$.

Need to find length of AC using Δ ADC

Using Cos Rule: $d^2 = a^2 + c^2 - 2ac\cos D$

$d^2 = 19^2 + 28^2 - 2\times19\times28\cos 67$

$d^2 = 1145 - 415.74$

$d^2 = 729.26$

$d = \sqrt{729.26} = 27.0$ cm

Using Sin Rule: $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{27} = \frac{\sin 74}{32}$

$\sin B = \frac{\sin 74}{32} \times 27$

$\sin B = 0.811$

$B = \sin^{-1}(0.811) = 54.2°$

[6]

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(4) The diagram shows triangle PQR.

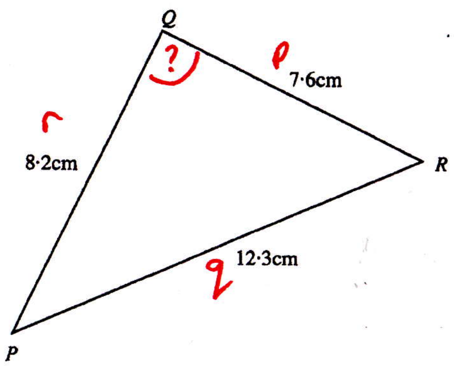

Diagram not drawn to scale.

The triangle PQR is such that $QR = 7{\cdot}6$ cm, $PR = 12{\cdot}3$ cm and $PQ = 8{\cdot}2$ cm.

Using Cos Rule

(a) Find the size of $P\hat{Q}R$.

$q^2 = r^2 + p^2 - 2rp\cos Q$

$2rp\cos Q = r^2 + p^2 - q^2$

$\cos Q = \dfrac{r^2+p^2-q^2}{2rp}$

$\cos Q = \dfrac{(8{\cdot}2^2 + 7{\cdot}6^2 - 12{\cdot}3^2)}{(2 \times 8{\cdot}2 \times 7{\cdot}6)} = \dfrac{-26{\cdot}29}{124{\cdot}64}$

$Q = \cos^{-1}\left(\dfrac{-26{\cdot}29}{124{\cdot}64}\right) = 102{\cdot}2^\circ$

[3]

(b) Find the area of triangle PQR.

$\text{Area} = \frac{1}{2} \times 8{\cdot}2 \times 7{\cdot}6 \times \sin 102{\cdot}2 = 30{\cdot}5\,\text{cm}^2$

[2]

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(5) The diagram shows triangle GHK.

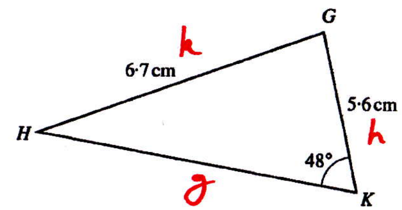

Diagram not drawn to scale.

Given that $GH = 6{\cdot}7$ cm, $GK = 5{\cdot}6$ cm and $G\hat{K}H = 48°$, calculate the area of the triangle GHK.

Need to find angle G

Can find angle H using Sine Rule:

$\dfrac{\sin H}{h} = \dfrac{\sin 48}{6{\cdot}7}$

$\sin H = \dfrac{\sin 48}{6{\cdot}7} \times 5{\cdot}6 = 0{\cdot}6211$

$H = \sin^{-1}(0{\cdot}6211) = 38{\cdot}4^\circ$

So $G = 180 - 48 - 38{\cdot}4 = 93{\cdot}6^\circ$

$\therefore$ Area of $\Delta = \frac{1}{2} \times 6{\cdot}7 \times 5{\cdot}6 \times \sin 93{\cdot}6$

$= 18{\cdot}7\,\text{cm}^2$

[6]