2

Formula List

Volume of prism = area of cross-section × length
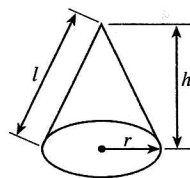

cross-section / length
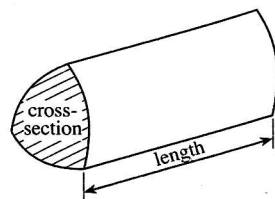
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$
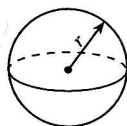
Volume of cone = $\frac{1}{3}\pi r^2 h$

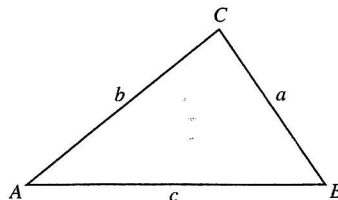**Curved surface area of cone** = $\pi r l$

In any triangle ABC

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

3

1. Clearly showing how you obtained your answer, ESTIMATE the value of:

$$\frac{87 \times 248}{52}$$

$$\frac{90 \times 200}{50} = \frac{18\,000}{50} = \frac{1800}{5} = 360$$

[2]

2. (a) Find 240 as a percentage of 600.

$$\frac{\overset{40}{\cancel{240}}}{\cancel{600}} \times \cancel{100} = 40\%$$

[2]

(b) A recipe for making 12 pancakes includes the following ingredients.

2 large eggs
200ml milk
110g flour

Calculate the quantities of these ingredients needed to make 30 pancakes.

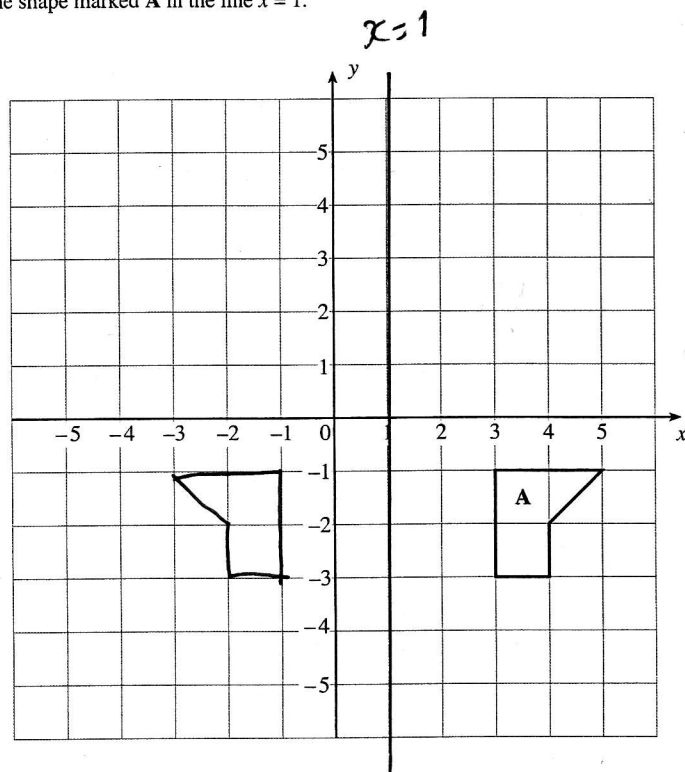30 pancakes = 24 + 6 pancakes
double + half quantities

5 eggs.
500 milk
275 flour

[3]

Turn over.

3. Reflect the shape marked **A** in the line $x = 1$.

$x = 1$

[2]

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4. In a game a player rolls a coin onto a squared board. The squares on the board are coloured red, blue, green or yellow. If the coin lands entirely within one of these coloured squares the player wins a prize, otherwise the player loses.

The table below shows the probabilities of the coin landing entirely within the coloured squares.

Colour	Red	Blue	Green	Yellow	Player loses
Probability	0·15	0·09	0·05	0·06	0·65

(a) One day 200 people play this game. Approximately how many would you expect to win a prize?

$0.35 \times 200 = \frac{35}{100} \times 200 = 70$

[2]

(b) It costs 80p to play the game once. The prize for winning is £2. If the 200 people play the game once, approximately how much profit do you expect the game to make?

game earns $200 \times 80p = £160$
game pays $70 \times £2 = £140$

profit £20.

[2]

Turn over.

5. *(a)* The angles of a quadrilateral are $x°$, $49°$, $3x°$ and $111°$.
Form an equation in x, and use your equation to find the value of x.

$$x + 49 + 3x + 111 = 360$$
$$4x + 160 = 360$$
$$4x = 360 - 160$$

$$4x = 200$$
$$x = \frac{200}{4} = 50°$$

[4]

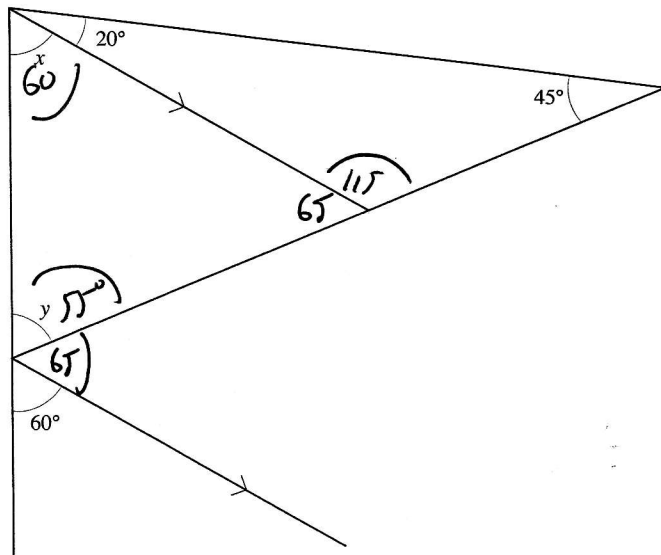*(b)* Find the size of **each** of the angles marked x and y in the following diagram.

Diagram not drawn to scale.

$x = 60$ ° $\quad y = 55$ °

[3]

6. *(a)* Solve **each** of the following equations.

(i) $7x + 4 = 3x + 16$

$$7x - 3x = 16 - 4$$
$$4x = 12$$
$$x = \frac{12}{4} = 3$$

(ii) $3x + 2 = 2(3 - 2x)$

$$3x + 2 = 6 - 4x$$
$$3x + 4x = 6 - 2$$
$$7x = 4$$
$$x = \frac{4}{7}$$

[6]

(b) Simplify **each** of the following.

(i) $2(3r + 1) + 5r$

$$6r + 2 + 5r$$
$$= 11r + 2$$

(ii) $3(2p + 3) - 2(p - 1)$

$$6p + 9 - 2p + 2$$
$$4p + 11$$

[4]

Turn over.

7. The following diagram shows a paved area with a circular pond of radius 10m.

40 m

40

50 m

1600

10 m

50

10 m

10 m

50

5 m

5 m
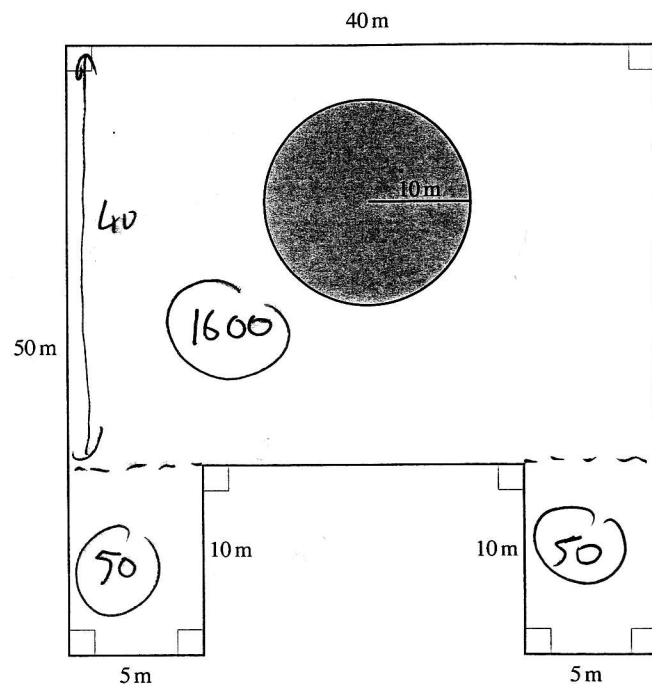
Using the value of π as 3·14, calculate the area of the paved surface clearly indicating the units of your answer.

Area of whole shape = 1600 + 50 + 50 = 1700

Area of pond = $\pi \times 10^2$ = 3·14 × 100 = 314
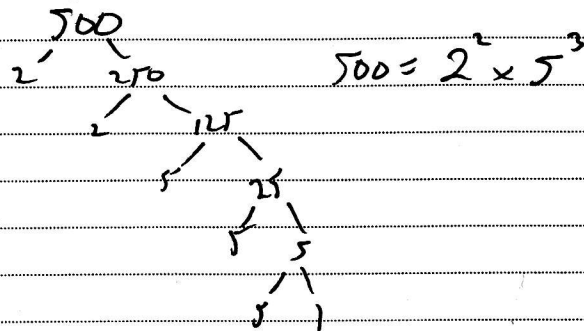
Area of paving = 1700 - 314 = 1386 m^2

[6]

8. Showing **all** your working, estimate the value of: $\dfrac{601·9 \times 19·94}{0·305}$

$\dfrac{600 \times 20}{0·3} = \dfrac{12000}{0·3} \times \dfrac{10}{10} = \dfrac{120000}{3} = 40000$

[3]

Turn over.

9. Express 500 as a product of prime numbers in index form.

$$500$$

$$2 \quad 250 \qquad 500 = 2^2 \times 5^3$$

$$2 \quad 125$$

$$5 \quad 25$$

$$5 \quad 5$$

$$5 \quad 1$$

[3]

10. Write down, in terms of n, the nth term of **each** of the following sequences.

(a)   3 $\overset{4}{}$ 7 $\overset{4}{}$ 11 $\overset{4}{}$ 15 $\overset{4}{}$ 19   ....

$$4n - 1$$

[2]

(b) 1×3 2×4 3×5 4×6 ....
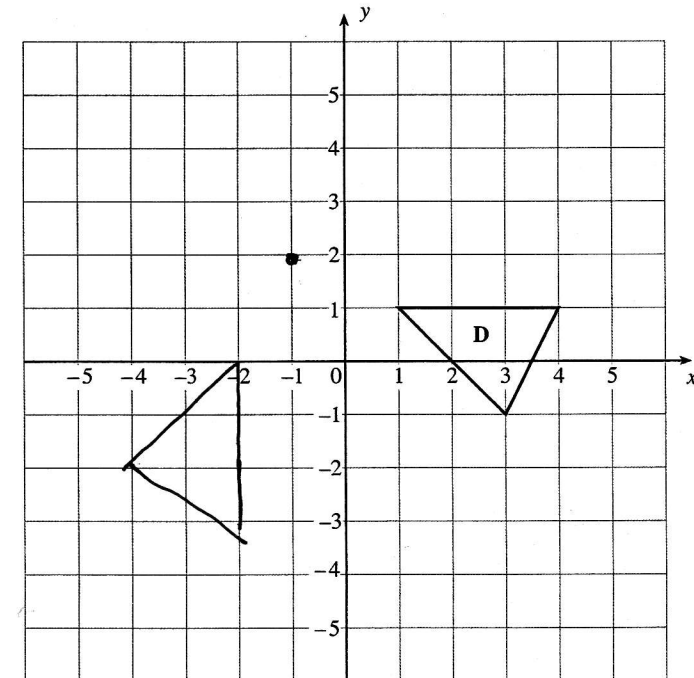
$$n(n+2)$$

[2]

11. *(a)* Draw the reflection of the triangle *ABC* in the line $y = x$.

[2]

$y = x$
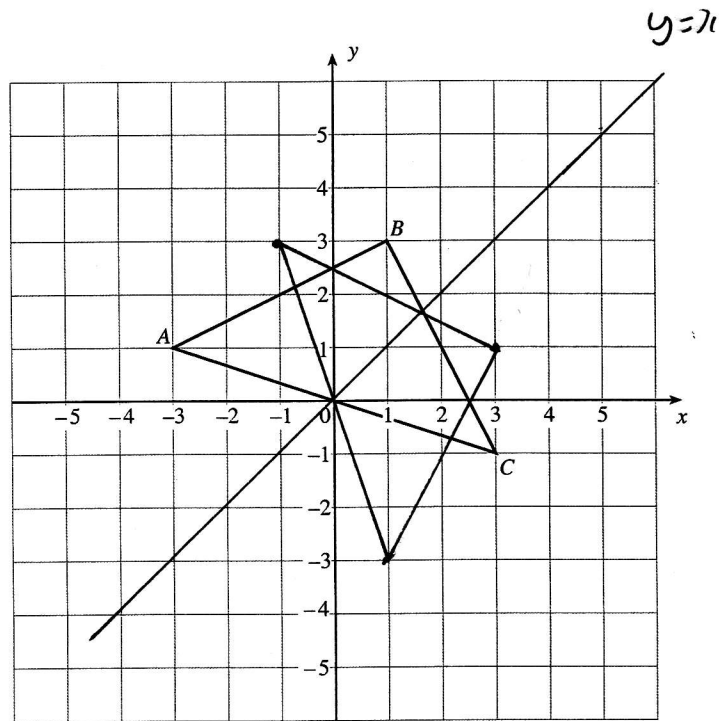
(b) Rotate the triangle **D** through 90° clockwise about the point (–1, 2).
Label the image **E**.

[2]

Turn over.

12. Solve the following simultaneous equations by an algebraic (not graphical) method.

$$2x + 5y = 4 \quad ①$$
$$3x + 4y = 13 \quad ②$$

① ×3 $\quad 6x + 15y = 12 \quad —③$

② ×2 $\quad 6x + 8y = 26 \quad —④$

③ − ④ $\quad 7y = -14$

$$y = \frac{-14}{7} = -2$$

In ① $\quad 2x + 5 \times -2 = 4$

$$2x - 10 = 4$$
$$2x = 14$$
$$x = \frac{14}{2} = 7$$

[4]

13. *(a)* Simplify $4p^3r^6 \times 3pr^2$.

$$12p^4r^8$$

[2]

(b) Factorise $6a^2b + 9a$.

$$3a(2ab + 3)$$

[2]

14. In each of the following formulae, every letter stands for the measurement of a length. By considering the dimensions implied by the formulae, write down, for each case, whether the formulae could be for a length, an area, a volume or none of these.
The first one has been done for you.

Formula could be for

$r^2 + dh$ $\qquad$ area

$r^2(2d - h) \qquad L^2(L - L) = L^2 \times L = L^3 = Vol$

$3d + 2h - r \qquad L + L - L = L = length$

$5r + 6dr + 2d \qquad L + L^2 + L = L + L^2 = nonsense$

[1]

15. *(a)* Solve the inequality

$$13 - 3x \geqslant 22 - 7x.$$

$$7x - 3x \geqslant 22 - 13$$
$$4x \geqslant 9$$
$$x \geqslant \frac{9}{4} \qquad x \geqslant 2\frac{1}{4}$$

[2]

(b) Write down the smallest whole number that satisfies this inequality.

$$3$$

[1]

16. In the diagram, *AB* is parallel to *DE*.
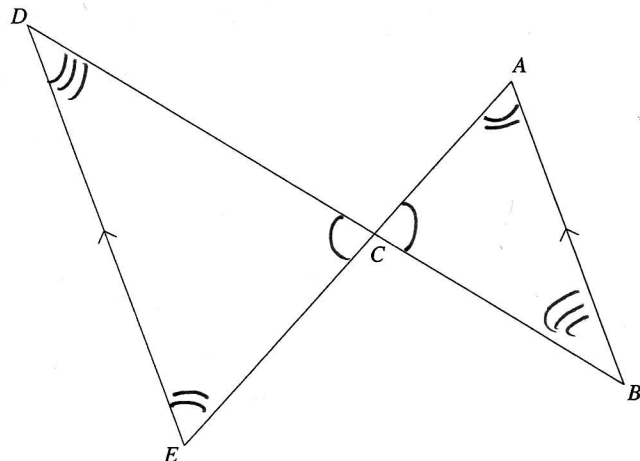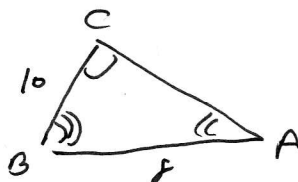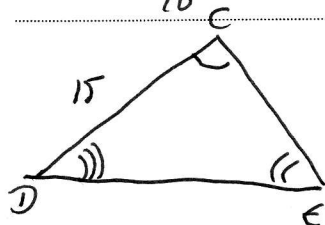
(a) Show that triangles *ABC* and *EDC* are similar.

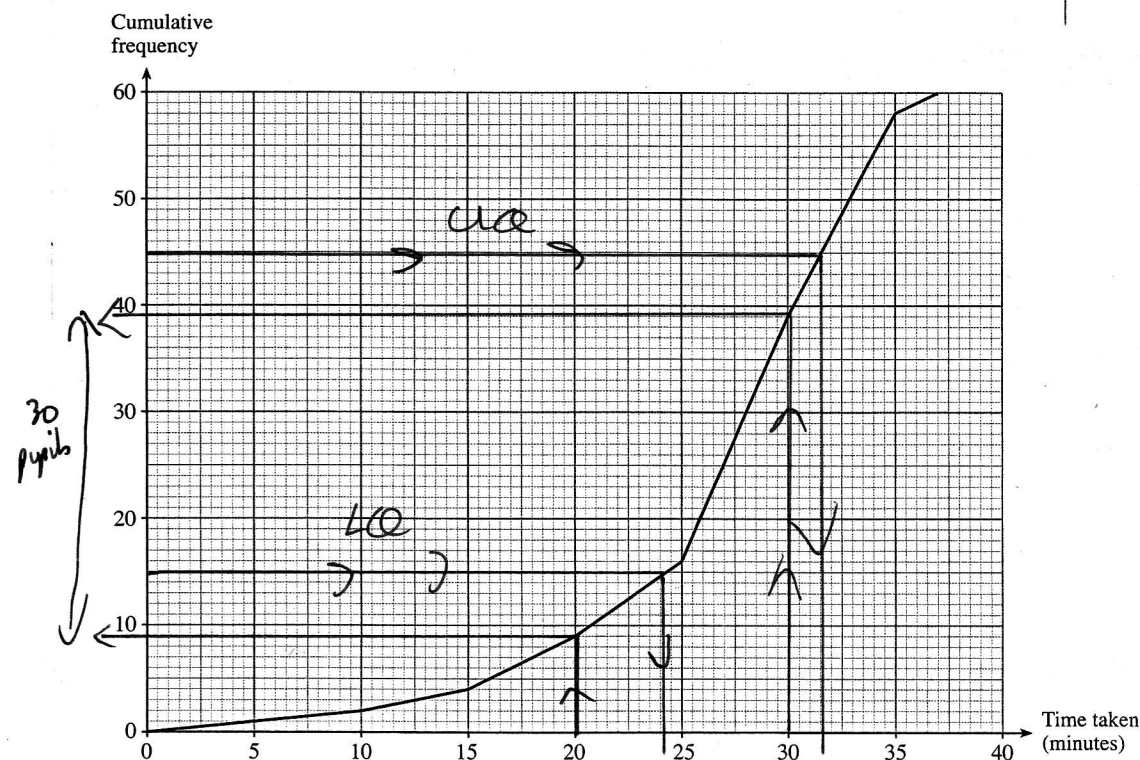*Diagram not drawn to scale.*

All angles are the same

[2]

(b) The lengths *AB* = 8cm, *BC* = 10cm and *DC* = 15cm.
Calculate the length of *DE*.

$$\frac{DE}{8} = \frac{15}{10}$$

$$DE = \frac{15 \times 8}{10} = \frac{120}{10} = 12cm$$

[2]

15
10

17. Each of 60 pupils is given a task and the time taken to complete the task is recorded. The results are summarised in the cumulative frequency polygon below.

Cumulative frequency

30 pupils

UQ

LQ

Time taken (minutes)

Use the cumulative frequency polygon to answer the following questions.

(a) Find an estimate for the interquartile range.

$$31.5 - 24 = 7.5$$

[2]

(b) Giving full details, find an estimate for the number of pupils that take between 20 and 30 minutes to complete the task.

$$39 - 9 = 30 \text{ pupils}$$

[2]

Turn over.

18. *(a)* Factorise $16p^2 - 25$.

$$(4p + 5)(4p - 5)$$

[2]

(b) Factorise $4q^2 + 3q - 10$.

$$(-40) + 8q, -5q$$

$$4q^2 + 8q - 5q - 10$$

$$4q(q+2) - 5(q+2) \qquad (4q-5)(q+2)$$

[2]

19. Make h the subject of the formula

$$10(h - 2e) = 7(h - k).$$

$$10h - 20e = 7h - 7k$$

$$10h - 7h = 20e - 7k$$

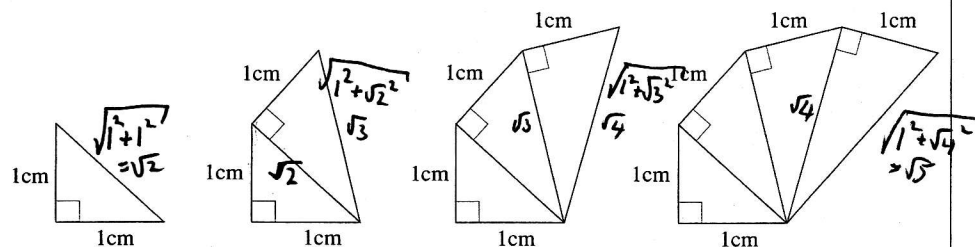$$3h = 20e - 7k$$

$$h = \frac{20e - 7k}{3}$$

[3]

20. Patterns are generated as shown in the diagram.

Pattern 1   Pattern 2   Pattern 3   Pattern 4

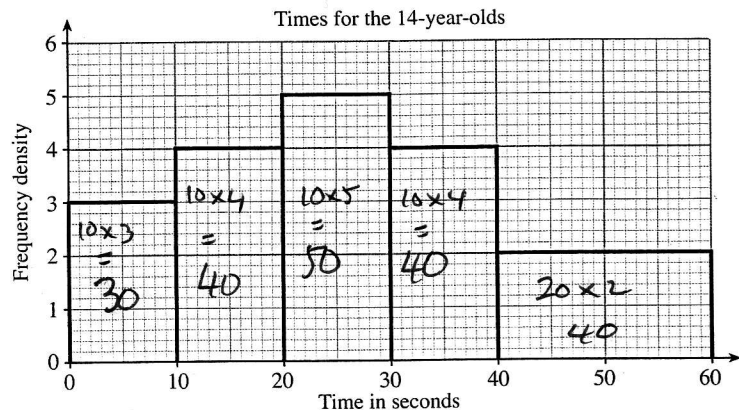$2 + \sqrt{2}$ $3 + \sqrt{3}$ $4 + \sqrt{4}$ $5 + \sqrt{5}$

Diagrams not drawn to scale.

Find the perimeter of Pattern 6 in the form $a + \sqrt{b}$, where a and b are whole numbers. Show your working.

$$7 + \sqrt{7}$$

[4]

Turn over.

21. As part of an investigation, the time taken to complete an obstacle course was measured for each pupil in a group of fourteen-year-olds.
The results are summarised in the histogram below.

Times for the 14-year-olds

(a) Use the histogram to calculate the number of fourteen-year-olds in this group.

$$30 + 40 + 50 + 40 + 40 = 200$$

[3]

(b) The time taken to complete the same obstacle course was measured for each student in a group of 200 eighteen-year-olds.
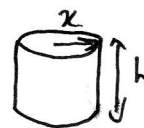The following grouped frequency distribution was obtained.

Time, t seconds	$0 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 60$
Numbers of students	10	20	30	40	100

Find an estimate for the median of this distribution.

Median = 100ᵗʰ person

40 seconds

[1]

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22. A cylinder, with base of radius x cm and height h cm, has the same volume as a cuboid with length x cm, width x cm and height $(x + 2)$ cm.
Find an expression for h in terms of x and π, simplifying your answer.

Volume of cylinder $= \pi x^2 h$

Volume of cuboid $= x \times x \times (x+2) = x^2(x+2)$

Volumes are equal so $\pi x^2 h = x^2(x+2)$

$$h = \frac{x^2(x+2)}{x^2 \pi} = \frac{x+2}{\pi}$$

[5]

23. *(a)* Write down a value of x (where $x > 1$) for which $x^{\frac{3}{2}}$ is rational.

④ $(4)^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

[1]

(b) Express $0.5\dot{4}\dot{1}$ as a fraction.

$x = 0.5414141\ldots$

$10x = 5.414141\ldots — ①$

$100x = 54.14141\ldots$

$1000x = 541.414141\ldots — ②$

② ① $990x = 536$

$$x = \frac{536}{990}$$

[2]

Turn over.

24. A box contains 2 strawberry yogurts, 4 vanilla yogurts and 6 cherry yogurts.
Three yogurts are selected at random from the box.
Calculate the probability that at least one of the selected yogurts is a cherry yogurt.

$P(\text{at least one cherry}) = 1 - \text{no cherry}$

$$= 1 - \left(\frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} \right)$$

$$= 1 - \frac{120}{1320}$$

$$= \frac{1320}{1320} - \frac{120}{1320}$$

[3]

$$= \frac{1200}{1320}$$

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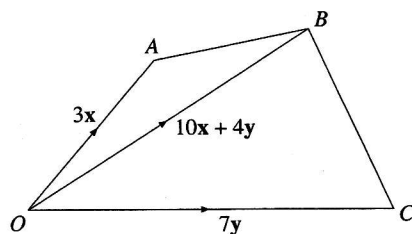
25. The diagram shows a quadrilateral *OABC*.

Diagram not drawn to scale.

In the quadrilateral *OABC*, the vectors **OA**, **OB** and **OC** are given by
OA = 3**x**, **OB** = 10**x** + 4**y** and **OC** = 7**y**.

(a) Express **CA** in terms of **x** and **y**.

$$\overrightarrow{CA} = -\overrightarrow{OC} + \overrightarrow{OA}$$
$$= -7y + 3x$$

[1]

(b) Given that *M* is the mid-point of *OB*, express each of the following in terms of **x** and **y** in their simplest form.

(i) **OM**

$$OM = \frac{1}{2}OB = \frac{1}{2}(10x+4y) = 5x+2y$$

(ii) **MA**

$$MA = -OM + OA$$
$$= -5x - 2y + 3x$$
$$= -2x - 2y$$

[2]

(c) Does *M* lie on the line *CA*? Give a reason for your answer.

No because if it did then CA would be a multiple of MA.

[1]