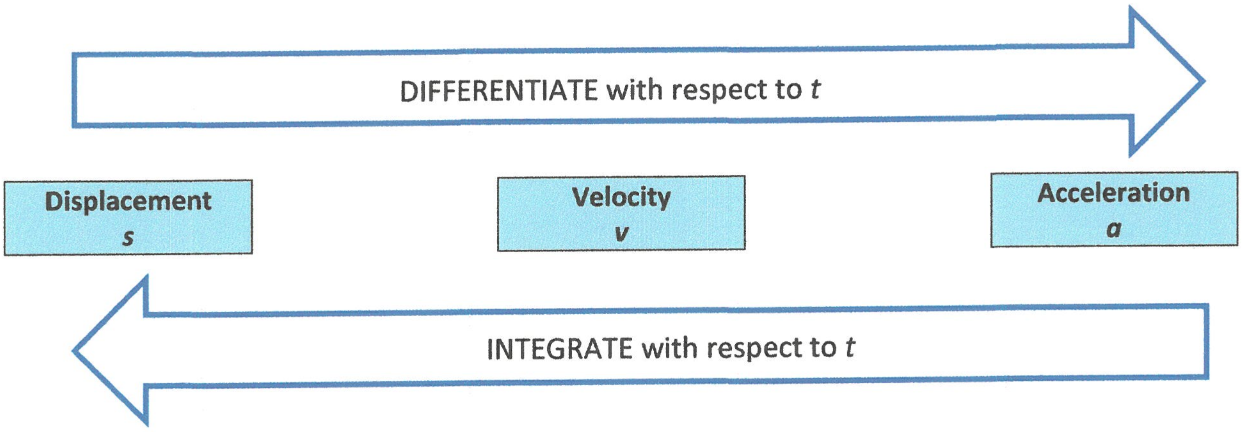
5. Managing Variable Acceleration Using Vectors

From Unit 2:

When acceleration is not uniform and the position, velocity and accelerations are given as functions of time then calculus is used to model particle motion:



Eg1 A particle P is moving along a straight line. At time t = 0, the particle is at point A and is moving with velocity 8ms^{-1} towards a point B on the line, where AB = 30m. At time t seconds (where t ≥ 0), the acceleration of P is (2 – 2t)ms^{-2} in the direction $\overrightarrow{AB}$.

a. Find an expression, in terms of t, for the displacement of P from A at t seconds
b. Show that P does not reach B.
c. Find the value of t when P returns to A, giving your answer to 3 significant figures.
d. Find the total distance travelled by P in the interval between the two instants when it passes through A.

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Eg2 A small metal ball moving in a magnetic field is modelled as a particle P of mass 0.2kg, moving in a straight line under the action of a single variable force F Newtons. At time t seconds, the displacement, x metres is given by $x = 3\sin 2t$.

Find the magnitude of F when $t = \frac{\pi}{6}$

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Unit 4:

When a particle is moving in a plane you can describe its position **r**, its velocity **v** and its acceleration **a** using vectors. The relationships between position (displacement), velocity and acceleration are the same in two dimensions as in one dimension.

Eg3 A particle P is moving in a plane. At time t seconds, its velocity **V**ms^{-1} is given by

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$$v = 3ti + \frac{1}{2}t^2j$$

When t = 0, the position vector of P is (2**i** - 3**j**)m. Find

a. the position vector of P at time t seconds
b. the acceleration of P when t = 3.

Eg4 A particle P is moving in a plane so that, at time t seconds, its acceleration is (4**i** – 2t**j**) ms^{-2}. When t = 3, the velocity of P is 6**i** ms^{-1} and the position vector of P is (20**i** + 3**j**)m with respect to the origin. Find

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a. the angle between the direction of motion of P and **i** when t = 2
b. the distance of P from O when t = 0.

Exercise 5.1 Qs 1 to 4

Eg5 A particle P of mass 0.5kg is moving under the action of a single force **F** newtons. At time t seconds, the position vector of P, **r** metres, is given by

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$$r = \left(\frac{3t^2}{2} - \frac{t^3}{3}\right)i + (2t^2 - 8t)j$$

Find

a. the value of t when P is moving parallel to the vector **i**
b. the magnitude of **F** when t = 3.5

Eg6 The velocity of a particle P at time t seconds is ((3t² – 8)**i** + 5**j**) ms^{-1}. When t = 0, the position vector of P with respect to a fixed origin is (2**i** – 4**j**)m.

a. Find the position vector of P after t seconds

A second particle Q moves with constant velocity (8**i** + 4**j**) ms^{-1}. When t = 0 the position vector of Q with respect to the origin is 2**i** m.

b. Prove that P and Q collide.

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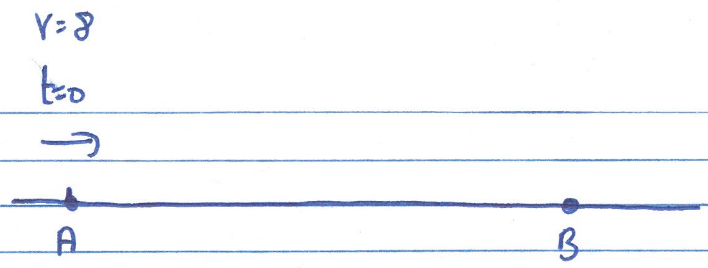
Exercise 5.1 Remaining Odds

Ex 5.1 Numerical Answers

1 a $(3\mathbf{i} + 23\mathbf{j})\,m\,s^{-1}$ b $18\mathbf{j}\,m\,s^{-2}$
2 a $(2t\mathbf{i} + 2\mathbf{j})\,m\,s^{-2}$ b $(3\frac{1}{3}\mathbf{i} + 2\mathbf{j})\,m$
3 a $16\mathbf{i}\,m\,s^{-1}$ b $(128\mathbf{i} - 104.8\mathbf{j})\,m$
4 a $20\,m\,s^{-1}$ b $10\,m\,s^{-2}$
5 a $6\sqrt{5}\,m\,s^{-1}$ b $t = 2$ c $(-16\mathbf{i} + 4\mathbf{j})\,m$
6 a $((3t^2 + 9)\mathbf{i} + (4t^3 - 10t + 6)\mathbf{j})\,m\,s^{-1}$
b 34° (nearest degree)
7 a 13 m b $(4\mathbf{i} + 6\mathbf{j})\,m\,s^{-2}$
8 a $\mathbf{v} = (t^2 - 40)\mathbf{i} + (6t - 24)\mathbf{j}$
b $7.81\,m\,s^{-1}$ (3 s.f.)
9 a $k = -0.5, -8.5$
b $10\,m\,s^{-2}$ for both values of k
10 a $\left(\left(\frac{5t^2}{2} - 3t + 2\right)\mathbf{i} + \left(8t - \frac{t^2}{2} - 5\right)\mathbf{j}\right)m\,s^{-1}$
b $t = \frac{1}{2}$ c $\frac{9\sqrt{2}}{8}\,m\,s^{-1}$
11 a $\left((t^2 + 6)\mathbf{i} + \left(2t - \frac{t^3}{3}\right)\mathbf{j}\right)m$ b $6\mathbf{j}\,m$
12 a $(45\mathbf{i} + 67.5\mathbf{j})\,m$ b $(135\mathbf{i} - 90\mathbf{j})\,m\,s^{-1}$

Eg1



(a) $a = 2 - 2t$

$v = \int a \, dt = \int 2 - 2t \, dt$

$v = 2t - t^2 + c$

when $t=0$, $v=8$ $\therefore c=8$

$v = 2t - t^2 + 8$

$s = \int v \, dt = \int 2t - t^2 + 8$

$s = t^2 - \frac{1}{3}t^3 + 8t + c$

Since when $t=0$, $s=0$ $\therefore c=0$

$s = t^2 - \frac{1}{3}t^3 + 8t$

(b) particle changes direction when $v=0$

$t^2 - 2t - 8 = 0$

$t = 4$

Now @ $t=4$, $s = 4^2 - \frac{1}{3}4^3 + 8(4) = 26\frac{2}{3}$ m $< \overrightarrow{AB}$

(c) time when $s=0$ $\quad t\left[t - \frac{1}{3}t^2 + 8\right] = 0$

either $t=0$

or $\frac{1}{3}t^2 - t - 8 = 0$

$t = 6.6$ sec

(d) total dist $= 2 \times 26\frac{2}{3} = 53\frac{1}{3}$ m.

Eg2 $x = 3\sin 2t$

$\underline{F} = m\underline{a}$

$x = 3\sin 2t$

$v = \frac{dx}{dt} = 6\cos 2t$

$a = \frac{dv}{dt} = -12\sin 2t$

@ $t = \frac{\pi}{6}$, $a = -12\sin\left(2\frac{\pi}{6}\right) = -6\sqrt{3}$

$\underline{F} = 0.2 \times -6\sqrt{3} = \frac{-6\sqrt{3}}{5}$

$|\underline{F}| = \frac{6\sqrt{3}}{5}$ N

Eg3 $\underline{v} = \begin{pmatrix} 3t \\ \frac{1}{2}t^2 \end{pmatrix}$

(a) $\underline{r} = \int \underline{v}\, dt$

$\underline{r} = \begin{pmatrix} \frac{3t^2}{2} \\ \frac{1}{6}t^3 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

When $t=0$, $r = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\therefore \underline{r} = \begin{pmatrix} \frac{3t^2}{2} + 2 \\ \frac{1}{6}t^3 - 3 \end{pmatrix}$ $\underline{r} = \left[\frac{3t^2}{2} + 2\right]\underline{i} + \left[\frac{1}{6}t^3 - 3\right]\underline{j}$

(b) $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 3 \\ t \end{pmatrix}$

@ $t=3$ $\underline{a} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$\underline{a} = 3\underline{i} + 3\underline{j}$

Eg4 $\quad \underline{a} = \begin{pmatrix} 4 \\ -2t \end{pmatrix}$

(a) for direction of motion, need $\underline{V}$

$\underline{V} = \int \underline{a}\,dt$

$\underline{V} = \begin{pmatrix} 4t \\ -t^2 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

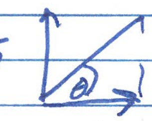
when $t=3$, $V = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 12 \\ -9 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 12 \\ -9 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}$

$\therefore \underline{V} = \begin{pmatrix} 4t-6 \\ 9-t^2 \end{pmatrix}$

@ $t=2$ $\quad \underline{V} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$



$\theta = \tan^{-1}\left(\frac{5}{2}\right) = 68.2^\circ$

(b) $\underline{s} = \int \underline{v}\,dt$

$\underline{s} = \begin{pmatrix} 2t^2 - 6t \\ 9t - \frac{t^3}{3} \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t=3$, $\underline{s} = \begin{pmatrix} 20 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 18-18 \\ 27-9 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 20 \\ 3 \end{pmatrix}$

$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 20 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 18 \end{pmatrix} = \begin{pmatrix} 20 \\ -15 \end{pmatrix}$

$\therefore \underline{s} = \begin{pmatrix} 2t^2 - 6t + 20 \\ 9t - \frac{t^3}{3} - 15 \end{pmatrix}$

@ $t=0$

$\underline{s} = \begin{pmatrix} 20 \\ -15 \end{pmatrix}$

$|\underline{s}| = \sqrt{20^2 + (-15)^2}$

$= 25$ metres

eg5 $\underline{r} = \begin{pmatrix} \frac{3t^2}{2} - \frac{t^3}{3} \\ 2t^2 - 8t \end{pmatrix}$

(a) when moving parallel to $\underline{i}$, $\underline{j}$ component of $\underline{V} = 0$

$$\underline{V} = \frac{dr}{dt} = \begin{pmatrix} 3t - t^2 \\ 4t - 8 \end{pmatrix}$$

Now $4t - 8 = 0$

$t = 2$

(b) Need $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 3 - 2t \\ 4 \end{pmatrix}$

@ $t = 3.5$ $\quad \underline{a} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$

$\underline{F} = m\underline{a}$

$\underline{F} = 0.5 \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$|\underline{F}| = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$ N.

Eg 6 $\underline{v} = \begin{pmatrix} 3t^2 - 8 \\ 5 \end{pmatrix}$

(a) ~~$\underline{v} = \frac{dr}{dt}$~~ $\underline{r} = \int \underline{v}\,dt = \begin{pmatrix} t^3 - 8t \\ 5t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t = 0$, $\underline{r} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

and $\underline{r}_p = \begin{pmatrix} t^3 - 8t + 2 \\ 5t - 4 \end{pmatrix}$

(b) $\underline{v}_Q = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$r_Q = \int v_Q\,dt = \begin{pmatrix} 8t \\ 4t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t = 0$ $r_Q = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\therefore r_Q = \begin{pmatrix} 8t + 2 \\ 4t \end{pmatrix}$

For P & Q to collide $r_P = r_Q$ at same time

$$\begin{pmatrix} t^3 - 8t + 2 \\ 5t - 4 \end{pmatrix} = \begin{pmatrix} 8t + 2 \\ 4t \end{pmatrix}$$

take $\underline{j}$ $5t - 4 = 4t$

$t = 4$ sec

for $\underline{i}$ $4^3 - 8(4) + 2 = 8(4) + 2$

$34 = 34$

So @ $t = 4$ P & Q both have position $34\underline{i} + 16\underline{j}$

So collision will occur.

Exercise 5.1

1 At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = (3t - 4)\mathbf{i} + (t^3 - 4t)\mathbf{j}.$$

Find

a the velocity of P when $t = 3$,

b the acceleration of P when $t = 3$.

2 A particle P is moving in a plane with velocity $\mathbf{v}$ m s^{-1} at time t seconds where

$$\mathbf{v} = t^2\mathbf{i} + (2t - 3)\mathbf{j}.$$

When $t = 0$, P has position vector $(3\mathbf{i} + 4\mathbf{j})$ m with respect to a fixed origin O. Find

a the acceleration of P at time t seconds,

b the position vector of P when $t = 1$.

3 A particle P starts from rest at a fixed origin O. The acceleration of P at time t seconds (where $t \geqslant 0$) is $(6t^2\mathbf{i} + (8 - 4t^3)\mathbf{j})$ m s^{-2}. Find

a the velocity of P when $t = 2$,

b the position vector of P when $t = 4$.

4 At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = 4t^2\mathbf{i} + (24t - 3t^2)\mathbf{j}.$$

a Find the speed of P when $t = 2$.

b Show that the acceleration of P is a constant and find the magnitude of this acceleration.

5 A particle P is initially at a fixed origin O. At time $t = 0$, P is projected from O and moves so that, at time t seconds after projection, its position vector $\mathbf{r}$ m relative to O is given by

$$\mathbf{r} = (t^3 - 12t)\mathbf{i} + (4t^2 - 6t)\mathbf{j},\ t \geqslant 0.$$

Find

a the speed of projection of P,

b the value of t at the instant when P is moving parallel to $\mathbf{j}$,

c the position vector of P at the instant when P is moving parallel to $\mathbf{j}$.

6 At time t seconds, the force $\mathbf{F}$ newtons acting on a particle P, of mass 0.5 kg , is given by

$$\mathbf{F} = 3t\mathbf{i} + (4t - 5)\mathbf{j}.$$

When $t = 1$, the velocity of P is $12\mathbf{i}$ m s^{-1}. Find

a the velocity of P after t seconds,

b the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$, giving your answer to the nearest degree.

7 A particle P is moving in a plane with velocity $\mathbf{v}$ m s^{-1} at time t seconds where

$$\mathbf{v} = (3t^2 + 2)\mathbf{i} + (6t - 4)\mathbf{j}.$$

When $t = 2$, P has position vector $9\mathbf{j}$ m with respect to a fixed origin O. Find

a the distance of P from O when $t = 0$,

b the acceleration of P at the instant when it is moving parallel to the vector $\mathbf{i}$.

8 At time t seconds, the particle P is moving in a plane with velocity $\mathbf{v}$ m s^{-1} and acceleration $\mathbf{a}$ m s^{-2}, where

$$\mathbf{a} = (2t - 4)\mathbf{i} + 6\mathbf{j}.$$

Given that P is instantaneously at rest when $t = 4$, find

a $\mathbf{v}$ in terms of t,

b the speed of P when $t = 5$.

9 A particle P is moving in a plane. At time t seconds, the position vector of P, $\mathbf{r}$ m, is given by

$$\mathbf{r} = (3t^2 - 6t + 4)\mathbf{i} + (t^3 + kt^2)\mathbf{j},$$ where k is a constant.

When $t = 3$, the speed of P is $12\sqrt{5}$ m s^{-1}.

a Find the two possible values of k.

b For both of these values of k, find the magnitude of the acceleration of P when $t = 1.5$.

10 At time t seconds (where $t \geq 0$), the particle P is moving in a plane with acceleration $\mathbf{a}$ m s^{-2}, where

$$\mathbf{a} = (5t - 3)\mathbf{i} + (8 - t)\mathbf{j}$$

When $t = 0$, the velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1}. Find

a the velocity of P after t seconds,

b the value of t for which P is moving parallel to $\mathbf{i} - \mathbf{j}$,

c the speed of P when it is moving parallel to $\mathbf{i} - \mathbf{j}$.

11 At time t seconds (where $t \geq 0$), a particle P is moving in a plane with acceleration $(2\mathbf{i} - 2t\mathbf{j})$ m s^{-2}. When $t = 0$, the velocity of P is $2\mathbf{j}$ m s^{-1} and the position vector of P is $6\mathbf{i}$ m with respect to a fixed origin P.

a Find the position vector of P at time t seconds.

At time t seconds (where $t \geq 0$), a second particle Q is moving in the plane with velocity $((3t^2 - 4)\mathbf{i} - 2t\mathbf{j})$ m s^{-1}. The particles collide when $t = 3$.

b Find the position vector of Q at time $t = 0$.

12 A particle P of mass 0.2 kg is at rest at a fixed origin O. At time t seconds, where $0 \leq t \leq 3$, a force $(2t\mathbf{i} + 3\mathbf{j})$ N is applied to P.

a Find the position vector of P when $t = 3$.

When $t = 3$, the force acting on P changes to $(6\mathbf{i} + (12 - t^2)\mathbf{j})$ N, where $t \geq 3$.

b Find the velocity of P when $t = 6$.

Ex5.1

(1) $\underline{r} = \begin{pmatrix} 3t - 4 \\ t^3 - 4t \end{pmatrix}$

(a) $\underline{v} = \frac{d\underline{r}}{dt} = \begin{pmatrix} 3 \\ 3t^2 - 4 \end{pmatrix}$

@ $t=3$ $\underline{v} = \begin{pmatrix} 3 \\ 27 - 4 \end{pmatrix} = 3\underline{i} + 23\underline{j}$

(b) $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 0 \\ 6t \end{pmatrix}$

@ $t=3$ $\underline{a} = 18\underline{j}$

(2) $\underline{v} = \begin{pmatrix} t^2 \\ 2t - 3 \end{pmatrix}$

(a) $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 2t \\ 2 \end{pmatrix}$

(b) $\underline{r} = \int \underline{v}\, dt = \begin{pmatrix} \frac{1}{3}t^3 \\ t^2 - 3t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

@ $t=0$, $\underline{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\therefore$ $r = \begin{pmatrix} \frac{1}{3}t^3 + 3 \\ t^2 - 3t + 4 \end{pmatrix}$

@ $t=1$ $r = \begin{pmatrix} \frac{1}{3} + 3 \\ 1 - 3 + 4 \end{pmatrix} = \begin{pmatrix} 10/3 \\ 2 \end{pmatrix}$

(3) $\underline{a} = \begin{pmatrix} 6t^2 \\ 8-4t^3 \end{pmatrix}$

(a) $\underline{V} = \int \underline{a}\,dt = \begin{pmatrix} 2t^3 \\ 8t - t^4 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t=0$, $V=0$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

when $t=2$ $\underline{V} = \begin{pmatrix} 2(2)^3 \\ 8(2) - 2^4 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \end{pmatrix}$

(b) $\underline{r} = \int \underline{v}\,dt = \begin{pmatrix} 0.5t^4 \\ 4t^2 - 0.2t^5 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

@ $t=0$, $r=0$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

@ $t=4$ $\underline{r} = \begin{pmatrix} 0.5(4)^4 \\ 4(4)^2 - 0.2(4)^5 \end{pmatrix} = \begin{pmatrix} 128 \\ 64 - 204.8 \end{pmatrix} = \begin{pmatrix} 128 \\ -140.8 \end{pmatrix}$ *

-104.8 given answer?

(4) $r = \begin{pmatrix} 4t^2 \\ 24t - 3t^2 \end{pmatrix}$

(a) $\underline{V} = \frac{dr}{dt} = \begin{pmatrix} 8t \\ 24 - 6t \end{pmatrix}$

@ $t=2$ $\underline{V} = \begin{pmatrix} 16 \\ 12 \end{pmatrix}$, $|\underline{V}| = \sqrt{16^2 + 12^2} = 20\ ms^{-1}$

(b) $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$ $\underline{a} \neq f(t) \therefore$ const.

$|\underline{a}| = \sqrt{8^2 + (-6)^2} = 10\ ms^{-2}$

(5) $\underline{r} = \begin{pmatrix} t^3 - 12t \\ 4t^2 - 6t \end{pmatrix}$

(a) $\underline{v} = \dfrac{dr}{dt} = \begin{pmatrix} 3t^2 - 12 \\ 8t - 6 \end{pmatrix}$

~~$@t=2,\ \underline{v} = \begin{pmatrix} 3(2)^2 - 12 \\ 8(2) - 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$~~

~~$|v| = 10\ ms^{-1}$~~

$@t=0 \quad \underline{v} = \begin{pmatrix} -12 \\ -6 \end{pmatrix}, \quad |\underline{v}| = \sqrt{(-12)^2 + (-6)^2} = 6\sqrt{5}\ ms^{-1}$

(b) when moving parallel to $\underline{j}$, $\underline{i}$ comp of $V = 0$

$3t^2 - 12 = 0$

$t = 2$ sec.

(c) $@t=2, \quad r_p = \begin{pmatrix} 2^3 - 12(2) \\ 4(2)^2 - 6(2) \end{pmatrix} = \begin{pmatrix} 8 - 24 \\ 16 - 12 \end{pmatrix} = \begin{pmatrix} -16 \\ 4 \end{pmatrix}$

$r_p = -16\underline{i} + 4\underline{j}$

(6) $F = \begin{pmatrix} 3t \\ 4t - 5 \end{pmatrix}$

$\underline{a} = \dfrac{F}{M} = \begin{pmatrix} 6t \\ 8t - 10 \end{pmatrix}$

(a) $\underline{V} = \int \underline{a}\, dt = \begin{pmatrix} 3t^2 \\ 4t^2 - 10t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

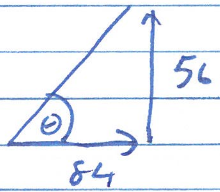
when $t = 1$, $V_P = \begin{pmatrix} 12 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 3 \\ -6 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \end{pmatrix}$

$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

(6) cont $\therefore$ $\underline{V} = \begin{pmatrix} 3t^2 + 9 \\ 4t^2 - 10t + 6 \end{pmatrix}$ * t^3 in answer.

(b) @ $t = 5$, $V = \begin{pmatrix} 3(5)^2 + 9 \\ 4(5)^2 - 10(5) + 6 \end{pmatrix} = \begin{pmatrix} 75 + 9 \\ 100 - 50 + 6 \end{pmatrix} = \begin{pmatrix} 84 \\ 56 \end{pmatrix}$



$\theta = \tan^{-1}\left(\frac{56}{84}\right) = 33.69\ldots = 34°$

(7) $\underline{V} = \begin{pmatrix} 3t^2 + 2 \\ 6t - 4 \end{pmatrix}$

(a) $\underline{r} = \int \underline{v} \, dt = \begin{pmatrix} t^3 + 2t \\ 3t^2 - 4t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t = 2$, $r = \begin{pmatrix} 0 \\ 9 \end{pmatrix}$

$\begin{pmatrix} 8 + 4 \\ 12 - 8 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \end{pmatrix}$

$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} - \begin{pmatrix} 12 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$

$\therefore \underline{r} = \begin{pmatrix} t^3 + 2t - 12 \\ 3t^2 - 4t + 5 \end{pmatrix}$

@ $t = 0$, $\underline{r} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$ $\quad |\underline{r}| = \sqrt{(-12)^2 + 5^2} = 13$ metres

(b) when moving parallel to i, j component of $V = 0$

ie $6t - 4 = 0$

$t = \frac{2}{3}$

(7)(b) now $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 6t \\ 6 \end{pmatrix}$

@ $t = \frac{2}{3}$ $\quad \underline{a} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(8) $\quad a = \begin{pmatrix} 2t - 4 \\ 6 \end{pmatrix}$

(a) $\quad \underline{V} = \int \underline{a}\,dt = \begin{pmatrix} t^2 - 4t \\ 6t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t = 4$, $V = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\therefore \begin{pmatrix} 4^2 - 4(4) \\ 6(4) \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 24 \end{pmatrix}$

$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ -24 \end{pmatrix}$

$\therefore \underline{V} = \begin{pmatrix} t^2 - 4t \\ 6t - 24 \end{pmatrix}$

(b) @ $t = 5$ $\quad \underline{V} = \begin{pmatrix} 5^2 - 4(5) \\ 6(5) - 24 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

$|\underline{V}| = \sqrt{5^2 + 6^2} = 7 \cdot 81 \ ms^{-1}$

(9) $r = \begin{pmatrix} 3t^2 - 6t + 4 \\ t^3 + kt^2 \end{pmatrix}$

(a) $\underline{v} = \frac{dr}{dt} = \begin{pmatrix} 6t - 6 \\ 3t^2 + 2kt \end{pmatrix}$

@ $t = 3$, $\underline{v} = \begin{pmatrix} 6(3) - 6 \\ 3(3)^2 + 2k(3) \end{pmatrix} = \begin{pmatrix} 12 \\ 27 + 6k \end{pmatrix}$

Now $|\underline{v}| = 12\sqrt{5}$

$\sqrt{12^2 + (27 + 6k)^2} = 12\sqrt{5}$

$144 + (27 + 6k)^2 = 720$

$(27 + 6k)^2 = 576$

$27 + 6k = \pm 24$

$\therefore$ either $27 + 6k = 24$ or $27 + 6k = -24$

$6k = -3$ $\qquad$ $6k = -51$

$k = -\frac{1}{2}$ $\qquad$ $k = -8.5$

(b) $\underline{a} = \frac{dv}{dt} = \begin{pmatrix} 6 \\ 6t + 2k \end{pmatrix}$

@ $t = 1.5$ $\quad \underline{a} = \begin{pmatrix} 6 \\ 9 + 2k \end{pmatrix}$

$|\underline{a}| = \sqrt{6^2 + (9 + 2k)^2}$

for $k = -0.5$, $|a| = \sqrt{36 + 64} = 10 \text{ ms}^{-2}$

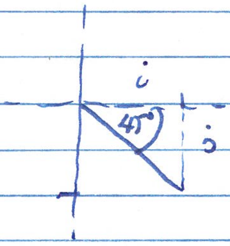
for $k = -8.5$, $|a| = \sqrt{36 + 64} = 10 \text{ ms}^{-2}$

(10) $\underline{a} = \begin{pmatrix} 5t-3 \\ 8-t \end{pmatrix}$

(a) $\underline{V} = \int \underline{a}\, dt = \begin{pmatrix} \frac{5t^2}{2} - 3t \\ 8t - \frac{t^2}{2} \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

@ $t=0$, $\underline{v} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$\therefore \underline{V} = \begin{pmatrix} \frac{5}{2}t^2 - 3t + 2 \\ 8t - \frac{t^2}{2} - 5 \end{pmatrix}$



(b) parallel to $i - j$ when $\frac{V_j}{V_i} = -1$, $\Rightarrow V_j = -V_i$

$8t - \frac{t^2}{2} - 5 = -\frac{5}{2}t^2 + 3t - 2$

$2t^2 + 5t - 3 = 0$

$t = 0.5$ sec

(c) @ $t = 0.5$ $\underline{V} = \begin{pmatrix} \frac{5}{2}(0.5)^2 - 3(0.5) + 2 \\ 8(0.5) - \frac{(0.5)^2}{2} - 5 \end{pmatrix} = \begin{pmatrix} 1.125 \\ -1.125 \end{pmatrix}$

Speed $|\underline{V}| = \sqrt{1.125^2 + (-1.125)^2} = \frac{9\sqrt{2}}{8}$ ms^{-1} (1.59).

(11) $\underset{\sim}{a} = \begin{pmatrix} 2 \\ -2t \end{pmatrix}$

(a) $\underline{V} = \int \underset{\sim}{a}\,dt = \begin{pmatrix} 2t \\ -t^2 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t=0$, $\underline{V} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

hence $\underline{V} = \begin{pmatrix} 2t \\ 2-t^2 \end{pmatrix}$

Now $\underline{r} = \int \underline{v}\,dt = \begin{pmatrix} t^2 \\ 2t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t=0$ $r = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ $\therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

hence $r_p = \begin{pmatrix} t^2+6 \\ 2t - \frac{1}{3}t^3 \end{pmatrix}$

(b) $V_Q = \begin{pmatrix} 3t^2 - 4 \\ -2t \end{pmatrix}$

Sub $t=3$ in r_p to find pos vect of Q @ $t=3$

$r_Q = r_p = \begin{pmatrix} 3^2+6 \\ 2(3) - \frac{1}{3}(3)^3 \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \end{pmatrix}$

Now $r_Q = \int V_Q\,dt = \begin{pmatrix} t^3 - 4t \\ -t^2 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

@ $t=3$ $\begin{pmatrix} 3^3 - 4(3) \\ -(3)^2 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \end{pmatrix}$

$\begin{pmatrix} 15 \\ -9 \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \end{pmatrix}$

(11) (b) contd $\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \end{pmatrix} - \begin{pmatrix} 15 \\ -9 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$

$\therefore \quad r_Q = \begin{pmatrix} t^3 - 4t \\ 6 - t^2 \end{pmatrix}$

@ $t = 0$ $r_Q = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$

(12) $\underline{F} = m \underline{a}$

(a) $\begin{pmatrix} 2t \\ 3 \end{pmatrix} = 0.2 \underline{a}$

$\underline{a} = \begin{pmatrix} 10t \\ 15 \end{pmatrix}$

$\underline{V} = \int \underline{a} \, dt = \begin{pmatrix} 5t^2 \\ 15t \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

@ $t = 0$, $\underline{V} = 0 \quad \therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$V = \begin{pmatrix} 5t^2 \\ 15t \end{pmatrix}$ — (1)

Now $r = \int \underline{v} \, dt = \begin{pmatrix} \frac{5t^3}{3} \\ \frac{15t^2}{2} \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$

when $t = 0$ $r = 0 \quad \therefore \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

hence $r = \begin{pmatrix} \frac{5t^3}{3} \\ \frac{15t^2}{2} \end{pmatrix}$

@ $t = 3$ $r = \begin{pmatrix} \frac{5(3)^3}{3} \\ \frac{15(3)^2}{2} \end{pmatrix} = \begin{pmatrix} 45 \\ 67.5 \end{pmatrix}$ metres

(12) (b) @ $t=3$, $\underline{F} = m\underline{a}$

$$\begin{pmatrix} 6 \\ 12-t^2 \end{pmatrix} = 0.2\underline{a}$$

$$\underline{a} = \begin{pmatrix} 30 \\ 60-5t^2 \end{pmatrix}$$

$$V = \int \underline{a}\, dt = \begin{pmatrix} 30t \\ 60t - \frac{5t^3}{3} \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$$

to find velocity @ $t=3$ in (1) $V = \begin{pmatrix} 5(3)^2 \\ 15(3) \end{pmatrix} = \begin{pmatrix} 45 \\ 45 \end{pmatrix}$

for journey described above $t=0$, $V = \begin{pmatrix} 45 \\ 45 \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix}$

$$V = \begin{pmatrix} 30t + 45 \\ 60t - \frac{5}{3}t^3 + 45 \end{pmatrix}$$

Now when $t=6$ overall, in this part of system, $t=3$

$$V = \begin{pmatrix} 30(3) + 45 \\ 60(3) - \frac{5}{3}(3)^3 + 45 \end{pmatrix} = \begin{pmatrix} 135 \\ 180 \end{pmatrix}$$

← * given answer different, worked solution not available.