C3/M1 Homework – Due 16/03/16

1. A car moves with constant acceleration along a straight horizontal road. It passes the point O with speed $12\,\text{ms}^{-1}$. It then passes point A, 4 seconds later, with speed $32\,\text{ms}^{-1}$.

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(a) Show that the acceleration of the car is $5\,\text{ms}^{-2}$. [3]

(b) Determine the distance OA. [3]

(c) The point M is the midpoint of OA. Calculate the speed of the car as it passes M. Give your answer correct to one decimal place. [3]

2. A train, travelling along a straight horizontal track, has a steady speed of $18\ \text{ms}^{-1}$ as it passes the point A. Fifteen seconds later, it begins to slow down at a uniform rate for 30 s until its speed is $10\ \text{ms}^{-1}$. The train then increases its speed uniformly for 45 s until it reaches a speed of $20\ \text{ms}^{-1}$ as it passes the point B.

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(a) Draw a sketch of the v-t graph for the motion of the train between A and B. [4]

(b) Calculate the acceleration of the train just before it reaches B. [2]

(c) Find the distance from A to B. [4]

3. A particle is projected vertically upwards with an initial speed of $15\,\text{ms}^{-1}$ from a point A, which is $1{\cdot}2\,\text{m}$ above horizontal ground.

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(a) Determine the time taken for the particle to reach the ground. Give your answer correct to one decimal place. [4]

(b) Suppose a heavier particle is projected vertically upwards from the same point A and with the same initial speed of $15\,\text{ms}^{-1}$. Would the time taken for the particle to reach the ground be greater, the same, or less than your answer in *(a)*?
Give a reason for your answer. [1]

(4)

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1. Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_1^2 \frac{1}{2+e^x}\,dx.$$

Show your working and give your answer correct to three decimal places. [4]

(5)

3. *(a)* Given that

$$x^3 + 5x^4y - 2y^3 + 7 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

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(b) Given that $x = t^3 - 5$, $y = t^4 + 7t^5$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t,

(ii) find an expression for $\frac{d^2y}{dx^2}$ in terms of t,

(iii) find the value of $\frac{d^2y}{dx^2}$ when $x = 3$. [9]

(6)

4. *(a)* On the same diagram, sketch the graphs of $y = \ln x$ and $y = 11 - 2x$.
Deduce the number of roots of the equation

$$\ln x + 2x - 11 = 0.$$ [3]

(b) **You may assume** that the equation

$$\ln x + 2x - 11 = 0$$

has a root α between 4 and 5.
The recurrence relation

$$x_{n+1} = \frac{11 - \ln x_n}{2},$$

with $x_0 = 4{\cdot}7$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

(7)

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5. *(a)* Differentiate each of the following with respect to x.

(i) $\sqrt{5x^2 - 3x}$ (ii) $\sin^{-1} 7x$ (iii) $e^{3x}\ln x$ [7]

(b) By first writing $\cot x = \frac{\cos x}{\sin x}$, show that $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [3]