Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Monday 20 June 2011 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C.

(4)

$$\frac{A(x-1)(2x+1) + B(2x+1) + C(x-1)^2}{(x-1)^2(2x+1)}$$

$$\frac{A(2x^2-x-1) + B(2x+1) + C(x^2-2x+1)}{(x-1)^2(2x+1)}$$

Compare coefficients: x^2: $2A + C = 9$ — (1)

x^1: $-A + 2B - 2C = 0$ — (2)

x^0: $-A + B + C = 0$ — (3)

From (1) $C = 9 - 2A$ — (4)

in (3) $-A + B + 9 - 2A = 0$

$B = 3A - 9$ — (5)

in (2) $-A + 2(3A-9) - 2(9-2A) = 0$

$-A + 6A - 18 - 18 + 4A = 0$

$9A = 36$

$A = 4$

in (4) $C = 9 - 8 = 1$

in (5) $B = 12 - 9 = 3$

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2.
$$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

$f(x) = (9+4x^2)^{-1/2}$

$= \left[9\left(1+\frac{4x^2}{9}\right)\right]^{-1/2}$

$= 9^{-1/2}\left(1+\frac{4}{9}x^2\right)^{-1/2}$

$= \frac{1}{3}\left[1 + \left(\frac{-1}{2}\right)\left(\frac{4x^2}{9}\right) + \left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{4}{9}x^2\right)^2 \cdot \frac{1}{2!} + \ldots\right]$

$= \frac{1}{3}\left[1 - \frac{4}{18}x^2 + \frac{48}{648}x^4 + \ldots\right]$

$= \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4 + \ldots$

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3.

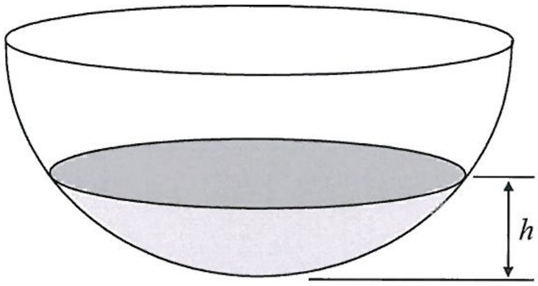

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume $V\text{m}^3$ is given by

$$V = \frac{1}{12}\pi h^2(3-4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.1$

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$

(2)

(a) $V = \frac{\pi}{4}h^2 - \frac{\pi}{3}h^3$

$\frac{dV}{dh} = \frac{2\pi}{4}h - \frac{3\pi}{3}h^2 = \frac{\pi h}{2} - \pi h^2$

when $h=0.1$ $\quad \frac{dV}{dh} = \frac{0.1}{2}\pi - \pi(0.1)^2 = \frac{\pi}{25}$

(b) $\frac{dh}{dt} = \frac{dh}{\bigcirc} \cdot \frac{\bigcirc}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

$= \frac{25}{\pi} \cdot \frac{\pi}{800}$

$= \frac{1}{32}$

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4.

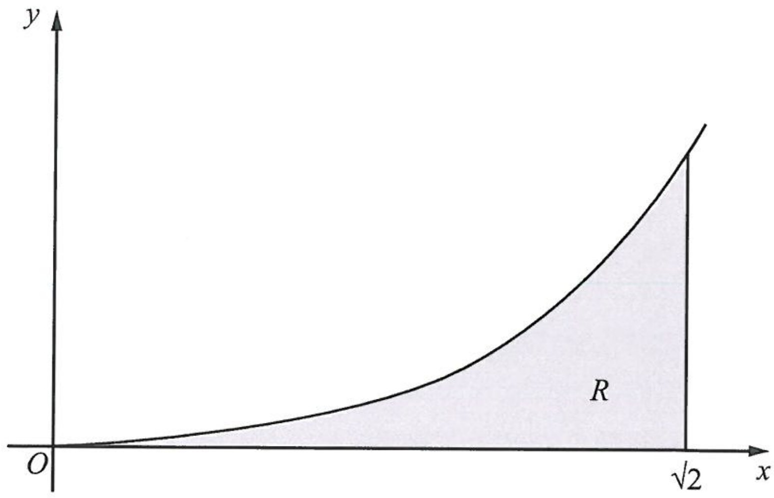

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geqslant 0$.
The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0	0.0333	0.3240	1.3596	3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.
(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.
(3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u - 2)\ln u \, du$$
(4)

(d) Hence, or otherwise, find the exact area of R.
(6)

Question 4 continued

(b) $A \approx \frac{\sqrt{2}/4}{2}\left[0 + 2(0.0333 + 0.3240 + 1.3596) + 3.9210\right]$

≈ 1.30

(c) $y = x^3 \ln(x^2+2)$

$u = x^2 + 2$

when $x=0$ $u=2$

$x=\sqrt{2}$ $u=4$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$\therefore I = \int_2^4 x^3 \ln u \frac{du}{2x}$

$= \frac{1}{2}\int_2^4 x^2 \ln u \, du$ but $x^2 = u - 2$

$= \frac{1}{2}\int_2^4 (u-2)\ln u \, du$ As required

(d) let $w = \ln u$ $\frac{dv}{dx} = u - 2$

$\frac{dw}{du} = \frac{1}{u}$ $v = \frac{u^2}{2} - 2u$

$2I = \ln u\left[\frac{u^2}{2} - 2u\right] - \int \frac{1}{u}\left(\frac{u^2}{2} - 2u\right) du$

$= \ln u\left[\frac{u^2}{2} - 2u\right] - \int \frac{u}{2} - 2 \, du$

Question 4 continued

$$2I = \left[\ln u\left[\frac{u^2}{2} - 2u\right] - \frac{u^2}{4} + 2u\right]_2^4$$

$$= 0\ln 4 - 4 + 8 - \left(-2\ln 2 - 1 + 4\right)$$

$$2I = 0 + 4 + 2\ln 2 - 3$$

$$\cancel{2I = 1 + 4\ln 2^2 + 2\ln 2}$$

$$\cancel{2I = 1 + 8\ln 2 + 2\ln 2}$$

$$\cancel{= 1 + 10\ln 2}$$

$$2I = 1 + 2\ln 2$$

$$I = \frac{1}{2}(1 + 2\ln 2)$$

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5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \qquad x > 0,\ y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

$\ln y = 2x\ln x$

$\frac{1}{y}\frac{dy}{dx} = 2x \cdot \frac{1}{x} + 2\ln x$

$\frac{1}{y}\frac{dy}{dx} = 2 + 2\ln x$ — 1

when $x = 2$ $\quad \ln y = 4\ln 2$

$\ln y = \ln 16$

$y = 16.$

@ $(2,16)$ $\quad \frac{1}{16}\frac{dy}{dx} = 2 + 2\ln 2$

$\frac{dy}{dx} = 32(1+\ln 2)$

Leave blank

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \ \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2: \ \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A. **(6)**

(b) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2. **(3)**

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1. **(1)**

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures. **(4)**

lines meet when $6-\lambda = -5+2\mu$ — (1)

$-3+2\lambda = 15-3\mu$ — (2)

$-2+3\lambda = 3+\mu$ — (3)

From (1) $\lambda = 11-2\mu$ — (4)

in (2) $-3+2(11-2\mu) = 15-3\mu$

$-3+22-4\mu = 15-3\mu$

$\mu = 4$

in (4) $\lambda = 11-8 = 3$

Subst for λ & μ in (3) $-2+9 = 3+4$

$7 = 7$

this is consistent $\therefore$ lines meet @ $\mathbf{r}_A = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$

Question 6 continued

(b) $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \sqrt{-1^2+2^2+3^2}\sqrt{2^2+-3^2+1^2}\ \text{Cos}\,\theta$

$-2-6+3 = \sqrt{14}\sqrt{14}\ \text{Cos}\,\theta$

$\frac{-5}{14} = \text{Cos}\,\theta$

$\text{Cos}\,\theta = \frac{-5}{14}$

$\theta = 110.9^\circ$

$= 69.1^\circ$

(c) $r_B = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6-\lambda \\ -3+2\lambda \\ -2+3\lambda \end{pmatrix}$ — ① — ② — ③

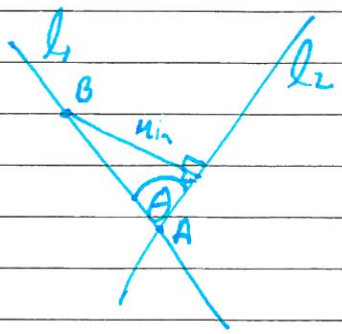
from ① $5 = 6-\lambda \Rightarrow \lambda = 1$

from ② $-1 = -3+2\lambda \Rightarrow \lambda = 1$

from ③ $1 = -2+3\lambda \Rightarrow \lambda = 1$

$\therefore$ B lies on l_1 when $\lambda = 1$

(d)



Shortest distance when line is perpendicular.

$\frac{\text{Min}}{|AB|} = \frac{0}{H} = \text{Sin}\,\theta$

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Question 6 continued

Now $|AB| = -r_A + r_B$

$$= -\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$$

$$|AB| = \sqrt{2^2 + (-4)^2 + (-6)^2} = \sqrt{56}$$

$$\frac{\text{Min}}{\sqrt{56}} = \sin 69.1^\circ$$

$\text{Min} = 6.98$ (3 s.f).

7.

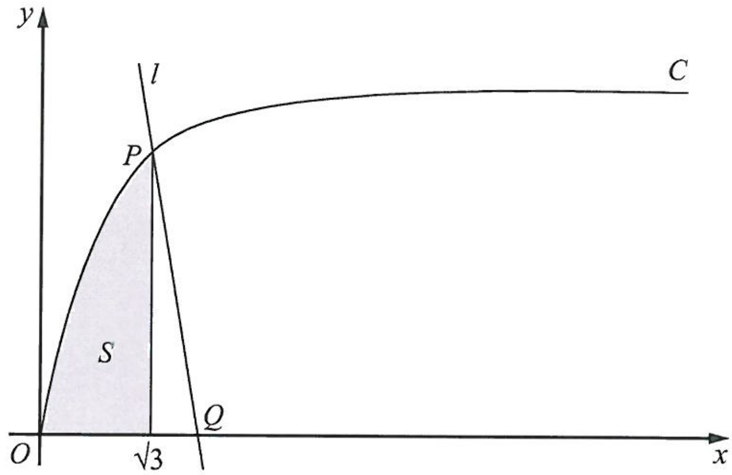

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

(a) @ P $\quad \tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

(b) Need eqⁿ of normal: $\quad \frac{dx}{d\theta} = \sec^2\theta \qquad \frac{dy}{d\theta} = \cos\theta$

$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta} = \cos^3\theta \quad @\ \theta = \frac{\pi}{3}, \quad \frac{dy}{dx} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

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Question 7 continued

$\therefore \quad M_{normal} = -8$

eq^n $\quad y - \frac{\sqrt{3}}{2} = -8(x - \sqrt{3})$

Now @Q, $y = 0$

$\therefore \quad -\frac{\sqrt{3}}{2} = -8x + 8\sqrt{3}$

$8x = 8\sqrt{3} + \frac{1}{2}\sqrt{3}$

$8x = \frac{17\sqrt{3}}{2}$

$x = \frac{17\sqrt{3}}{16}$

$\therefore \quad k = \frac{17}{16}$

(c) $\quad x = \tan\theta \qquad y = \sin\theta$

$y^2 = \sin^2\theta$

$y^2 = 1 - \cos^2\theta$

$\cos^2 = 1 - y^2$

$x^2 = \frac{\sin^2\theta}{\cos^2\theta}$

$x^2 = \frac{y^2}{1 - y^2}$

$x^2 - x^2y^2 = y^2$

$y^2(1 + x^2) = x^2$

$y^2 = \frac{x^2}{1 + x^2}$

Question 7 continued

$$\cancel{1+x^2 \overline{)\, x^2} \quad 1 \quad x^2+1 \quad -1}$$

$$\cancel{\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}}$$

(c) $V = \pi \int y^2 \frac{dx}{d\theta}\, d\theta$

$V = \pi \int \sin^2\theta . \sec^2\theta \, d\theta$

$V = \pi \int \tan^2\theta \, d\theta$

$V = \pi \int \sec^2\theta - 1 \, d\theta$

$V = \pi \left[\tan\theta - \theta \right]_0^{\frac{\pi}{3}}$

$V = \pi \left[\tan\frac{\pi}{3} - \frac{\pi}{3} - (0-0) \right]$

$V = \pi \left[\sqrt{3} - \frac{\pi}{3} \right]$

$V = \sqrt{3}\pi - \frac{\pi^2}{3}$

So $p = 1$ + $q = -\frac{1}{3}$

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8. (a) Find $\int (4y+3)^{-\frac{1}{2}}\,dy$

(2)

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y = f(x)$.

(6)

(a) $\frac{1}{\frac{1}{2}}(4y+3)^{\frac{1}{2}} \cdot \frac{1}{4} + c$

$= \frac{1}{2}(4y+3)^{\frac{1}{2}} + c$

(b) $\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2}$

$\int (4y+3)^{-\frac{1}{2}}\,dy = \int \frac{1}{x^2}\,dx$

$\frac{1}{2}(4y+3)^{\frac{1}{2}} = \frac{1}{-1}x^{-1} + c$

$\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + c$

@ $(-2, 1.5)$

$\frac{1}{2}(4(1.5)+3)^{\frac{1}{2}} = -\frac{1}{-2} + c$

$\frac{3}{2} = \frac{1}{2} + c$

$c = 1.$

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Question 8 continued

$\therefore \quad \frac{1}{2}(4y+3)^{\frac{1}{2}} = 1 - \frac{1}{x}$

$\frac{1}{2}(4y+3)^{\frac{1}{2}} = \frac{x-1}{x}$

$(4y+3)^{\frac{1}{2}} = \frac{2(x-1)}{x}$

$4y+3 = \frac{4(x-1)^2}{x^2}$

$4y = \frac{4(x-1)^2}{x^2} - 3$

$y = \frac{(x-1)^2}{x^2} - \frac{3}{4}$

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