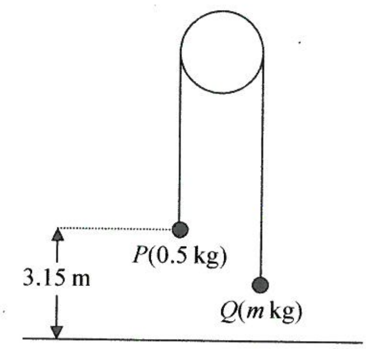
M1 – Nasty Connected Particles PPQ's

M1 – June 07

6.

Figure 4



Two particles P and Q have mass 0.5 kg and m kg respectively, where $m < 0.5$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially P is 3.15 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 4. After P has been descending for 1.5 s, it strikes the ground. Particle P reaches the ground before Q has reached the pulley.

(a) Show that the acceleration of P as it descends is 2.8 m s^{-2}.

(3)

(b) Find the tension in the string as P descends. 3.5N

(3)

(c) Show that $m = \frac{5}{18}$.

(4)

(d) State how you have used the information that the string is inextensible.

(1)

When P strikes the ground, P does not rebound and the string becomes slack. Particle Q then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

(e) Find the time between the instant when P strikes the ground and the instant when the string becomes taut again. $t = \frac{6}{7}$ s

(6)

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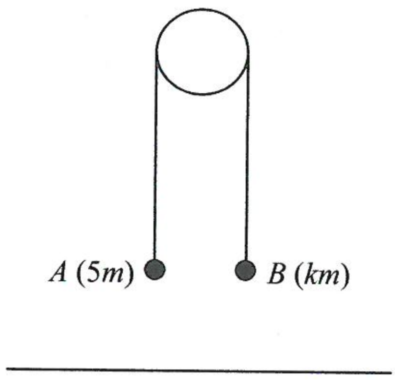

Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.
(3)

(b) Find the value of k. $k=3$
(3)

(c) State how you have used the information that the pulley is smooth.
(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane. 4.0 m
(7)

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7.

Figure 4

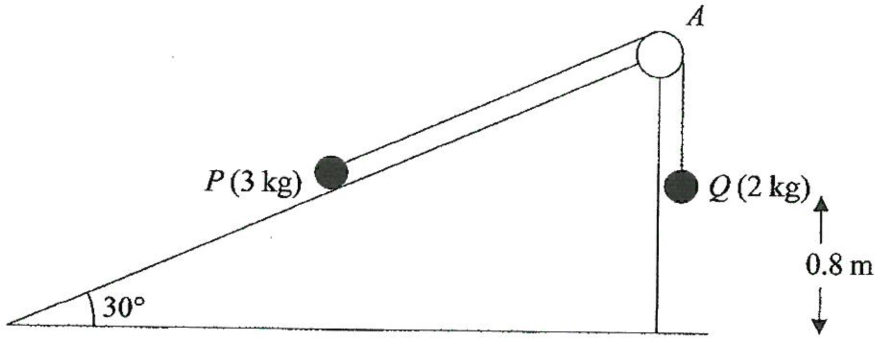

Figure 4 shows two particles P and Q, of mass 3 kg and 2 kg respectively, connected by a light inextensible string. Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth light pulley A fixed at the top of the plane. The part of the string from P to A is parallel to a line of greatest slope of the plane. The particle Q hangs freely below A. The system is released from rest with the string taut.

(a) Write down an equation of motion for P and an equation of motion for Q.

(4)

(b) Hence show that the acceleration of Q is 0.98 m s^{-2}.

(2)

(c) Find the tension in the string. 18N

(2)

(d) State where in your calculations you have used the information that the string is inextensible.

(1)

On release, Q is at a height of 0.8 m above the ground. When Q reaches the ground, it is brought to rest immediately by the impact with the ground and does not rebound. The initial distance of P from A is such that in the subsequent motion P does not reach A. Find

(e) the speed of Q as it reaches the ground, 1.3 ms^{-1}

(2)

(f) the time between the instant when Q reaches the ground and the instant when the string becomes taut again. 0.51s

(5)

N 2 3 5 6 0 A 0 1 4 1 6

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Figure 3

Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough horizontal table. The string passes over a small smooth pulley P fixed on the edge of the table. The particle B hangs freely below the pulley, as shown in Figure 3. The coefficient of friction between A and the table is μ. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of A and B is $\frac{4}{9}g$. By writing down separate equations of motion for A and B,

(a) find the tension in the string immediately after the particles begin to move,

$\frac{10mg}{9}$

(3)

(b) show that $\mu = \frac{2}{3}$.

(5)

When B has fallen a distance h, it hits the ground and does not rebound. Particle A is then a distance $\frac{1}{3}h$ from P.

(c) Find the speed of A as it reaches P.

$\frac{2}{3}\sqrt{gh}$

(6)

(d) State how you have used the information that the string is light.

(1)

M 2 6 3 3 1 A 0 2 2 2 8

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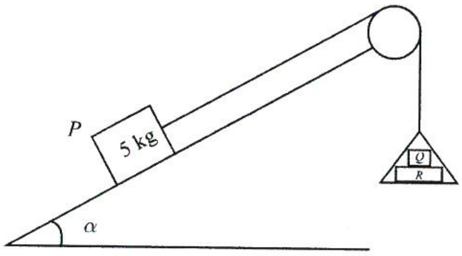

Figure 3

One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R, as shown in Figure 3. The mass of block Q is 5 kg and the mass of block R is 10 kg. The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find

(a) (i) the acceleration of the scale pan, 0.6g

(ii) the tension in the string, 6g

(8)

(b) the magnitude of the force exerted on block Q by block R, 2g

(3)

(c) the magnitude of the force exerted on the pulley by the string. 105N

(5)

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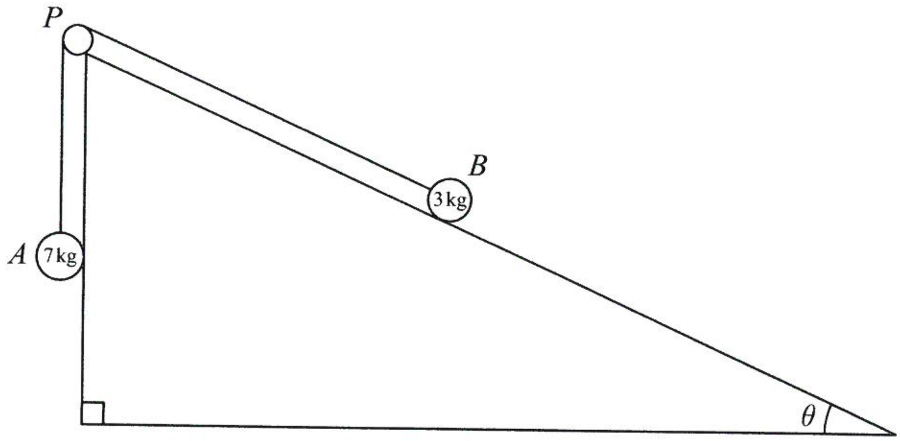

Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release.

$3.9\,\mathrm{ms^{-2}}$

(10)

(b) Find the speed of B when it has moved 1 m up the plane.

$2.8\,\mathrm{ms^{-1}}$

(2)

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest.

$\frac{2}{7}$ s

(4)

N 3 5 4 0 7 A 0 2 0 2 4

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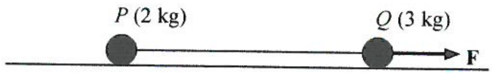


Figure 4

Two particles P and Q, of mass 2 kg and 3 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. A constant force **F** of magnitude 30 N is applied to Q in the direction PQ, as shown in Figure 4. The force is applied for 3 s and during this time Q travels a distance of 6 m. The coefficient of friction between each particle and the plane is μ. Find

(a) the acceleration of Q, $\frac{4}{3}\ \text{ms}^{-2}$
(2)

(b) the value of μ, $\frac{10}{21}$
(4)

(c) the tension in the string. 12N
(4)

(d) State how in your calculation you have used the information that the string is inextensible.
(1)

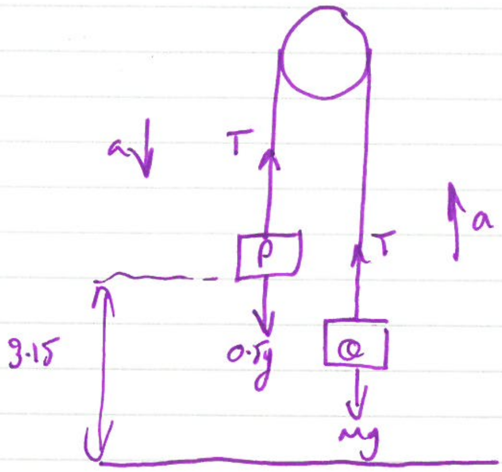
When the particles have moved for 3 s, the force **F** is removed.

(e) Find the time between the instant that the force is removed and the instant that Q comes to rest.
$t = \frac{6}{7}\ \text{s}$
(4)

H 2 9 4 9 2 A 0 2 0 2 4

M1 - JUNE 07

Q6
5.



(a) $u=0$ $t=1.5$ $s=3.15$ $a=?$

$3.15 = 0 + \frac{1}{2} a \times 1.5^2$

$a = \frac{3.15}{1.125} = 2.8 \text{ ms}^{-2}$

(b) N2L on P: $0.5g - T = 0.5a$ — (1)

N2L on Q: $T - Mg = Ma$ — (2)

(1) + (2) $0.5g - Mg = 0.5a + Ma$

$0.5g - Mg = 1.4 + 2.8M$

$0.5g - 1.4 = 2.8M + Mg$

$0.5g - 1.4 = M(2.8 + g)$

$3.5 = 12.6M$

$\therefore M = \frac{3.5}{12.6}$

in (2) $T = M(a+g) = \frac{3.5}{12.6}(2.8 + 9.8) = 3.5\text{N}.$

(c) $M = \frac{3.5}{12.6} = \frac{35}{126} = \frac{5}{18}$ As required.

(d) acceleration is the same for both particles

M1 - JUNE 07

Q6(e) string goes slack, Q moves freely under gravity, time taken to rise & fall back to same point, ie displacement of O.

Need initial speed of Q = speed P hits ground

For P: $u=0,\ a=2.8,\ t=1.5,\ V=?$

$$V=0+1.5\times 2.8=4.2\,ms^{-1}$$

$\therefore$ For Q: $u=4.2\uparrow \quad a=9.8\downarrow \quad s=0 \quad t=?$

$= -9.8\uparrow$

$$0=4.2t+\frac{1}{2}\times -9.8t^2$$

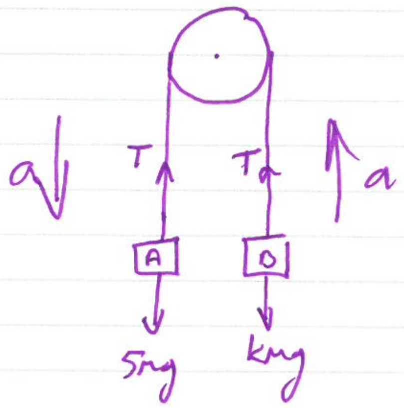
$$0=4.2t-4.9t^2$$

$$t(4.9t-4.2)=0$$

$t=0$ and $t=\dfrac{4.2}{4.9}=\dfrac{6}{7}$ seconds.

M1 - January 10

Q3



(a) N2L on A : $5mg - T = 5ma$ —— (1)

N2L on B : $T - kmg = kma$ —— (2)

$a = \frac{g}{4}$ in (1) $5mg - T = 5m\frac{g}{4}$

$\therefore T = 5mg - \frac{5}{4}mg = \frac{20}{4}mg - \frac{5}{4}mg = \frac{15}{4}mg$ as reqd.

(b) (1)+(2) $5\cancel{m}g - k\cancel{m}g = (5\cancel{m} + k\cancel{m})\frac{g}{4}$

×4

$20\cancel{g} - 4k\cancel{g} = 5\cancel{g} + k\cancel{g}$

$20 - 5 = 4k + k$

$15 = 5k$

$k = 3.$

(c) Tensions are equal throughout system.

(d) B is twice distance A falls when string goes slack, then travells with an inital speed that A hits ground under gravity until it comes instantaneously to rest.

For A $u = 0, t = 1.2, a = \frac{g}{4}, V = ?, s = ?$

$V = u + at$

$= 0 + \frac{g}{4} \times 1.2$

$= 2.94 \text{ ms}^{-1}$

$s = 0 + \frac{1}{2} \times \frac{g}{4} \times 1.2^2$

$= 1.764$ metres

So for B: $u = 2.94\uparrow$ $a = -9.8\uparrow$ $V = 0$, $s = ?$

$0^2 = 2.94^2 - 2 \times 9.8 s$

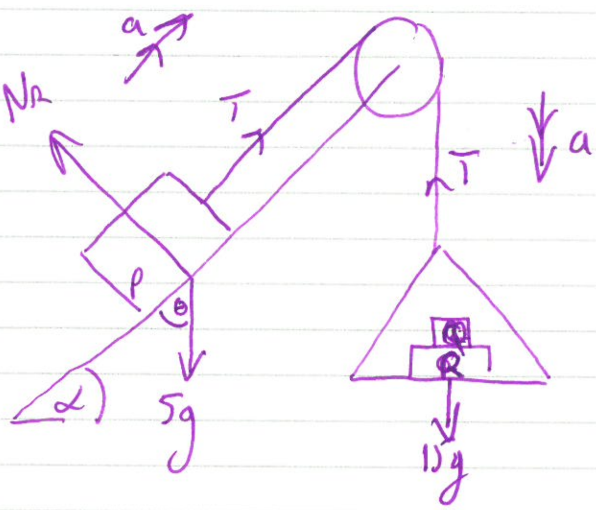
$s = 0.441$

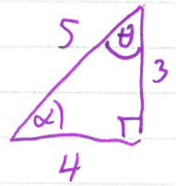
$\therefore$ Max height of B $= 0.441 + 1.764 + 1.764$

$= 4.0$ (1dp)

M1 - JAN 09

Q7.





$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

(a)(i) N2L on pan $\quad 15g - T = 15a \quad$ — ①

N2L on P $\quad T - 5g\cos\theta = 5a \quad$ — ②

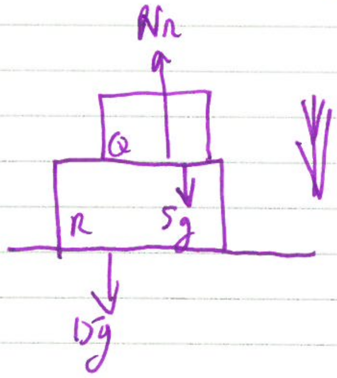
① + ② $\quad 15g - 5g \times \frac{3}{5} = 20a$

$12g = 20a$

$a = \frac{12g}{20} = 0.6g \text{ ms}^{-2}$

(ii) In ② $\quad T = 5(0.6g) + 5g \times \frac{3}{5} = 3g + 3g = 6g\text{ N}.$

(iii)

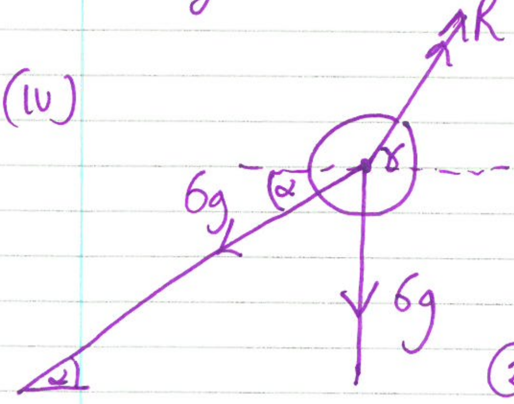


$a = 0.6g \quad$ N2L on Q: $\quad 5g - N_R = 5a$

$5g - N_R = 5(0.6g)$

$N_R = 5g - 3g = 2g\text{ N}.$

(iv)



$\Sigma F_x: R\cos\gamma - 6g\cos\alpha = 0 \Rightarrow R\cos\gamma = \frac{24g}{5}$

$= 47.04$ — ①

$\Sigma F_y: R\sin\gamma - 6g - 6g\sin\alpha = 0 \Rightarrow R\sin\gamma = 6g + \frac{18g}{5}$

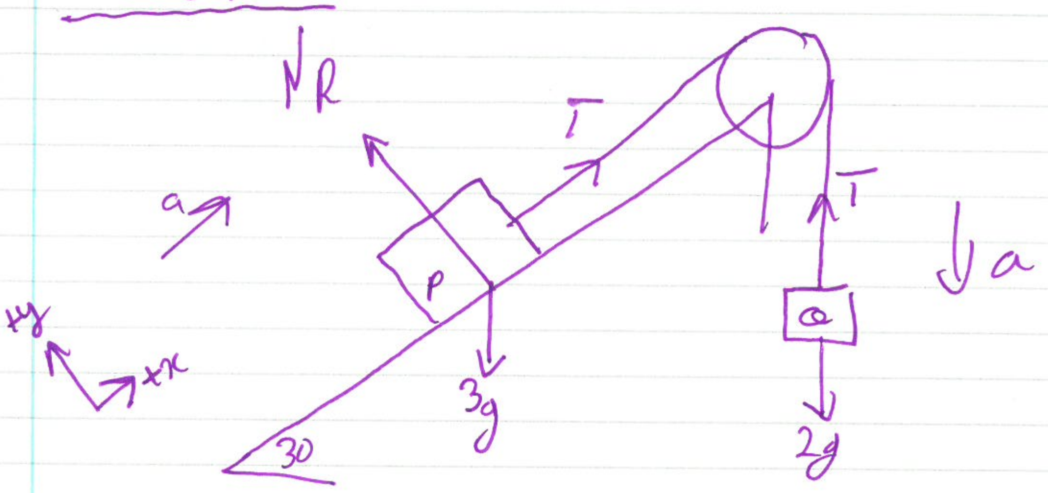
$= 94.08$ — ②

② ÷ ① $\quad \tan\gamma = 2 \Rightarrow \gamma = 63.4^\circ$

in ① $\quad R = \frac{47.04}{\cos 63.4} = 105.2\text{ N}.$

M1 – JAN 07

Q7



(a) ~~for~~ N2L for P: $T - 3g\cos 60 = 3a$ — ①

N2L for Q: $2g - T = 2a$ — ②

(b) ① + ② $2g - 3g\cos 60 = 5a$

$2g - 1.5g = 5a$

$0.5g = 5a$

$a = 0.98\,ms^{-2}$ As required.

(c) In ① $T = 3(0.98) + \cancel{3g}\,1.5g = 17.64 = 18\,N$

(d) Both P & Q have same acceleration

(e) $u = 0,\ s = 0.8,\ a = 0.98,\ V = ?$

$V^2 = 0^2 + 2 \times 0.98 \times 0.8 = 1.568$

$V = \sqrt{1.568}$

$V = 1.3\,ms^{-1}$

(f) When Q hits ground, string goes slack & P will continue to move up slope, but decelerate

① becomes $-1.5g = 3a \Rightarrow a = -g/2 = -4.9\,ms^{-2}$

~~P will come to a halt after $u = 1.3,\ a = -4.9,\ v = 0,\ t = ?$~~

~~$0 = 1.3 - 4.9t$~~

~~$t = \frac{13}{49}$ sec~~

M1 - JAN 07

Q7(f) Now P will move with $u = \sqrt{1.568}$ $a = -4.9$

We want to know times when displacement $= 0$

Using $s = ut + \frac{1}{2}at^2$

$$0 = \sqrt{1.568}\,t + \frac{1}{2} \times -4.9\,t^2$$

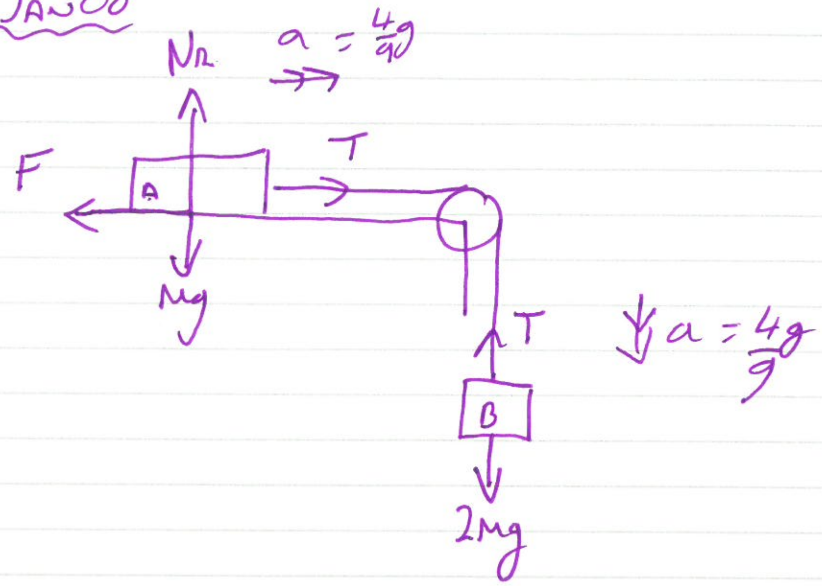
$$2.45t^2 - \sqrt{1.568}\,t = 0$$

$$t\left(2.45t - \sqrt{1.568}\right) = 0$$

$\therefore$ either $t = 0$ or $t = \frac{\sqrt{1.568}}{2.45} = 0.51$ sec.

M1 - JAN 08

Q7.



(a) N2L on B: $2Mg - T = 2M \cdot \frac{4}{9}g$

$T = 2Mg - \frac{8}{9}Mg$

$T = \frac{10}{9}Mg$ N

(b) N2L on A: $T - F = Ma$

$\frac{10}{9}Mg - M \cdot \frac{4g}{9} = F$

$F = \frac{6}{9}Mg = \frac{2}{3}Mg$

Now $F = \mu N_R$ & $N_R = Mg$

$\therefore \mu = \frac{F}{N_R} = \frac{\frac{2}{3}Mg}{Mg} = \frac{2}{3}$ As required.

M1 – JAN 08

~~Q7~~ (c) Initial speed of A when string goes slack = speed of B on impact

For B $u = 0$, $a = \frac{4g}{9}$, $s = h$, $v = ?$

$$v^2 = 0^2 + 2 \times \frac{4g}{9} h$$

$$v^2 = \frac{8gh}{9}$$

$$v = \sqrt{\frac{8gh}{9}}$$

Now for A: $u = \sqrt{\frac{8gh}{9}}$ $s = \frac{1}{3}h$

new acceleration: $-F = m a$

$$-\mu N_2 = ma$$

$$-\frac{2}{3} \cdot mg = ma$$

$$a = -\frac{2g}{3}$$

$\therefore$ Speed of A when reaches P $= V$

$$V^2 = u^2 + 2as$$

$$V^2 = \frac{8gh}{9} + 2 \times -\frac{2}{3}g \times \frac{1}{3}h$$

$$V^2 = \frac{8gh}{9} - \frac{4}{9}gh$$

$$V^2 = \frac{4}{9}gh$$

$$V = \frac{2}{3}\sqrt{gh}$$

(d) Equal tension throughout system.

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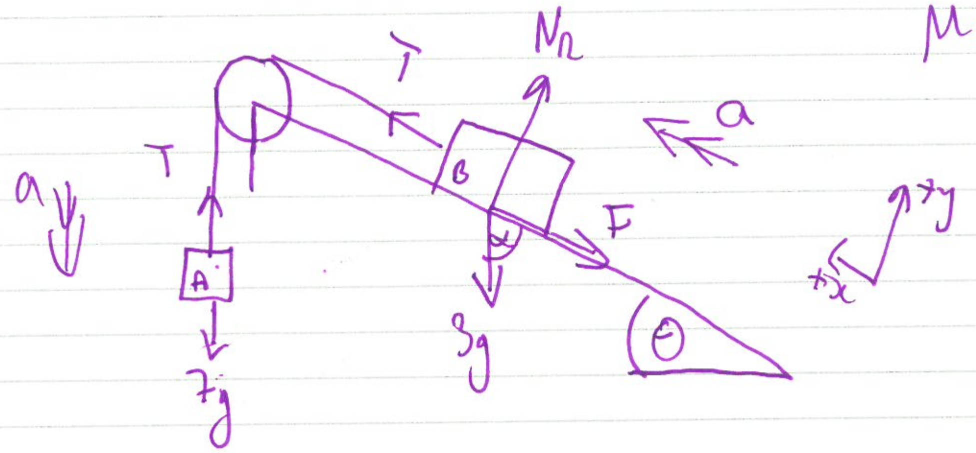
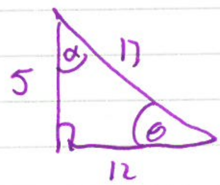
Q7.

$\mu = \frac{2}{3}$





$\tan\theta = \frac{5}{12}$ $\quad \sin\alpha = \frac{12}{13}$ $\quad \cos\alpha = \frac{5}{13}$

(a) N2L on A: $7g - T = 7a$ — ①

N2L on B: $T - F - 3g\cos\alpha = 3a$

$N_R - 3g\sin\alpha = 0$

$N_R = 3g \times \frac{12}{13} = \frac{36g}{13}$

$F = \mu N_R = \frac{2}{3} \times \frac{36}{13}g = \frac{24g}{13}$

$\therefore T - \frac{24g}{13} - 3g \times \frac{5}{13} = 3a$

$T - 3g = 3a$ — ②

① + ② $\quad 4g = 10a$

$a = \frac{4g}{10} = 3.9 \text{ ms}^{-2}$

(b) $u = 0,\ a = 3.9,\ s = 1,\ v = ?$

$v^2 = 0^2 + 2 \times 3.9 \times 1 \Rightarrow v^2 = 7.8 \Rightarrow v = 2.8 \text{ ms}^{-1}$

M1 - Jan 11

Q7(c) when string breaks, accel changes

eq^n ② becomes $-3g = 3a \Rightarrow a = -g \text{ ms}^{-2}$

$\therefore u = 2.8,\ a = -g,\ v = 0,\ t = ?$

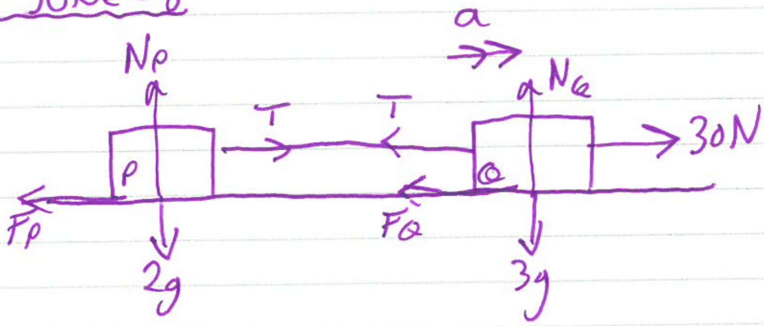
$v = u + at$

$0 = 2.8 - 9.8t$

$t = 0.29$ sec.

M1 - JUNE 08

Q8.



(a) $u=0, a=?, t=3, s=6$

$$6 = 0 + \frac{1}{2} \times a \times 3^2$$

$$6 = \frac{9a}{2}$$

$$a = \frac{12}{9} = \frac{4}{3} \text{ ms}^{-2}$$

(b) N2L on P: $T - F_P = 2a$

$$F_P = \mu N_P = \mu 2g$$

$$\therefore \quad T - 2\mu g = 2 \times \frac{4}{3}$$

$$T - 2\mu g = \frac{8}{3} \quad — (1)$$

N2L on Q: $30 - T - F_Q = 3a$

$$F_Q = \mu N_Q = \mu 3g$$

$$\therefore \quad 30 - T - 3\mu g = 3 \times \frac{4}{3}$$

$$-T - 3\mu g = -26 \quad — (2)$$

(1) + (2) $\quad -5\mu g = -\frac{70}{3}$

$$\mu = \frac{-70}{-15g} = \frac{70}{147} = \frac{10}{21}$$

M1 – JUNE 08

Q8. (c) in ① $T = \frac{8}{3} + 2\mu g = \frac{8}{3} + 2g\left(\frac{10}{21}\right) = \frac{8}{3} + \frac{28}{3} = 12N.$

(d) acceleration the same in P & Q.

(e) F removed, new acceln, string goes slack. speed at this instant $V = 0 + \frac{4}{3} \times 3 = 4ms^{-1}$

eqn ② becomes $-3\mu g = 3a$

$a = -\mu g = -\frac{14}{3}$

Now $u = 4, V = 0, a = -\frac{14}{3}, t = ?$

$0 = 4 - \frac{14}{3}t$

$t = \frac{12}{14} = \frac{6}{7}$ sec.