Components of forces

We have seen that two forces can be combined into a single force which is called their *resultant*.

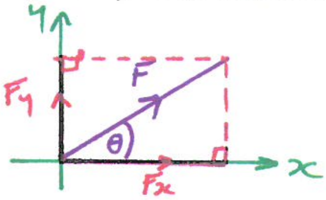
There is a reverse process which consists of expressing a single force in terms of its two *components*. These components are sometimes referred to as the *resolved parts* of the force.

It is particularly useful to find two mutually perpendicular components of a force.

The directions may, for example, be horizontal and vertical, or parallel and perpendicular to the surface of an inclined plane.

The component of the force F in any given direction is a measure of the effect of the force F in that direction.

Consider a force F acting at and angle θ to the x-axis as shown below. The components F_x and F_y being the horizontal and vertical components of F respectively.

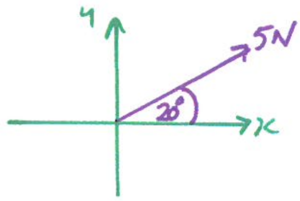


$$\frac{F_y}{F} = \frac{opp}{hyp} = \sin\theta \qquad \frac{F_x}{F} = \frac{adj}{hyp} = \cos\theta$$

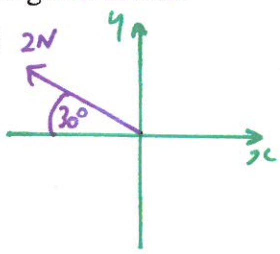
$$F_y = F\sin\theta \qquad F_x = F\cos\theta$$

Eg13 Find the components F_x and F_y of the given forces:

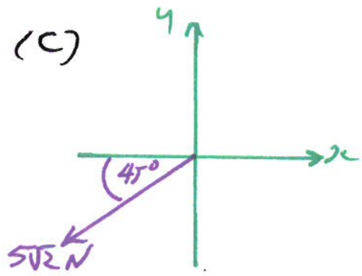
(a)



(b)



(c)

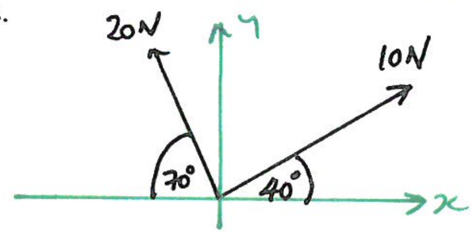


Exercise Q's 1 & 2

Eg14 A body of mass 4kg rests on an incline of 35°. Find the component of the weight of the body parallel and perpendicular to the plane.

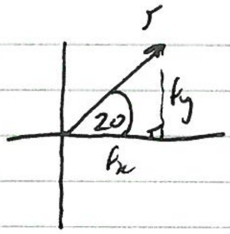
Exercise Q3

Eg15 Find the sum of the components of the given forces in the direction of (i) x-axis (ii) y-axis.



Exercise Q's 4 & 5

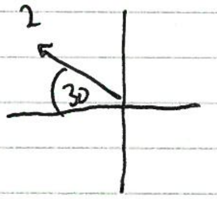
Eg13 (a)



$F_x = 5\cos 20 = 4.70\,N$

$F_y = 5\sin 20 = 1.71\,N$

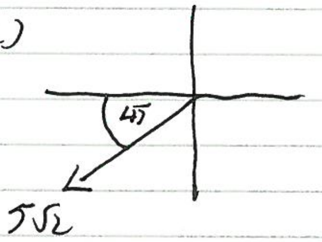
(b)



$F_x = -2\cos 30 = -2 \times \frac{\sqrt{3}}{2} = -\sqrt{3}\,N$

$F_y = +2\sin 30 = 2 \times \frac{1}{2} = 1\,N$

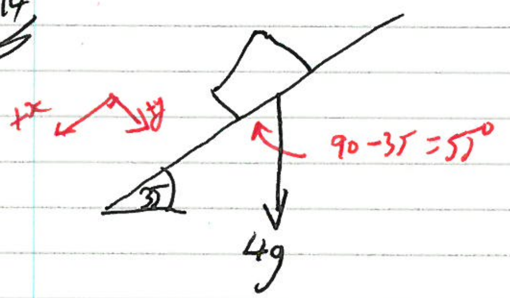
(c)



$F_x = -5\sqrt{2}\cos 45 = -5\sqrt{2} \times \frac{1}{\sqrt{2}} = -5\,N$

$F_y = -5\sqrt{2}\sin 45 = -5\sqrt{2} \times \frac{1}{\sqrt{2}} = -5\,N$

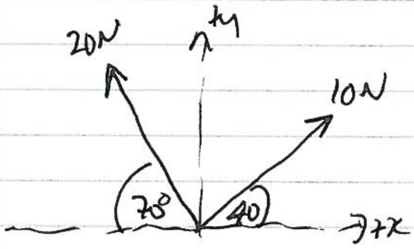
Eg14



$F_x = 4g\cos 55 = 22.5\,N$

$F_y = 4g\sin 55 = 32.1\,N$

Eg15

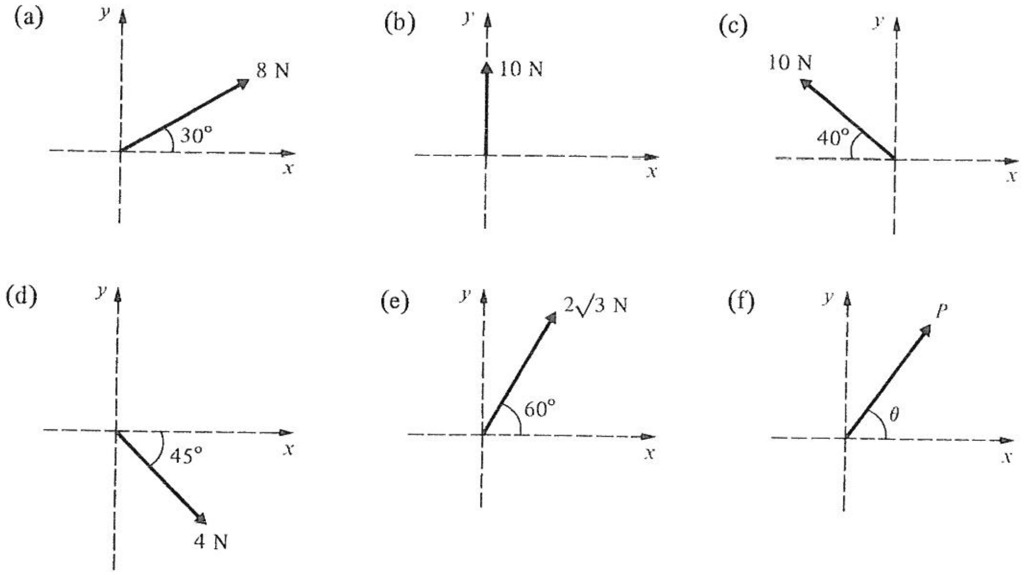


$\Sigma F_x = 10\cos 40 - 20\cos 70 = 0.82\,N$

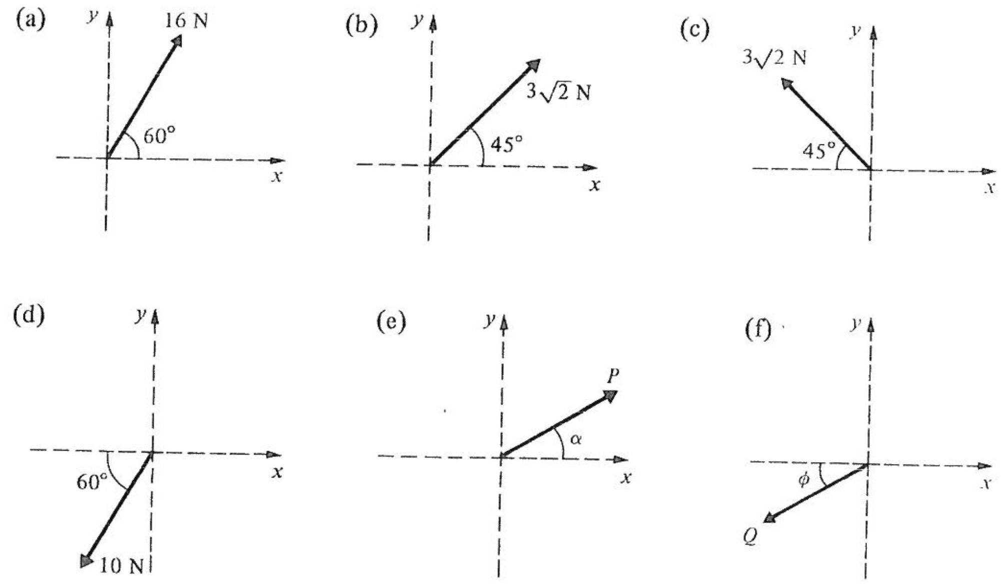
$\Sigma F_y = 10\sin 40 + 20\sin 70 = 25.2\,N$

Components of Forces Exercise

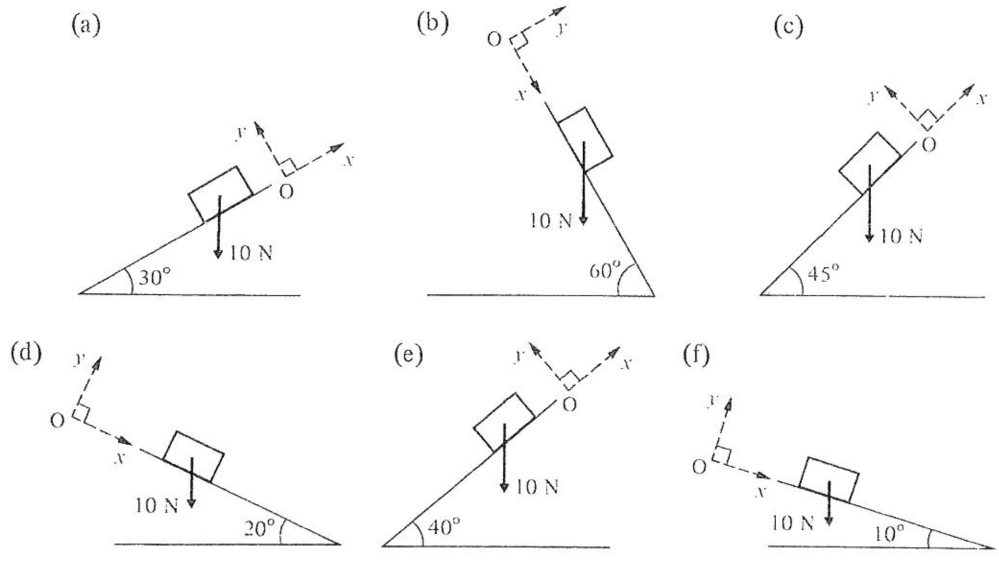
1. For each of the forces shown below, find the components in the direction of (i) the x-axis and (ii) the y-axis.



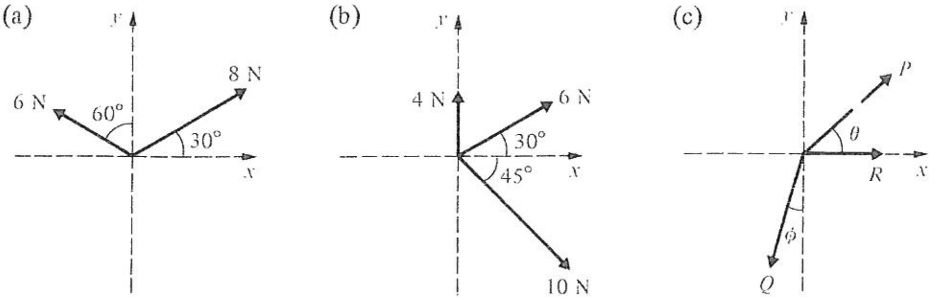
2. Express each of the following forces in the form $a\mathbf{i} + b\mathbf{j}$.



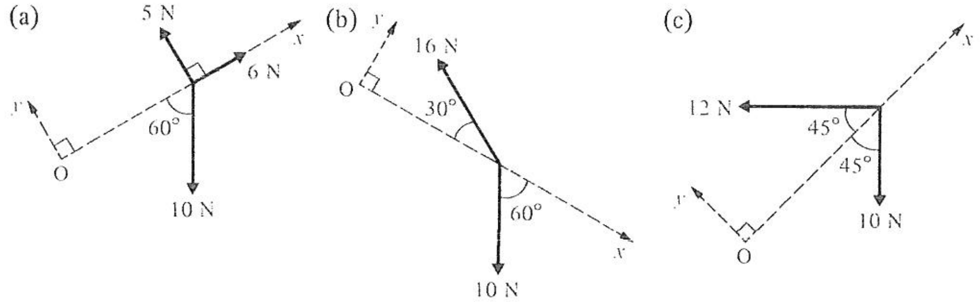
3. Each of the following diagrams shows a body of weight 10 N on an incline. In each case find the component of the weight of the body (i) in the Ox direction and (ii) in the Oy direction.

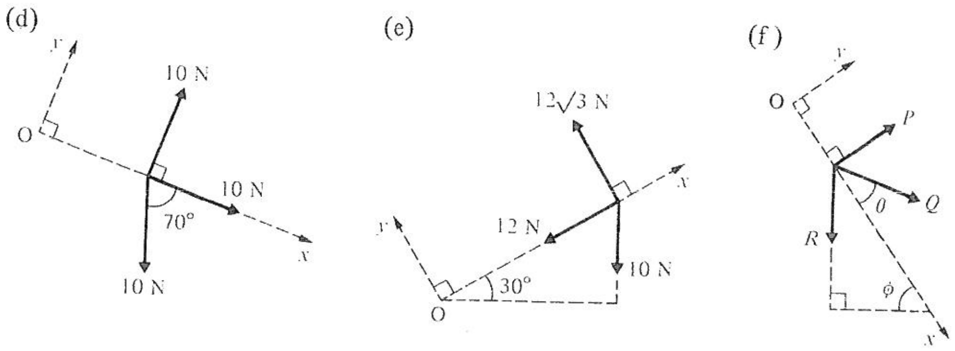


4. For each of the following systems of forces, find the sum of the components in the direction of (i) the x-axis and (ii) the y-axis.



5. For each of the following systems of forces, find the sum of the components (i) in the Ox direction and (ii) in the Oy direction.





ANSWERS

1. (a) (i) $4\sqrt{3}$ N (ii) 4 N (b) (i) 0 (ii) 10 N
(c) (i) $-7{\cdot}66$ N (ii) $6{\cdot}43$ N (d) (i) $2\sqrt{2}$ N (ii) $-2\sqrt{2}$ N
(e) (i) $\sqrt{3}$ N (ii) 3 N (f) (i) $P\cos\theta$ (ii) $P\sin\theta$

2. (a) $(8\mathbf{i} + 8\sqrt{3}\mathbf{j})$ N (b) $(3\mathbf{i} + 3\mathbf{j})$ N (c) $(-3\mathbf{i} + 3\mathbf{j})$ N (d) $(-5\mathbf{i} - 5\sqrt{3}\mathbf{j})$ N
(e) $P\cos\alpha\mathbf{i} + P\sin\alpha\mathbf{j}$ (f) $-Q\cos\phi\mathbf{i} - Q\sin\phi\mathbf{j}$

3. (a) (i) -5 N (ii) $-5\sqrt{3}$ N (b) (i) $5\sqrt{3}$ N (ii) -5 N
(c) (i) $-5\sqrt{2}$ N (ii) $-5\sqrt{2}$ N (d) (i) $3{\cdot}42$ N (ii) $-9{\cdot}40$ N
(e) (i) $-6{\cdot}43$ N (ii) $-7{\cdot}66$ N (f) (i) $1{\cdot}74$ N (ii) $-9{\cdot}85$ N

4. (a) (i) $\sqrt{3}$ N (ii) 7 N (b) (i) $12{\cdot}3$ N (ii) $-0{\cdot}071$ N
(c) (i) $P\cos\theta + R - Q\sin\phi$ (ii) $P\sin\theta - Q\cos\phi$

5. (a) (i) 1 N (ii) $-3{\cdot}66$ N (b) (i) $-8{\cdot}86$ N (ii) $-0{\cdot}66$ N
(c) (i) $-11\sqrt{2}$ N (ii) $\sqrt{2}$ N (d) (i) $13{\cdot}4$ N (ii) $0{\cdot}60$ N
(e) (i) -17 N (ii) $7\sqrt{3}$ N (f) (i) $R\sin\phi + Q\cos\theta$ (ii) $P + Q\sin\theta - R\cos\phi$

Further N2L Problems

Now that we can resolve forces, we can extend the complexity of the situations which require Newton's second law to solve.

Eg16 A body of mass 4kg has an acceleration a when it is acted upon by a force of $25\sqrt{2}N$ which is inclined at 45° to the smooth horizontal surface on which the body rests. By resolving the forces parallel and perpendicular to the plane, calculate the normal reaction between the body and the plane and the acceleration, a, of the body.

Exercise Q's 1 to 5

Eg17 A body of mass $3\sqrt{3}kg$ on the surface of a smooth plane inclined at 60° is acted on by a horizontal force of 15g N. Calculate the normal reaction of the plane on the body, and the acceleration of the body up the surface of the smooth inclined plane.

Exercise Q's 6 to 9

Rough Surfaces

In the previous examples, motion has taken place on smooth surfaces. In practice this does not happen; all surfaces tend to impede motion. The resistance to motion is an external force acting upon the body, parallel to the surfaces in contact. It will be considered to be a constant force.

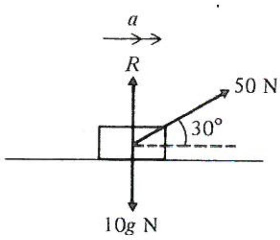
Eg18 A body of mass 5kg is released from rest on the surface of a rough plane which is inclined at $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If the body takes 3 seconds to reach a speed of 4.9ms^{-1} from rest, find the resistance to motion which the body must be experiencing.

Exercise Q's 12 to 17

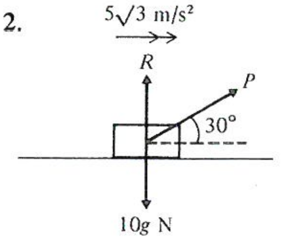
Exercise 5C

Each of the diagrams in questions **1** to **9** shows a body of mass 10 kg accelerating along a surface in the direction indicated. All of the forces acting are as shown. In each case
(a) obtain an equation by resolving perpendicular to the direction of motion,
(b) obtain an equation by applying $F = ma$ parallel to the direction of motion,
(c) use your equations to (a) and (b) to find the unknown forces, accelerations and angles.

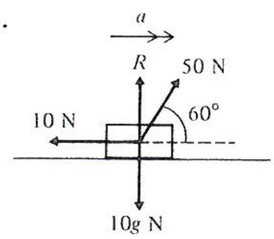
1.



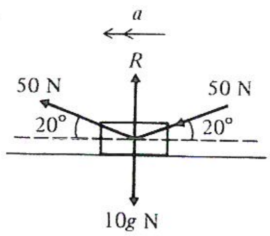
2.



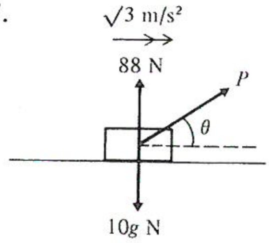
3.



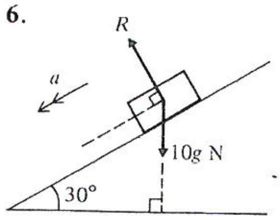
4.



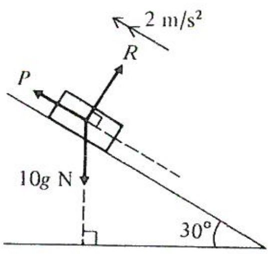
5.



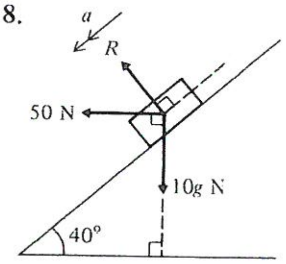
6.



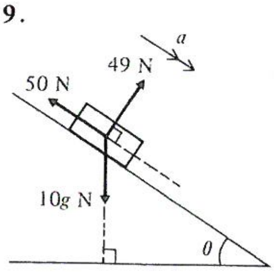
7.



8.



9.



12. A body of mass 10 kg is initially at rest on a rough horizontal surface. It is pulled along the surface by a constant force of 60 N inclined at 60° above the horizontal. If the resistance to motion totals 10 N, find the acceleration of the body and the distance travelled in the first 3 s.

13. A body of mass 5 kg, initially at rest on a smooth horizontal surface, is pulled along the surface by a constant force P inclined at 45° above the horizontal. In the first 5 seconds of motion the body moves a distance of 10 m along the surface. Find the acceleration of the body, the magnitude of P and the normal reaction between the body and the surface.

14. A mass of 5 kg is initially at rest at the bottom of a smooth slope which is inclined at $\sin^{-1}\frac{3}{5}$ to the horizontal. The mass is pushed up the slope by a horizontal force of 50 N. Find the normal reaction between the mass and the plane and the acceleration up the slope. How far up the slope will the mass travel in the first 4 s?

15. A body of mass 100 kg is released from rest at the top of a smooth plane which is inclined at 30° to the horizontal. Find the velocity of the body when it has travelled 20 m down the slope. What would your answer be if the mass had been 50 kg?

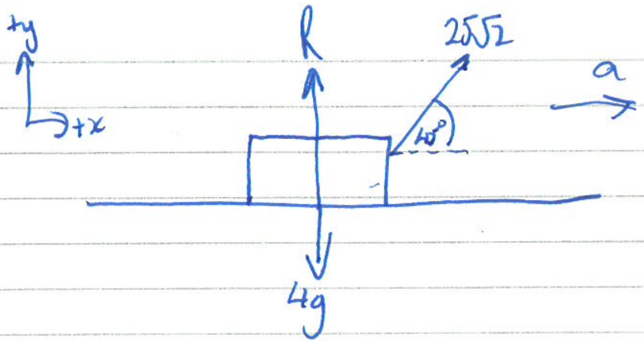
16. A body of mass 20 kg is released from rest at the top of a rough slope which is inclined at 30° to the horizontal. If the body accelerates down the slope at 3 m/s², find the resistance to motion experienced by the body. (Assume this resistance to be constant throughout).

17. A body of mass 20 kg is released from rest at the top of a rough slope which is inclined at 30° to the horizontal. Six seconds later the body has a velocity of 21 m/s down the slope. Find the resistance to motion experienced by the body. (Assume this resistance to be constant throughout).

ANSWERS

1. (a) $R + 50\sin 30° = 10g$ (b) $50\cos 30° = 10a$ (c) $R = 73$ N, $a = \frac{5\sqrt{3}}{2}$ m/s²
2. (a) $R + P\sin 30° = 10g$ (b) $P\cos 30° = 50\sqrt{3}$ (c) $P = 100$ N, $R = 48$ N
3. (a) $R + 50\sin 60° = 10g$ (b) $50\cos 60° - 10 = 10a$ (c) $R = 54{\cdot}7$ N, $a = 1{\cdot}5$ m/s²
4. (a) $R + 50\sin 20° = 10g + 50\sin 20°$ (b) $50\cos 20° + 50\cos 20° = 10a$
   (c) $R = 98$ N, $a = 9{\cdot}40$ m/s²
5. (a) $88 + P\sin\theta = 10g$ (b) $P\cos\theta = 10\sqrt{3}$ (c) $P = 20$ N, $\theta = 30°$
6. (a) $R = 10g\cos 30°$ (b) $10g\sin 30° = 10a$ (c) $R = 84{\cdot}9$ N, $a = 4{\cdot}9$ m/s²
7. (a) $R = 10g\cos 30°$ (b) $P - 10g\sin 30° = 20$ (c) $R = 84{\cdot}9$ N, $P = 69$ N
8. (a) $R + 50\sin 40° = 10g\cos 40°$ (b) $50\cos 40° + 10g\sin 40° = 10a$
   (c) $R = 42{\cdot}9$ N, $a = 10{\cdot}1$ m/s²
9. (a) $49 = 10g\cos\theta$ (b) $10g\sin\theta - 50 = 10a$ (c) $a = 3{\cdot}49$ m/s², $\theta = 60°$
10. 65·3 N, 1·63 m/s², 84·9 N
11. 34·6°, 80·7 N, 25·6 N
12. 2 m/s², 9 m
13. 0·8 m/s², $4\sqrt{2}$ N, 45 N
14. 69·2 N, 2·12 m/s², 16·96 m
15. 14 m/s, 14 m/s
16. 38 N
17. 28 N
18. 0·74 s
19. 3·8 m/s², 7·6 m
20. 31 N
21. 30°
22. 0·663 s

Eg16



No accel perp to plane $\therefore$ $\Sigma F_y = 0$ $\quad 25\sqrt{2}\sin 45 + R - 4g = 0$ — (1)

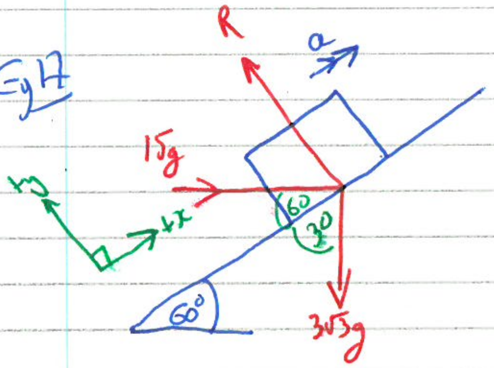
accel parallel to plane $\therefore$ N2L applies $\Sigma F_x = ma$ $\quad 25\sqrt{2}\cos 45 = 4a$ — (2)

From (1) $\quad R = 4g - 25\sqrt{2} = 4g - 25\sqrt{2} \times \frac{1}{\sqrt{2}} = 14.2N$

From (2) $\quad 25\sqrt{2} \times \frac{1}{\sqrt{2}} = 4a$

$$a = \frac{25}{4} = 6.25 \text{ ms}^{-2}$$

Eg17



No accel perp to plane $\therefore$ $\Sigma F_y = 0$ $\quad R - 15g\sin 60 - 3\sqrt{3}g\sin 30 = 0$ — (1)

accel parallel to plane $\therefore$ $\Sigma F_x = ma$ $\quad 15g\cos 60 - 3\sqrt{3}g\cos 30 = 3\sqrt{3}a$ — (2)

From (1) $\quad R = 15g \times \frac{\sqrt{3}}{2} + 3\sqrt{3}g \times \frac{1}{2} = \frac{18g\sqrt{3}}{2} = 9\sqrt{3}g$ N

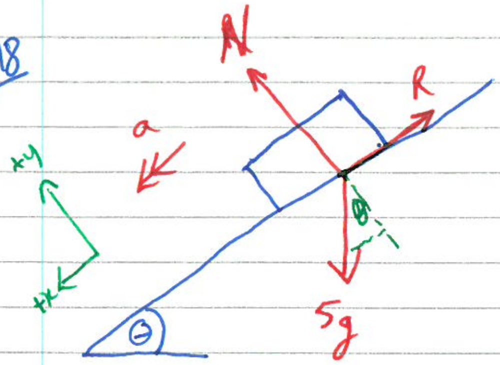
From (2) $\quad 15g \times \frac{1}{2} - 3\sqrt{3}g \times \frac{\sqrt{3}}{2} = 3\sqrt{3}a$

$$3\sqrt{3}a = \frac{15g}{2} - \frac{9g}{2}$$

$$3\sqrt{3}a = 3g$$

$$a = \frac{3g}{3\sqrt{3}} = \frac{g}{\sqrt{3}} = \frac{g\sqrt{3}}{3} = 5.66 \text{ ms}^{-2}$$

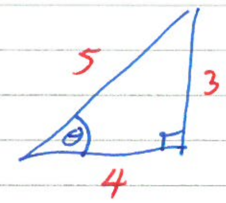
Eg 18



$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin\theta = \frac{3}{5}$ opp / hyp

$\therefore \cos\theta = \frac{4}{5}$



$\sum F_y = 0 \quad N - 5g\cos\theta = 0$

$N = 5g \times \frac{4}{5} = 4g \text{ N}$

$\sum F_x = Ma \quad 5g\sin\theta - R = 5a$

$5g \times \frac{3}{5} - R = 5a$

$R = 3g - 5a$ — ①

Now $u = 0 \quad v = 4.9 \quad t = 3 \quad a = ?$

$v = u + at$

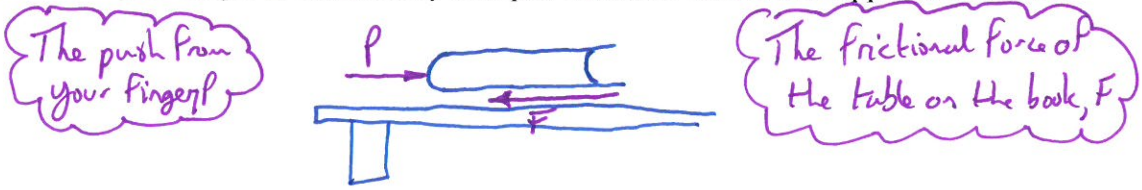
$4.9 = 3a$

$a = \frac{4.9}{3}$

in ① $R = 3g - 5\left(\frac{4.9}{3}\right) = 21.2 \text{ N}$

Friction

Place a heavy book on a table and push it lightly with your finger. Nothing happens. The force from your finger is balanced by an equal frictional force in the opposite direction.



Now increase the force P with which your finger is pushing the book. As P increases, so does the frictional force F opposing it. They balance each other, so

$$P = F$$

Until … the book moves. At that point the frictional force F has reached the greatest value it can take, and it is no longer able to balance P.

So the frictional force F between an object and surface is not constant, but increases as the applied force P increases until the force F reaches a value F_{max} beyond which it cannot increase. The book is then on the point of moving and is said to be in a state of ***limiting equilibrium.***

In our situation with the book, whilst $P < F_{max}$, the book will not move. When $P = F_{max}$, the book is in limiting equilibrium (on the point of moving). When $P > F_{max}$, the book moves.

A frictional force will always act in the direction opposed to motion. If an object is moving, the frictional force will take its greatest possible value.

Coefficient of friction

The magnitude of the maximum frictional force is a fraction of the normal reaction, R. This fraction is called the coefficient of friction (μ) for the two surfaces in contact.

$$F_{\text{max}} = \mu R$$

From Wikipedia…

> Most dry materials in combination have friction coefficient values between 0.3 and 0.6. Values outside this range are rarer, but teflon, for example, can have a coefficient as low as 0.04. A value of zero would mean no friction at all, an elusive property – even magnetic levitation vehicles have drag. Rubber in contact with other surfaces can yield friction coefficients from 1 to 2. Occasionally it is maintained that μ is always < 1, but this is not true. While in most relevant applications μ < 1, a value above 1 merely implies that the force required to slide an object along the surface is greater than the normal force of the surface on the object. For example, silicone rubber or acrylic rubber-coated surfaces have a coefficient of friction that can be substantially larger than 1.

Eg19 A block of mass 5kg rests on a rough horizontal plane, the coefficient of friction between the block and the plane being 0.6. Calculate the frictional force acting on the block when a horizontal force P is applied to the block and the magnitude of P is (a) 12N, (b) 28N, (c) 36N. Also calculate the magnitude of any acceleration that may occur.

Eg20 A 10kg trunk lies on a rough horizontal floor. The coefficient of friction between the trunk and the floor is $\frac{\sqrt{3}}{4}$. Calculate the magnitude of the force P which is necessary to pull the trunk horizontally if P is applied (a) horizontally, (b) at 30° above the horizontal.

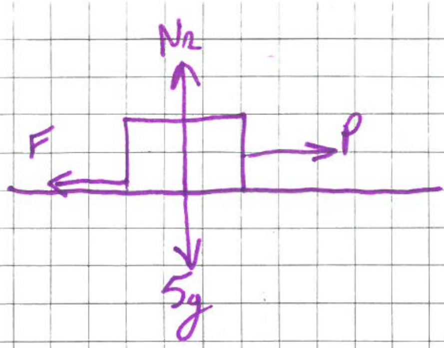
Exercise 3D Pg 52 Q2

Eg21 A mass of 6kg rests in limiting equilibrium on a rough plane inclined at 30° to the horizontal. Find the coefficient of friction between the mass and the plane.

Eg22 A mass of 0.5kg is resting on a rough plane. The coefficient of friction between the mass and the plane is $\frac{1}{\sqrt{2}}$, and the plane is inclined at angle θ to the horizontal such that $\sin\theta = \frac{1}{3}$. A mass then experiences a force of 6N applied up the plane along a line of greatest slope. Calculate the magnitude of the acceleration of the mass up the slope.

Exercise 3E Pg 55

Eg 19



$\Sigma F_y = 0$ $\quad N_R - 5g = 0$

$N_R = 5g = 49N$

Now $F_{max} = \mu N_R = 0.6 \times 49 = 29.4N$

(a) when $P = 12N$, this is less than the Max Friction force $\therefore$ block remains stationary

(b) when $P = 28$, again remains stationary

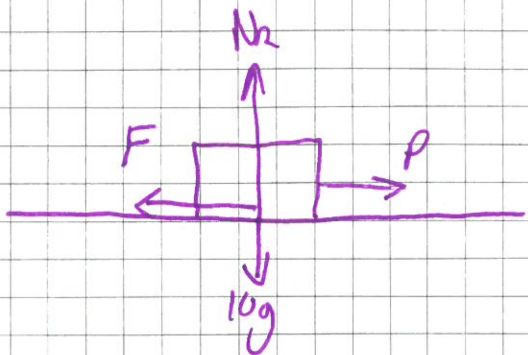
(c) when $P = 36$, motion will occur

N2L $\quad P - F = 5a$

$36 - 29.4 = 5a$

$a = 1.32\,ms^{-2}$

Eg 20 (a)



$\Sigma F_y \quad N_R - 10g = 0 \qquad N_R = 98N$

$F_{max} = \mu N_R = \frac{\sqrt{3}}{4} \times 98 = 24.5\sqrt{3}$

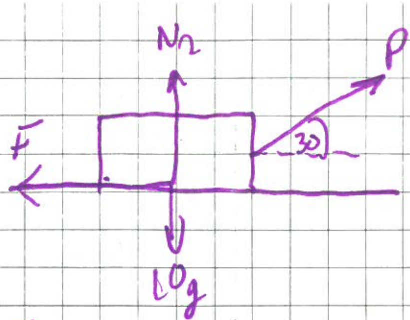
$\cancel{P - F = 10a}$

At limiting equilib $\quad P - F_{max} = 0$

$P = 42.4N$

$\therefore$ P must exceed $42.4N$

eg20 (b)



~~Fhdh~~ Now

$$N_R + P\sin 30 - 10g = 0 \quad \text{— (1)}$$
$$P\cos 30 - F = 0 \quad \text{— (2)}$$
$$F = \frac{\sqrt{3}}{4} N_R \quad \text{— (3)}$$

Sub (3) in (2)

$$P\cos 30 - \frac{\sqrt{3}}{4} N_R = 0$$

$$N_R = \cancel{4}\ \frac{4}{\sqrt{3}} P \times \frac{\sqrt{3}}{2} = 2P$$

In (1)

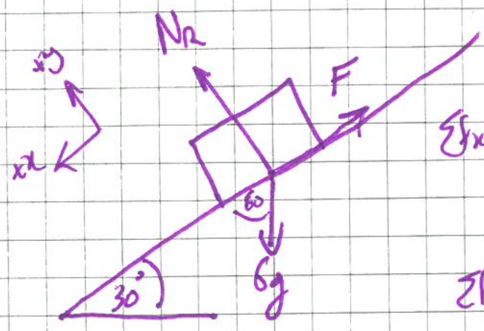
$$2P + \frac{P}{2} = 10g$$

$\times 2$

$$4P + P = 20g$$
$$5P = 20g$$
$$P = 4g = 39.2N$$

$\therefore$ P must exceed $39.2N$.

eg21



On point of sliding down plane.

$\Sigma F_x: \; 6g\cos 60 - F = 0$

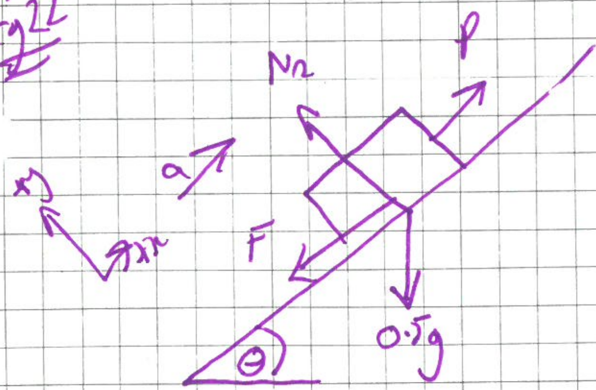
$F = 3g$

$\Sigma F_y: \; N_R - 6g\sin 60 = 0$

$N_R = 3\sqrt{3}g$

$\mu = \frac{F}{N_R} = \frac{3g}{3\sqrt{3}g} = \frac{1}{\sqrt{3}}$

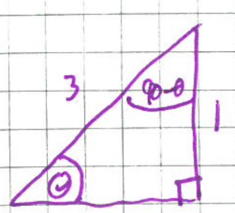
Eg22



$P - F - 0.5g\cos(90-\theta) = 0.5a$ — ①

$N_R - 0.5g\sin(90-\theta) = 0$ — ②

$F = \mu N_R$ — ③



$\sqrt{3^2 - 1^2}$

$= 2\sqrt{2}$

$\cos(90-\theta) = \frac{1}{3}$

$\sin(90-\theta) = \frac{2\sqrt{2}}{3}$

From ② $N_R = 0.5g \times \frac{2\sqrt{2}}{3} = \frac{g\sqrt{2}}{3}$

in ③ $F = \frac{1}{\sqrt{2}} \times \frac{g\sqrt{2}}{3} = \frac{g}{3}$

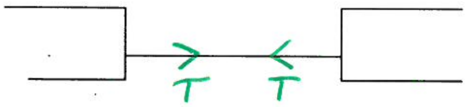
in ① $6 - \frac{g}{3} - 0.5g \times \frac{1}{3} = 0.5a$

$0.5a = 1.1$

$a = 2.2\,ms^{-2}$

The Effect of Newton's Laws on Connected Particles

When two moving particles are connected by a string which is light (no weight to consider) and inextensible (no elasticity to consider), there will be a tension in the string. By N3L, the forces acting on the particles will have the same magnitude but will act in opposite directions as shown below:



It is important on diagrams to show these separate tensions clearly.

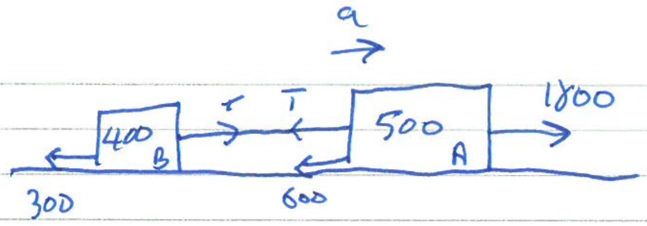
When working with problems involving connected particles, the forces acting on each particle can be considered separately, and ***if all parts of the system are moving in the same straight line***, then the whole system can also be treated as a single particle.

Eg23 A car of mass 500kg tows a trailer of mass 400kg along a horizontal road. The resistance to motion of the car is 600N, that of the trailer is 300N. The driving force of the car is 1800N. Find the acceleration of the car and the trailer and the tension in the towbar.

Eg24 A body of mass 2kg is hanging freely at the end of a light inextensible string. A second body of mass 5kg is then hung from the base of the first using a second string. A force is then applied to the upper string, and the particles begin to accelerate constantly upwards at $2ms^{-2}$. Calculate the tensions in the strings.

Exercise 3F Pg 64 No's 1, 2, 3, 8, 10

Eg 23



On whole: $1800 - 600 - 300 = 900a$

$900 = 900a$

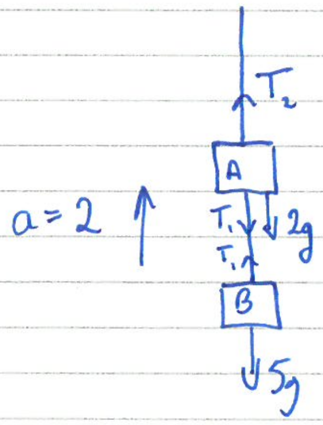
$a = 1ms^{-1}$

On CAR: $1800 - T - 600 = 500a$

$1200 - T = 500$

$T = 700N$

Eg 24



on A: $T_2 - T_1 - 2g = 2a$ — (1)

on B: $T_1 - 5g = 5a$ — (2)

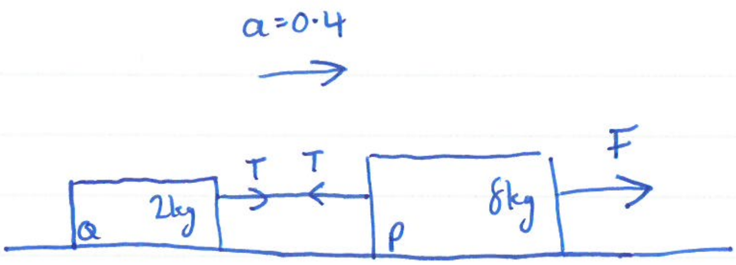
on all: $T_2 - 7g = 7a$ — (3)

From (2) $T_1 = 5\times2 + 5\times9.8 = 59N$

From (3) $T_2 = 7\times2 + 7\times9.8 = 82.6N$

Ex3F

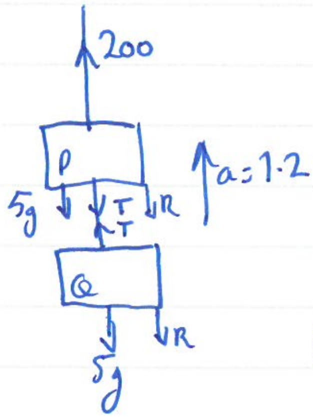
(1)



(a) All together: $F = 10 \times 0.4 = 4N$

(b) Q: $T = 2 \times 0.4 = 0.8\ N$

(2)



(a) All together: $200 - 10g - 2R = 10 \times 1.2$

$2R = +90$

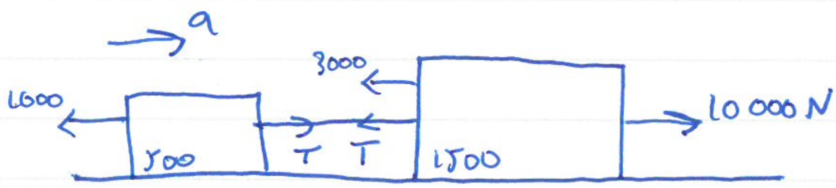
$R = +45N$

(b) Q: $T - 5g - 45 = 5 \times 1.2$

$T = 6 + 45 + 5g$

$= 100\ N$

(3)



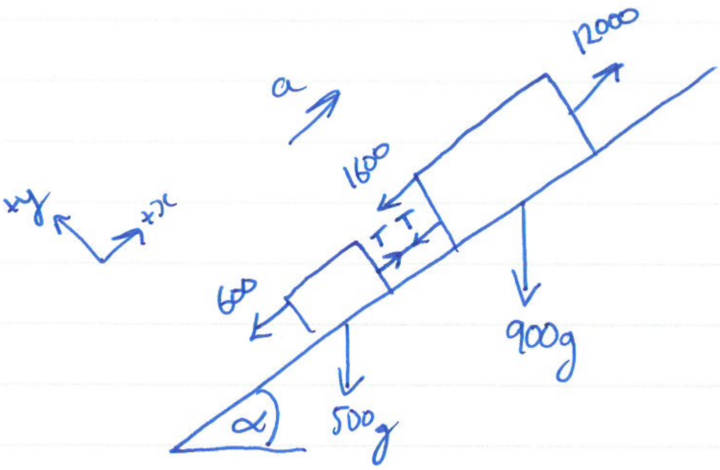
as a whole: $10000 - 3000 - 1000 = 2000a$

$a = 3 ms^{-2}$

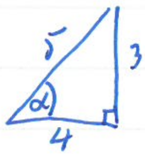
on trailer: $T - 1000 = 500 \times 3$

$T = 2500N$

(8)



$\tan\alpha = \frac{3}{4}$



$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

Whole system: $12000 - 1600 - 600 - 500g\sin\alpha - 900g\sin\alpha = 1400a$

$9800 - 1400g \times \frac{3}{5} = 1400a$

$9800 - 8232 = 1400a$

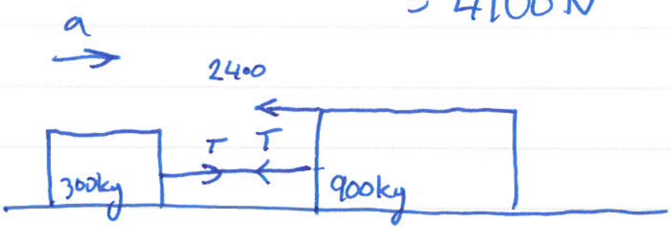
$a = \frac{1568}{1400} = 1.12\,ms^{-2}$

Consider trailer: $T - 600 - 500g\sin\alpha = 500 \times 1.12$

$T = 560 + 600 + 500 \times 9.8 \times 0.6$

$= 4100\,N$

(10)



(a) whole system: $-2400 \cancel{} = 1200a$

$a = -2\,ms^{-2}$

(b) on trailer $T = 300 \times -2 = -600N$ ie thrust of $600N$

(c) $u = 20,\ v = 0,\ a = -2,\ s = ?$

$v^2 = u^2 + 2as \implies 0^2 = 20^2 + 2 \times -2 \times s$

$4s = 400$

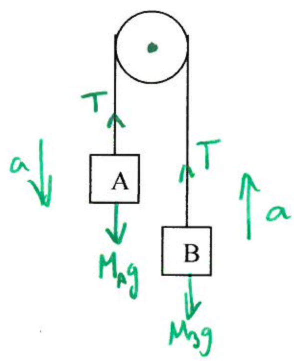
$s = 100$ metres.

The Effect of N2L on Pulleys

When connected particles pass over a pulley and released, the resulting motion will produce the same acceleration in each body. However, it is not possible to consider the system as a whole as the particles will be travelling in different directions.

A smooth pulley means that the tensions in the string are equal on both sides of the pulley.

If $M_A > M_B$



Eg 25 Two particles of mass 7kg and 3 kg are connected by a light, inextensible string passing over a smooth fixed pulley. Find the acceleration of the particles, the tension in the string and the force exerted on the pulley.

Eg 26 Bodies of mass 3Mkg and Mkg are connected by a light inextensible string which passes over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find the acceleration of the system and the distance moved by the 3Mkg mass in the first 2 seconds of motion. After the 3M kg mass hits the floor 10metres below the point of release, how much farther will the Mkg body travel before beginning to fall again?

Exercise 3F Pg 64 Q4

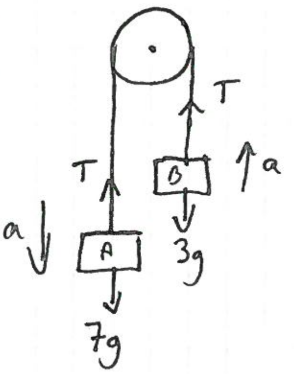
Eg 27 A particle of mass 3kg rests on a rough horizontal table ($\mu = 0.2$). It is connected by a light, inextensible string passing over a smooth pulley fixed at the edge of the table to a particle of mass 2kg which hangs freely. Find the acceleration of the system when it is released from rest. Find also the force exerted on the pulley.

Ex 3F Q5

Eg 28 A particle of mass Mkg rests on a smooth plane inclined at an angle of 30° to the horizontal. It is connected by a light, inextensible string passing over a smooth pulley fixed at the top of the plane to a particle of mass 4Mkg which hangs freely. Find the acceleration of the system when it is released from rest, the tension in the string and also the force exerted by the string on the pulley.

Ex 3F Q's 6, 7, 9

Eg 25



A: $7g - T = 7a$ — ①

B: $T - 3g = 3a$ — ②

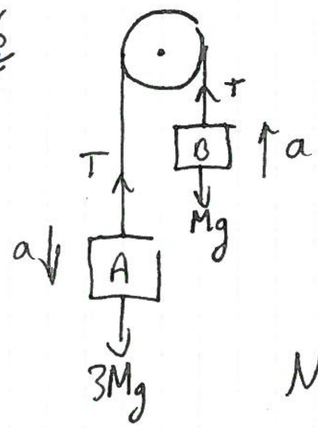
①+② $4g = 10a$

$a = \frac{4g}{10} = 3.92\,ms^{-2}$

in ② $T = 3(3.92) + 3(9.8) = 41.16\,N$

$R_y = T + T = 2T = 82.32\,N$

Eg 26



A: $3Mg - T = 3Ma$ — ①

~~Mg ra~~ $T - Mg = Ma$ — ②

① + ② $2\cancel{M}g = 4\cancel{M}a$

$a = \frac{g}{2} = 4.9\,ms^{-2}$

Now $s = ?$ $a = \frac{g}{2}\downarrow$ $u = 0$ $t = 2$

Using ~~[illegible]~~ $s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2} \times 4.9 \times 2^2 = 9.8$ metres.

Particle B now travels as projectile. Speed at this point: $u = 0$ $v = ?$ $a = 4.9$ $s = 12$

$v^2 = 0^2 + 2 \times 4.9 \times 12 = 117.6$

$v = 10.8\,ms^{-1}$

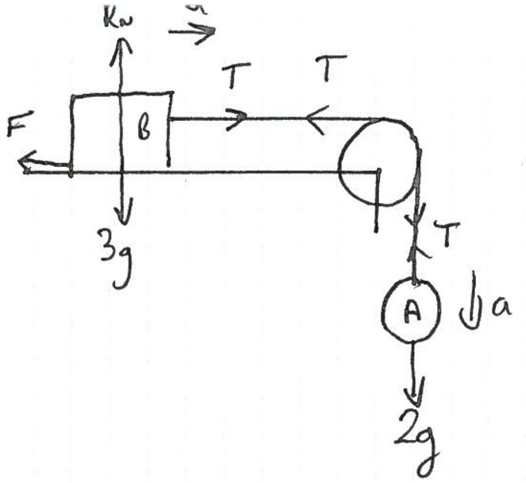
Now $u = 10.8\uparrow$ $v = 0$ $a = 9.8\downarrow = -9.8\uparrow$ $s = ?$

$0^2 = 10.8^2 - 19.6s$

$s =$ ~~6 metres~~ 5.95

$\therefore$ B will continue to rise for 6 metres.

eg 27



$$2g - T = 2a \quad (1)$$

$$T - F = 3a \quad (2)$$

$$R_N - 3g = 0 \quad (3)$$

$$F = \mu R_N \quad (4)$$

$$F = 0.2 \times 3g = 5.88$$

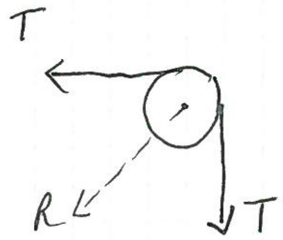
(1) + (2) $\quad 2g - F = 5a$

$$2g - 5.88 = 5a$$

$$a = 2.74\,ms^{-2}$$

In (2) $\quad T = 3 \times 2.74 + 5.88$

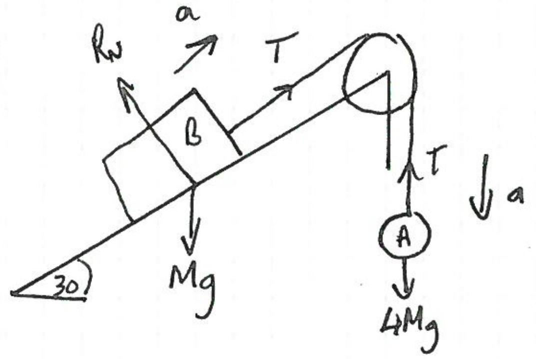
$$T = 14.1\,N$$



$R_x = T \qquad R_y = T$

$\therefore R = \sqrt{T^2 + T^2} = 19.9\,N$

eg 28



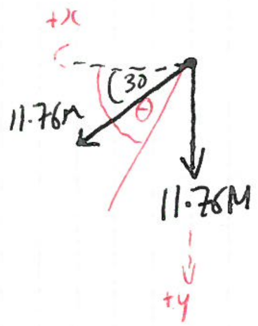
A: $\quad 4Mg - T = 4Ma \quad (1)$

B: $\quad T - Mg\sin 30 = Ma \quad (2)$

(1) + (2) $\quad 4\cancel{M}g - \frac{1}{2}\cancel{M}g = 5\cancel{M}a$

$$a = \frac{3.5g}{5} = 6.86\,ms^{-2}$$

In (2) $\quad T = 6.86M + 4.9M = 11.76M\ N.$



$R_x = 11.76M\cos 30 = 10.18M$

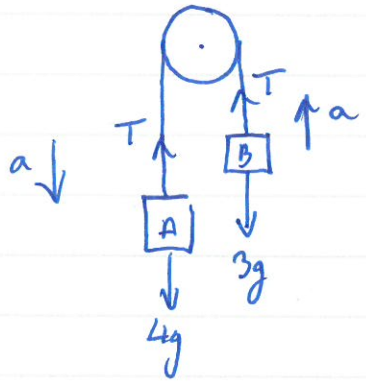
$R_y = 11.76M + 11.76M\sin 30 = 17.64M$

$$R = \sqrt{(10.18M)^2 + (17.64M)^2} = \sqrt{414.802M^2} = 20.37M\ N$$

$$\theta = \tan^{-1}\left(\frac{17.64\cancel{M}}{10.18\cancel{M}}\right) = 60^\circ$$

Ex3F (Pulley Q's)

(4) a)



A: $4g - T = 4a$ — (1)

B: $T - 3g = 3a$ — (2)

(1)+(2) $g = 7a$

$a = \frac{g}{7}$

in (2) $T = \frac{3g}{7} + 3g = 33.6$ N.

(b) speed of A on impact: $u=0$ $a=g/7$ $s=2$ $v=?$

$v^2 = 0^2 + 2 \times 2 \times \frac{g}{7} = \frac{4g}{7}$

$v = \sqrt{\frac{4g}{7}}$

Speed of A on impact = initial speed of B when travelling freely under gravity

$u = \sqrt{\frac{4g}{7}} \uparrow$ $a = g\downarrow = -g\uparrow$ $v=0$ $s=?$

$0 = \sqrt{\frac{4g}{7}}^{\,2} + 2 \times -g \times s$

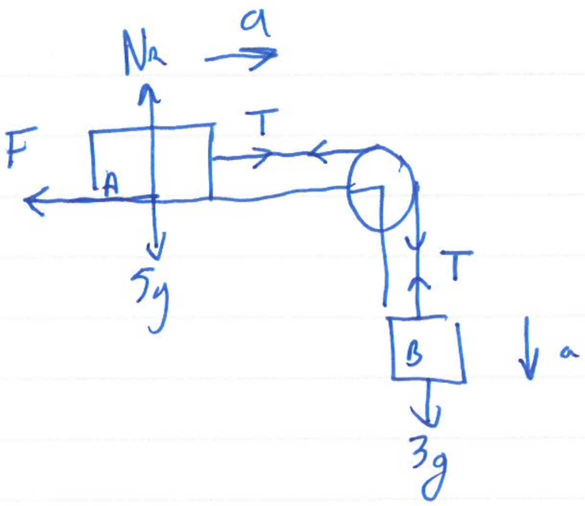
$2gs = \frac{4g}{7}$

$s = 2/7$ m

but B had travelled 2m before string A hit ground ∴

dist travelled $= 2\frac{2}{7}$ m

5



(a) Forces on A: $T - F = 5a$

$F = 0.5 N_R$

$N_R = 5g$

$\therefore \quad T - 2.5g = 5a \quad — ①$

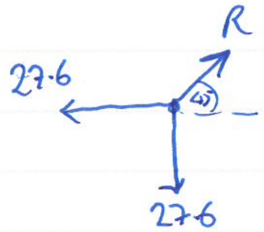
Forces on B: $3g - T = 3a \quad — ②$

① + ② $\quad 0.5g = 8a$

$a = \frac{0.5g}{8} = 0.61 \ ms^{-2}$

(b) ~~In~~ ① $T = 5 \times \frac{0.5g}{8} + 2.5g = 27.6 N$

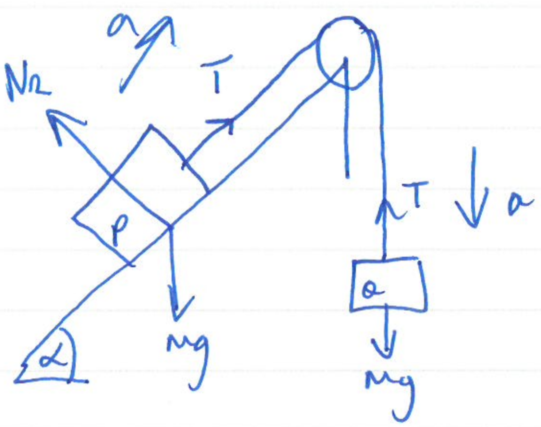
(c)



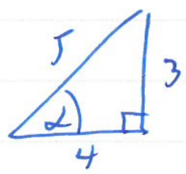
ΣF_y: $R \sin 45 = 27.6$

$R = 27.6\sqrt{2} = 39.0 N$

(6)



$\tan\alpha = \frac{3}{4}$



$\sin\alpha = \frac{3}{5}$

Forces on P: $T - Mg\sin\alpha = Ma$ — (1)

Forces on Q: $Mg - T = Ma$ — (2)

(1) + (2) $\cancel{M}g - \cancel{M}g\sin\alpha = 2\cancel{M}a$

$$\frac{2}{5}g = 2a$$

$\therefore a = g/5 \ \mathrm{ms}^{-2}$

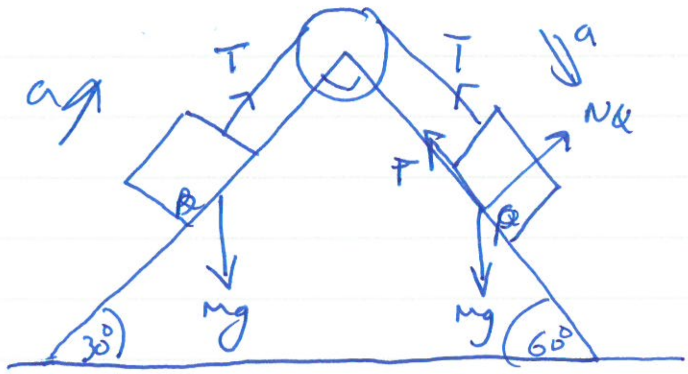
Now P hits pulley after travelling 2m.

$u = 0 \quad v = ? \quad a = g/5 \quad s = 2$

$v^2 = 0^2 + 2 \times \frac{g}{5} \times 2$

$v = \sqrt{\frac{4g}{5}} = 2.8 \ \mathrm{ms}^{-1}$

(7).



Forces on Q: $T - Mg\sin 30 = Ma$ — (1)

Forces on P: $Mg\sin 60 - F - T = Ma$ — (2)

$F = 0.5 N_Q$ — (3)

$N_Q - Mg\cos 60 = 0$ — (4)

From (4) $N_Q = \frac{Mg}{2}$

in (3) $F = 0.5\left(\frac{Mg}{2}\right) = \frac{Mg}{4}$

From (1) $T = Ma + \frac{Mg}{2}$ — (5)

Sub in (2) $\frac{\cancel{M}g\sqrt{3}}{2} - \frac{\cancel{M}g}{4} - \cancel{M}a - \frac{\cancel{M}g}{2} = Ma$

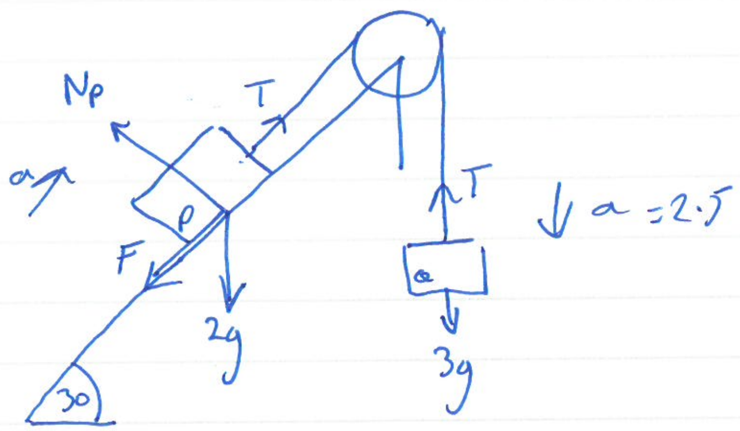
$2\cancel{M}a = \frac{g\sqrt{3}}{2} - \frac{g}{4} - \frac{g}{2}$

$2a = 1.14$

$a = 0.57\ ms^{-2}$

in (5) $T = M\left[0.57 + \frac{g}{2}\right] = 5.5M\ N$

(9)



Forces on P: $T - F - 2g\sin 30 = 2 \times 2.5$

$$T - F = 5 + g$$
$$T - F = 14.8 \quad — (1)$$

$$N_P - 2g\cos 30 = 0$$
$$N_P = g\sqrt{3} \quad — (2)$$

$$F = \mu N_P = \mu g\sqrt{3}$$

in (1) $\quad T - \mu g\sqrt{3} = 14.8 \quad — (3)$

(a) Forces on Q: $3g - T = 3 \times 2.5$

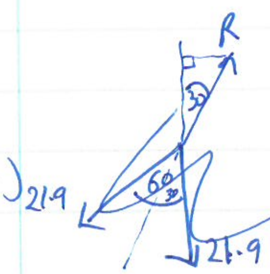
$$T = 3g - 7.5$$
$$T = 21.9\text{ N}$$

(b) in (3) $\quad 21.9 - \mu g\sqrt{3} = 14.8$

$$\mu g\sqrt{3} = 21.9 - 14.8$$

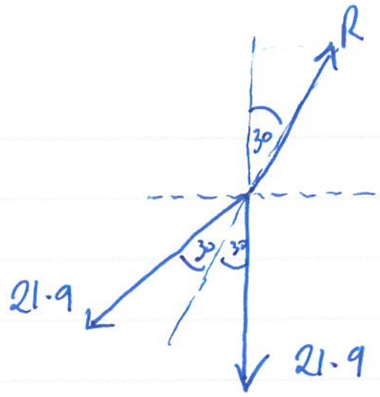
$$\mu = \frac{7.1}{g\sqrt{3}} = 0.418$$

(c)



~~ΣF_y: $R\cos 30 - 21.9 = 21.9$ R...~~

(9)(c)



$\Sigma F_y \quad R\cos 30 - 21.9 - 21.9\cos 60 = 0$

$$R = \frac{21.9 + 21.9\cos 60}{\cos 30} \simeq 38\,N$$