Momentum & Impulse

The momentum of a body of mass m kg and velocity v ms^{-1} is mv. The units of momentum are Newton-seconds (Ns).

Since the momentum of a body depends on the velocity with which it is moving, momentum is a vector quantity, ie its direction of motion must be considered.

Eg1 Find the magnitude of the momentum of
(a) a cricket ball of mass 420g thrown at 20ms^{-1}
(b) a steam-roller of mass 6 tonnes moving at 0.4 ms^{-1}

Changes in momentum

If the velocity of a body changes from u to v, then its momentum also changes. The change in momentum can be found by considering the initial momentum mu and the final momentum mv.

Eg2 Find the change in the momentum of a body of mass 2kg when its speed changes from
(a) 6ms^{-1} to 15ms^{-1} in the same direction
(b) 5ms^{-1} to 3ms^{-1} in the opposite direction.

Impulse

The impulse of a constant force F is defined as $F \times t$, where t is the time for which the force is acting.

Impulse, $I = Ft$

but N2L, $F = ma \rightarrow I = mat$

using $v = u + at \rightarrow a = \frac{v-u}{t}$

$\therefore I = m\frac{(v-u)}{t} \times t \longrightarrow I = mv - mu$ Impulse = change in momentum

Eg3 A body of mass 4kg is initially at rest on a smooth horizontal surface. A horizontal force of 3.5N acts on the body for 8 seconds. Find
(a) the magnitude of the impulse given to the body,
(b) the magnitude of the final momentum of the body,
(c) the final speed of the body.

Eg4 A ball of mass 0.25kg hits a vertical wall with a horizontal speed 30ms^{-1}. It rebounds with a speed of 20ms^{-1}. Find the impulse exerted by the wall on the ball.

Eg5 A ball of mass 0.3kg is released from a point at a height of 10m above horizontal ground. After hitting the ground the ball rebounds to a height of 2.5m. Calculate the magnitude of the impulse of the force exerted on the ball by the ground during the impact.

Exercise 3G Pg 68

Eg1 (a) $M = 0.42 \quad V = 20$

$M = 8.4\ Ns$

(b) $M = 6000 \quad V = 0.4$

$M = 6000 \times 0.4 = 2400\ Ns.$

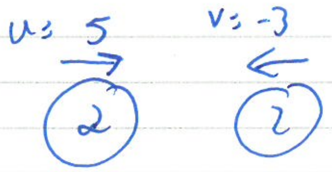
Eg2 (a)

change in mom $= M(V-u)$

$= 2(15-6)$

$= 18\ Ns.$

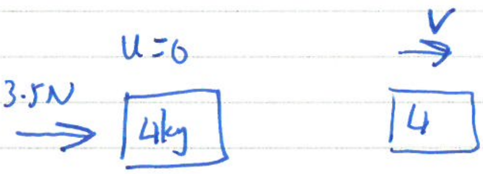
(b)



change in mom $= 2(-3-5)$

$= -16\ Ns$

Eg3



(a) ~~$8 \times 3.5 = 4(V-0)$~~ Impulse $= 8 \times 3.5 = 28\ Ns$

~~$V = 7ms^{-1}$~~

(b) $28\ Ns = mV - mu$

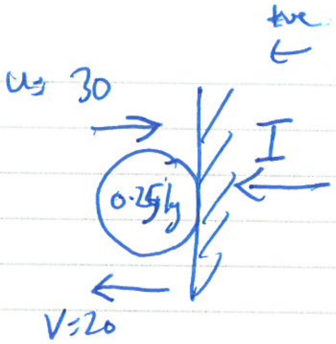
$28 = mV - 0$

$mV = 28\ Ns$

(c) $4V = 28$

$V = 7ms^{-1}$

Eg 4.



$I = m(v - u)$

$= 0.25(20 - (-30))$

$= 12.5 \text{ Ns}$

Impulse exerted by wall on ball = 12.5 Ns

but by N3L Impulse exerted by ball on wall is the same.

Eg 5. Need to find speed of impact $u = 0$ $s = 10\downarrow$ $v = ?\downarrow$ $a = 9.8\downarrow$

$v^2 = 0^2 + 2 \times 9.8 \times 10$

$v^2 = 196$

$v = 14\downarrow$.

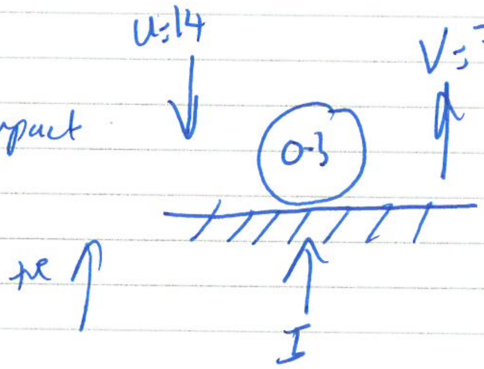
Need to find speed after impact $u = ?\uparrow$ $s = 2.5\uparrow$ $v = 0$ $a = 9.8\downarrow = -9.8\uparrow$

$0^2 = u^2 + 2 \times -9.8 \times 2.5$

$u^2 = 49$

$u = 7\uparrow$

∴ Impact



$I = 0.3(7 - -14)$

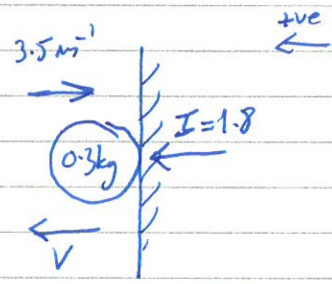
$= 6.3 \text{ Ns}$

Ex3G

(1) $I = m(v - u)$

$15 = 0.5(v - 0)$

$v = 30\ ms^{-1}$

(2) 

$1.8 = 0.3(v - (-3.5))$

$6 = v + 3.5$

$v = 2.5\ ms^{-1}$

(3). Need speed @ impact : $u = 0$ $v = ?\downarrow$ $s = 2.5\downarrow$ $a = 9.8\downarrow$

$v^2 = 0^2 + 2 \times 9.8 \times 2.5$

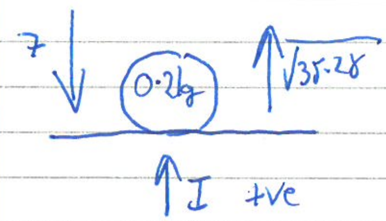
$v^2 = 49$

$v = 7\downarrow$

Need speed @ departure : $u = ?\uparrow$ $v = 0$ $s = 1.8\uparrow$ $a = 9.8\downarrow = -9.8\uparrow$

$0^2 = u^2 + 2 \times -9.8 \times 1.8$

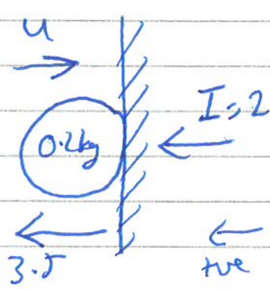
$u^2 = 35.28$ $\quad u = \sqrt{35.28}$



$I = m(v - u)$

$I = 0.2(\sqrt{35.28} - -7)$

$I = 2.59\ Ns.$

(4) 

$2 = 0.2(3.5 - -u)$

$10 = 3.5 + u$

$u = 6.5\ ms^{-1}$

(5) Impulse = $F \times t = 0.4 \times 1.5 = 0.6 Ns$

Now $I = m(v-u)$

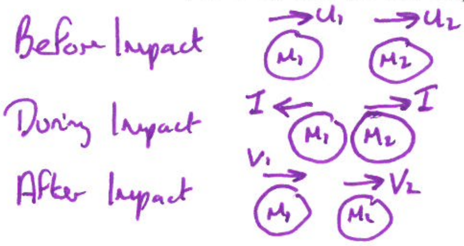
$0.6 = 0.2(v-0)$

$v = 3ms^{-1}$

Collisions and Conservation of Momentum

When two particles collide, by N3L they exert equal and opposite forces, and hence impulses, on each other.

Consider two particles of masses m_1 and m_2 moving in the same direction in a straight line on a smooth horizontal surface with speeds u_1 and u_2 respectively where $u_1 > u_2$. The particles collide. Let their speeds after impact be v_1 and v_2 respectively in the same direction as u_1 and u_2 and let the impulse created by the impact be I.



Impulse: $I = MV - MU$

For M_1: $-I = M_1V_1 - M_1U_1$ — (1)

For M_2: $I = M_2V_2 - M_2U_2$ — (2)

(1) + (2) $0 = M_1V_1 - M_1U_1 + M_2V_2 - M_2U_2$

$M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

or Total Mom b4 Impact = Total Mom After Impact

Principle of Conservation of Momentum

Eg6 A van of mass 2500kg and travelling with a velocity of $10ms^{-1}$ collides head on with a car of mass 1000kg travelling in the opposite direction with a velocity of $20ms^{-1}$. As a result of the collision, the van comes to rest. Find

(i) the final velocity of the car;
(ii) the impulse on each vehicle

If it is assumed that the impact lasts for one twentieth of a second, find

(iii) the force on each vehicle and its acceleration.

Eg7 In an experiment on lorry bumper design, the Transport Research Laboratory arranged for a car and a lorry, of masses 1 and 3.5 tonnes to travel towards each other, both with speed $9ms^{-1}$. After a head-on collision both vehicles move together at approximately $5ms^{-1}$ in the direction that the lorry was originally moving. Show that the total momentum is conserved during the collision.

Eg8 A child of mass 30kg running through a supermarket at $4ms^{-1}$ leaps onto a stationary shopping trolley of mass 15kg. Find the speed of the child and trolley together, assuming that the trolley is free to move easily.

Explosions

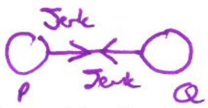
Conservation of momentum also applies when explosions take place provided there are no external forces. For example when a bullet is fired from a rifle, or a rocket is launched.

Eg9 A rifle of mass 8kg is used to fire a bullet of mass 80g at a speed of $200ms^{-1}$. Calculate the initial recoil speed of the rifle.

Jerk in a string

Consider two particles P and Q which are at rest on a smooth horizontal surface and are connected by a light inextensible string which is initially slack.

Suppose Q is given a velocity in the direction PQ. In time, the string will become taut. At the instant when the string becomes taut both particles will experience a jerk from the string.

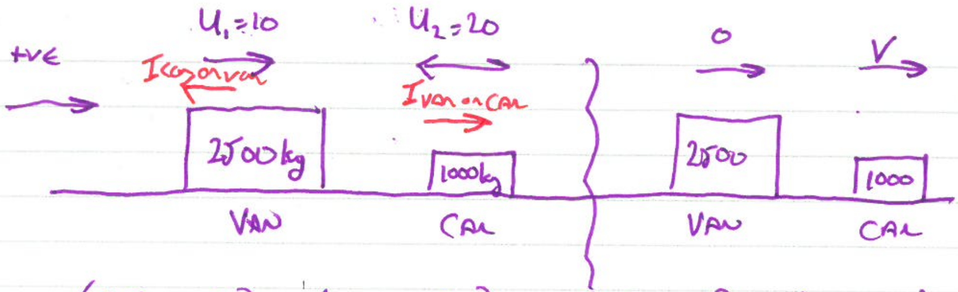


By N3L, the jerk experienced by P will be equal in magnitude but opposite in direction to the jerk experienced by Q. Therefore momentum is conserved.

Eg10 Two particles P and Q of mass 3kg and 6kg respectively are connected by a light inextensible string. Initially they are at rest on a smooth table with the string slack. Q is projected directly away from P with a speed of $3ms^{-1}$. Find their common speed when the string becomes taut.

Exercise 3H Pg 72

eg6.



(i) $(2500 \times 10) + (1000 \times {}^{-}20) = (2500 \times 0) + (1000 \times V)$

$5000 = 1000V$

$V = 5ms^{-1}$

(ii) Impulse = change in Momentum

for Car $\quad I = 1000(V - -20) = 1000(5+20) = 25000Ns$

for Van $\quad -I = 2500(0 - 10) = -25000 \qquad I = 25000\ Ns.$

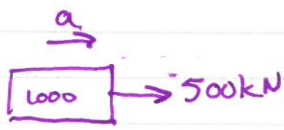
ie impulses are equal + opposite

(iii) $I = F \times t$

$25000 = F \times \frac{1}{20}$

$F = 500kN$ in direction of impulse

N2L on CAR



$500\ 000 = 1000a$

$a = 500ms^{-2}$

N2L on VAN

$-500\ 000 = 2500a$

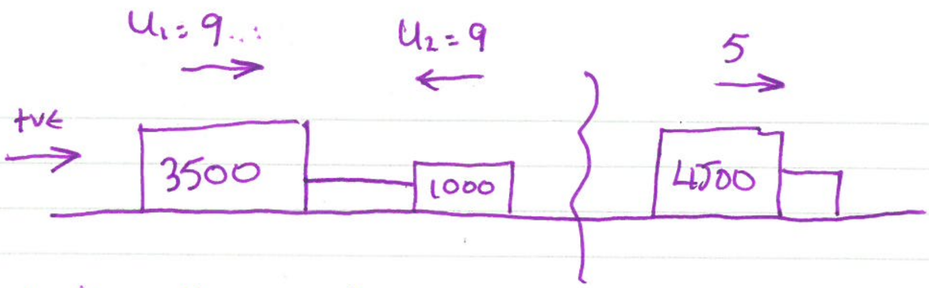
$a = -200ms^{-2}$

$10ms^{-1} \Rightarrow 22mph$

$20ms^{-1} \Rightarrow 45mph$

$5ms^{-1} \Rightarrow 11mph.$

Eg7

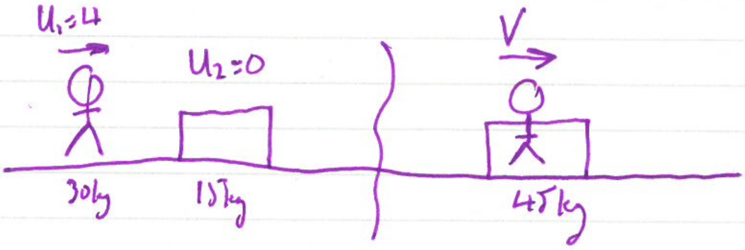


Mom b4 collision = $(3500 \times 9) + (1000 \times {}^{-}9)$ = 22500 Ns

Mom after collision = 4500×5 = ~~[illegible]~~ 22500 Ns.

Mom b4 + collision are equal ∴ Momentum is conserved.

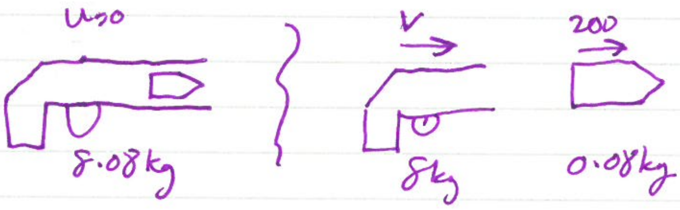
Eg8



$(30 \times 4) + (15 \times 0) = 45V$

$120 = 45V \implies V = 2\frac{2}{3}\ ms^{-1}$

Eg9.



$8.08 \times 0 = 8V + (200 \times 0.08)$

$V = -2ms^{-1}$

∴ recoil speed of $2ms^{-1}$

Eg10

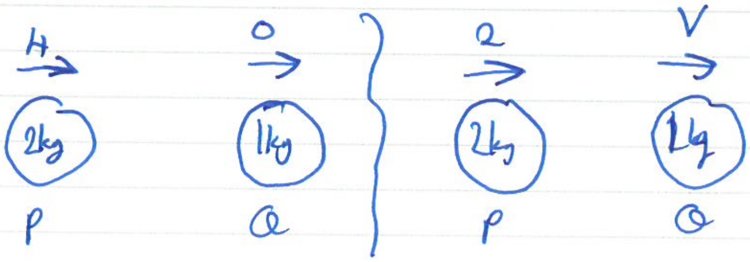
$(3 \times 0) + (6 \times 3) = 3V + 6V$

$18 = 9V$

$V = 2ms^{-1}$.

Ex3H

(1)

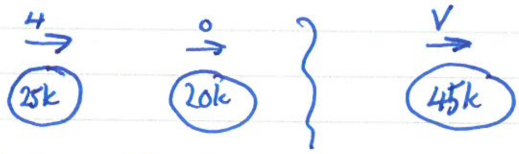


Con Mom $(4\times2)+(1\times0) = (2\times2) + 1\times V$

$8 = 4 + 1V$

$V = 4ms^{-1}$

(2)

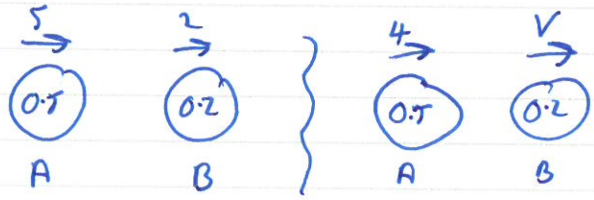


$(25000\times4)+(20000\times0) = 45000V$

$100k = 45kV$

$V = 2\tfrac{2}{9}\ ms^{-1}\ (2.\dot{2})$

(3)



$(5\times0.5)+(2\times0.2) = (4\times0.5) + 0.2V$

$2.9 = 2 + 0.2V$

$V = 4.5ms^{-1}$

(4)

(a) $(2\times4)+(1\times0) = 3V$

$V = \frac{8}{3}\ ms^{-1}$

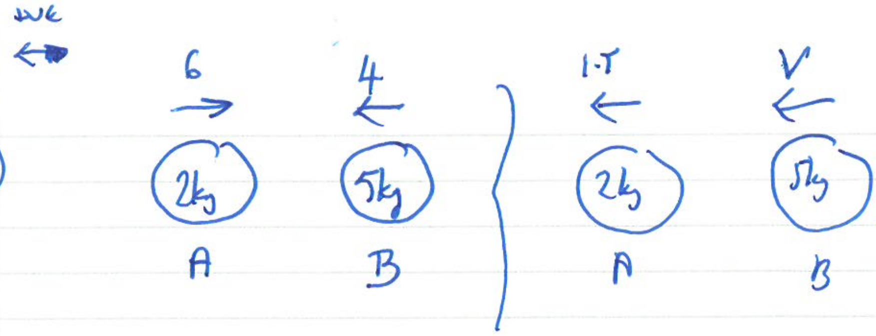
(b) Impulse = change in mom on one particle

$= 2(\tfrac{8}{3}-4)$

$= -2\tfrac{2}{3}\ Ns$

Magnitude $2\tfrac{2}{3}\ Ns$

(5)



(a) $(-6\times2)+(+4\times5)=(1.5\times2)+5V$

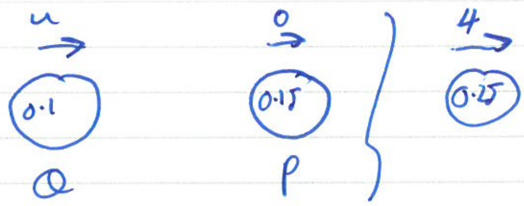
$-12+20=3+5V$

$V=1\,ms^{-1}$ same direction.

(b) Impulse = change in mom of one particle

A: $I=2(1.5--6)=15\,Ns.$

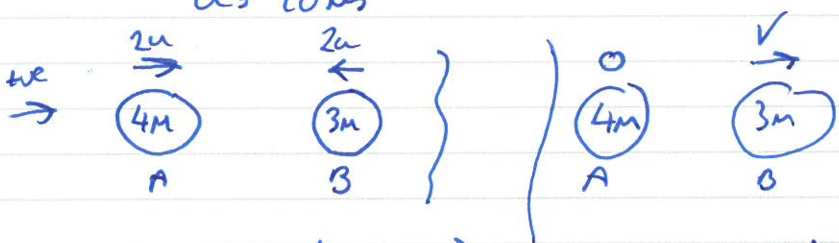
(6)



$0.1u=0.25\times4$

$u=10\,ms^{-1}$

(7)



(a) $(4m\times2u)+(3m\times{}^{-}2u)=(4m\times0)+3mV$

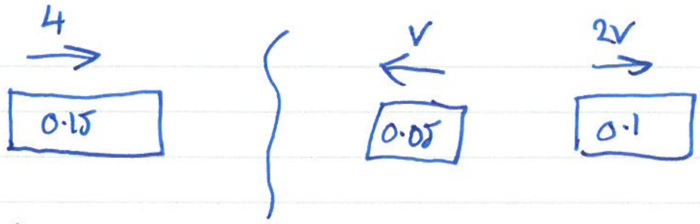
$8mu-6mu=3mV$

$2u=3V$

$V=\frac{2}{3}u$, direction reversed

(b) Impulse $=4m(0-2u)=-8mu$ Magnitude $=8mu$ Ns.

(8)



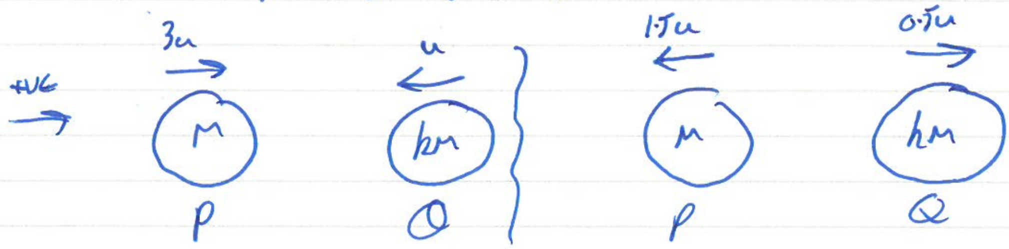
$(4 \times 0.15) = (2V \times 0.1) + (-V \times 0.05)$

$0.6 = 0.2V - 0.05V$

$0.6 = 0.15V$

$V = 4\ ms^{-1}$ & $8\ ms^{-1}$

(9)



(a) $3uM + -ukM = -1.5uM + 0.5ukM$

$3 - k = -1.5 + 0.5k$

$4.5 = 1.5k$

$k = 3$

(b) Impulse $= M(-1.5u - 3u)$

$= -4.5uM$ Mag $= 4.5uM$

(10)

$4u = 6V$ — (1)

(a) Impulse $4(V - u) = -6$ — (2)

$4V - 4u = -6$

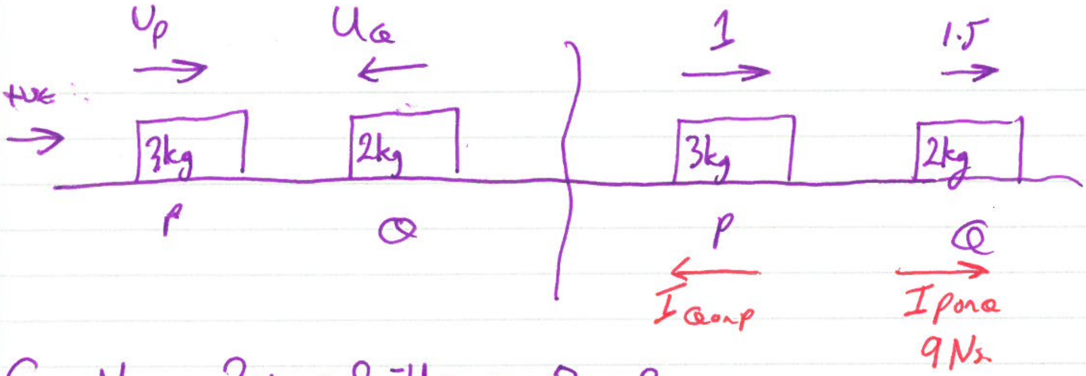
but $4u = 6V$ $\quad 4V - 6V = -6$

$-2V = -6$

$V = +3\ ms^{-1}$

(b) $u = \frac{6}{4}V = \frac{3}{2}V = \frac{3}{2}.3 = 4.5\ ms^{-1}$

(11)



Cons Mom $3u_P + 2\times{-u_Q} = 3 + 3$

$3u_P - 2u_Q = 6$ — (1)

Impulse on Q $9 = 2(1.5 - -u_Q)$

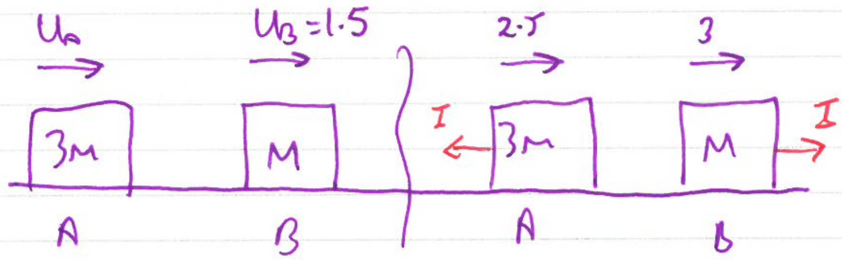
$4.5 = 1.5 + u_Q$

$u_Q = 3\,ms^{-1}$ in opposite direction to after collision

In (1) $3u_P - 6 = 6$

$u_P = 4\,ms^{-1}$ in same direction to after collision.

(12)



(a) Cons Mom $3m u_A + 1.5m = (3m \times 2.5) + 3m$

$3u_A + 1.5 = 10.5$

$3u_A = 9$

$u_A = 3\,ms^{-1}$

(b) Impulse on A $-3 = 3m(2.5 - 3)$

$-1 = -0.5m$

$m = 2$

∴ Mass of A = 3m = 6kg.