Moments

The Moment of a Force

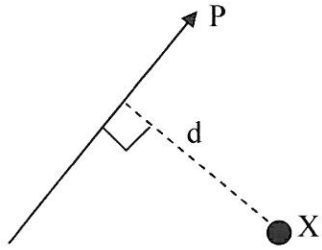
From our everyday experience we know that:

- it is easier to undo a tight nut using a long spanner when the force is applied to the end of the spanner, rather than by using a short spanner.
- If a boy sits at one end of a see-saw which is pivoted at its centre, he can be balanced by a heavier boy sitting nearer to the centre of the see-saw.
- A door is more easily closed by pushing on the edge further from the hinges, rather than by pushing at a point part way across the door.

In each of these examples, the application of the force is causing a body to rotate about an axis, ie rotational motion. Previously, only motion along a line has been considered, ie translational motion.

Definition

The moment of a force about a point is found by multiplying the magnitude of the force by the perpendicular distance from the point to the line of action of the force.



The moment of the force P about the point X is $P \times d$

A force will have no moment about a point on its line of action as d = 0.

If the force is measured in Newtons and the distance in metres, the moment of the force is measured in Newton metres (Nm).

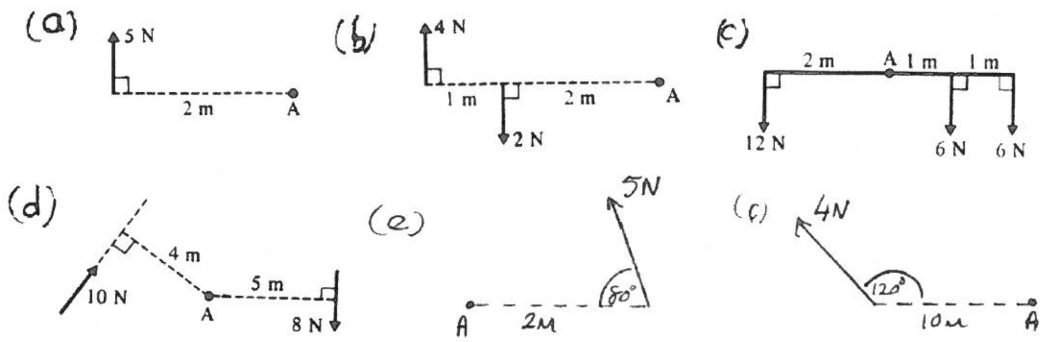
Sense of Rotation

A nut is usually rotated in an anticlockwise direction when being undone. All rotations should have their sense clearly stated (eg, +ve clockwise, -ve anticlockwise): the moment of a force about a point has both magnitude and direction.

Algebraic Sum of Moments

If a number of coplanar forces act on a body, their moments about any point may be added provided due regard is given to the sense of each moment.

Eg1 For each of the situations below, find the total moment about the point A.



Exercise 5B Pg 121Q1 consonants, Q2 all

Parallel Forces in Equilibrium

For parallel forces to be in equilibrium, two conditions must hold true:

1. the component of the resultant force in any direction must be zero,
2. the algebraic sum of the moments about any point must be zero, ie the sum of the anti-clockwise moments must equal the sum of the clockwise moments.

Eg2 A uniform beam, of length 2m and mass 4kg, has a mass of 3kg attached at one end and a mass of 1kg attached at the other end. Find the position of the support if the beam rests in a horizontal position.

Eg3 A light horizontal beam of length 2m rests with ends A and B on smooth supports. The beam carries masses of 5kg and 2kg at distances of 60cm and 150cm respectively from A. Find the reaction at each support.

Exercise 5C Pg 125 Q's 3 to 11 odds

Non-Uniform Rods (rod – assumes no thickness and no bending)

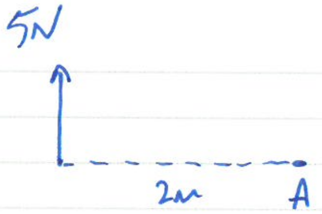
The centre of mass of a non-uniform rod is located at some point other than the midpoint of the rod.

Eg4 Two children are sitting on a non-uniform plank AB of mass 24kg and length 2m. This plank is pivoted at M, the midpoint of AB. The centre of mass of AB is at C, where AC is 0.8m. Anne has mass 24kg and sits at A. John has mass 30kg. Find where John must sit for the plank to be horizontal.

Eg5 A non-uniform rod AB of length 4m and mass 5kg is in equilibrium in a horizontal position resting on two supports at points C and D where AC = 1m and AD = 2m. The magnitude of the reaction at C is half the magnitude of the reaction at D. Find the distance of the centre of mass of the rod from A.

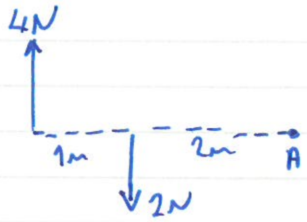
Exercise 5D Pg 128 All

Eg1 (a)



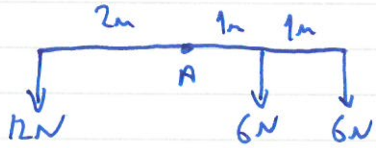
$\curvearrowright_A : \quad 5 \times 2 = 10 \text{ Nm}$

(b)



$\curvearrowright_A : \quad (4 \times 3) + (2 \times {}^{-}2) = 12 - 4 = 8 \text{ Nm}$

(c)

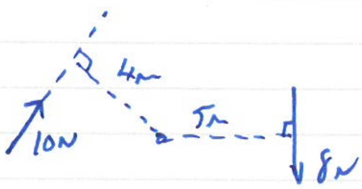


$\curvearrowright_A : \quad (2 \times {}^{-}12) + (1 \times 6) + (2 \times 6)$

$= -24 + 6 + 12$

$= -6 \text{ Nm}$

(d)

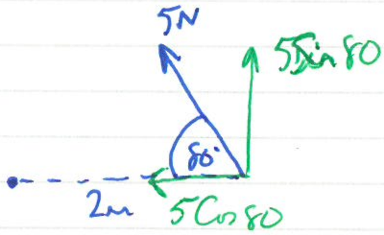


$\curvearrowright_A : \quad (4 \times 10) + (5 \times 8)$

$= 40 + 40$

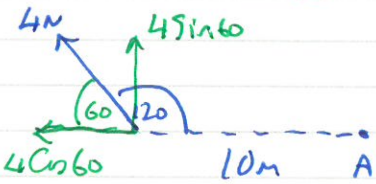
$= 80 \text{ Nm}$

(e)



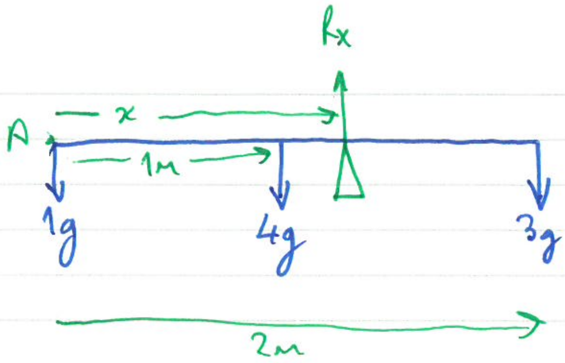
$\curvearrowright_A : \quad (2 \times {}^{-}5\sin 80) + (0 \times 5\cos 80) = -9.8 \text{ Nm}$

(f)



$\curvearrowright_A : \quad (10 \times 4\sin 60) + (0 \times 4\cos 60) = 20\sqrt{3} \text{ Nm}$

Eg2



Beam in equilibrium $\therefore \sum F_y = 0$ $\quad R_x - 1g - 4g - 3g = 0$

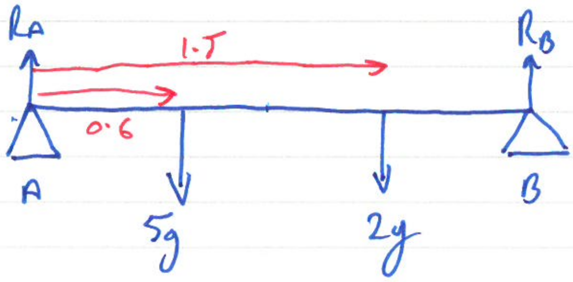
$R_x = 8g$ N

$\circlearrowright_A$: $(1\times 4g) + (R_x \times -x) + (2\times 3g) = 0$

$8g\,x = 10g$

$x = \frac{10}{8} = 1.25$m.

Eg3



$\sum F_y$: $R_A + R_B = 7g$ — (1)

$\circlearrowright_A$: $(0.6\times 5g) + (1.5\times 2g) + (2\times -R_B) = 0$

$2R_B = 6g$

$R_B = 3g$

in (1) $R_A = 7g - 3g = 4g$ N.