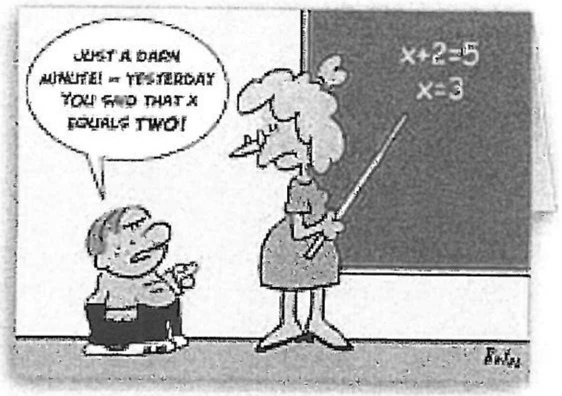
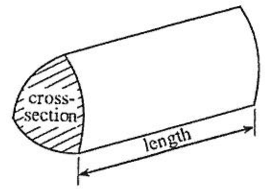
Algebra Booklet
Non Calculator
Higher



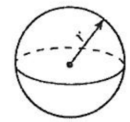
Formula List

Volume of prism = area of cross-section × length



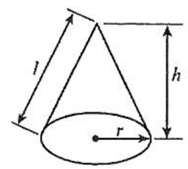
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

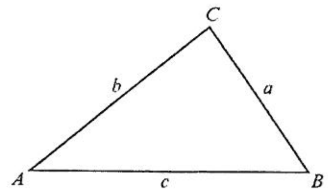


In any triangle *ABC*

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

6. *(a)* Solve **each** of the following equations.

(i) $7x + 4 = 3x + 16$

$7x - 3x = 16 - 4$

$4x = 12$

$x = 3$

(ii) $3x + 2 = 2(3 - 2x)$

$3x + 2 = 6 - 4x$

$3x + 4x = 6 - 2$

$7x = 4$

$x = \frac{4}{7}$

[6]

(b) Simplify **each** of the following.

(i) $2(3r + 1) + 5r$

$6r + 2 + 5r$

$11r + 2$

(ii) $3(2p + 3) - 2(p - 1)$

$6p + 9 - 2p + 2$ $-2x-1=2$

$4p + 11$

[4]

13. *(a)* Simplify $4p^3r^6 \times 3pr^2$.

$12p^4r^8$ | $4 \times 3 = 12$

$p^3 \times p = p^4$

$r^6 \times r^2 = r^8$

[2]

(b) Factorise $6a^2b + 9a$.

$3a(2ab + 3)$

[2]

4. Solve **each** of the following equations.

(a) $6x - 11 = 17 + 2x$

$6x - 2x = 17 + 11$

$4x = 28$

$x = 7$

[3]

(b) $3(x - 7) = 27$

$3x - 21 = 27$

$3x = 27 + 21$

$3x = 48$

$x = 16$

[3]

(c) $\frac{2x}{3} = 6$

$2x = 6 \times 3$

$2x = 18$

$x = 9$

[2]

14. *(a)* Simplify $5c^6d^4 \times 4c^3d$.

$20c^9d^5$

$5 \times 4 = 20 \quad c^6 \times c^3 = c^9$

$d^4 \times d = d^5$

[2]

(b) Factorise $6ab - 2a^2$.

$2a(3b - a)$

[2]

7. *(a)* Simplify $5(2x + 5) - 2(3x - 4)$.

$-2x - 4 = +8$

$\boxed{10x} + 25 \boxed{-6x} + 8$

$\underline{4x + 33}$

[2]

(b) Expand $2y(y^2 + 3)$.

$2y^3 + 6y$

[2]

(c) Solve $\frac{240}{x} = 30$.

$240 = 30 \times x$

$240 = 30x$

$x = \frac{240}{30} = 8$

[1]

9. *(a)* Factorise $x^2 - 2x - 8$.

$(x - 4)(x + 2)$

$\times -8 \quad + -2$

$-4 \times 2 = -8$

$-4 + 2 = -2$

So -4 & 2.

Check

$x^2 - 4x + 2x - 8$

$x^2 - 2x - 8$ ✓

[2]

(b) Expand and simplify $(2x + 1)(x - 5)$.

FOIL

$2x^2 + x - 10x - 5$

$\underline{2x^2 - 9x - 5}$

[2]

(c) Solve $\frac{21 - 2x}{5} = 4 - x$.

$21 - 2x = 5(4 - x)$

$21 - 2x = 20 - 5x$

$-2x + 5x = 20 - 21$

$3x = -1$

$\underline{x = -1/3}$

[3]

(b) Solve the following equation.

$$\frac{x+3}{6} + \frac{2x-5}{3} = \frac{2}{9}$$

Kiss & a smile

$$\frac{3(x+3)+6(2x-5)}{18} = \frac{2}{9}$$

$$\frac{3x+9+12x-30}{18} = \frac{2}{9}$$

$$15x-21 = \frac{2}{9}\times 18$$

$$15x-21 = 4$$

$$15x = 25$$

$$x = 25/15 = 5/3$$

[4]

24. Express $\frac{n}{n-3} - \frac{n}{n+2}$ as a single fraction in its simplest

$$\frac{n(n+2)-n(n-3)}{(n-3)(n+2)}$$

don't alter the bottom

$$\frac{n^2+2n-n^2+3n}{(n-3)(n+2)} = \frac{5n}{(n-3)(n+2)}$$

[3]

19. (a) Express the following as a single fraction in its simplest form.

$$\frac{8}{3-x} - \frac{2}{x-5}$$

$$= \frac{8(x-5)-2(3-x)}{(3-x)(x-5)}$$

$$= \frac{8x-40-6+2x}{(3-x)(x-5)}$$

$$= \frac{10x-46}{(3-x)(x-5)}$$

[3]

(b) Hence, or otherwise, solve the following equation.

$$\frac{8}{3-x} - \frac{2}{x-5} = 0$$

$$\frac{8(x-5)-2(3-x)}{(3-x)(x-5)} = 0$$

$$\frac{10x-46}{(3-x)(x-5)} = 0$$ (Fraction is 0 when top is 0)

$$10x-46 = 0$$

$$10x = 46 \quad x = 4.6$$

[3]

5. The table shows some of the values of $y = x^3 - 8$ for values of x from -2 to 4.

(*a*) Complete the table by finding the values of y for $x = -1$ and $x = 3$.

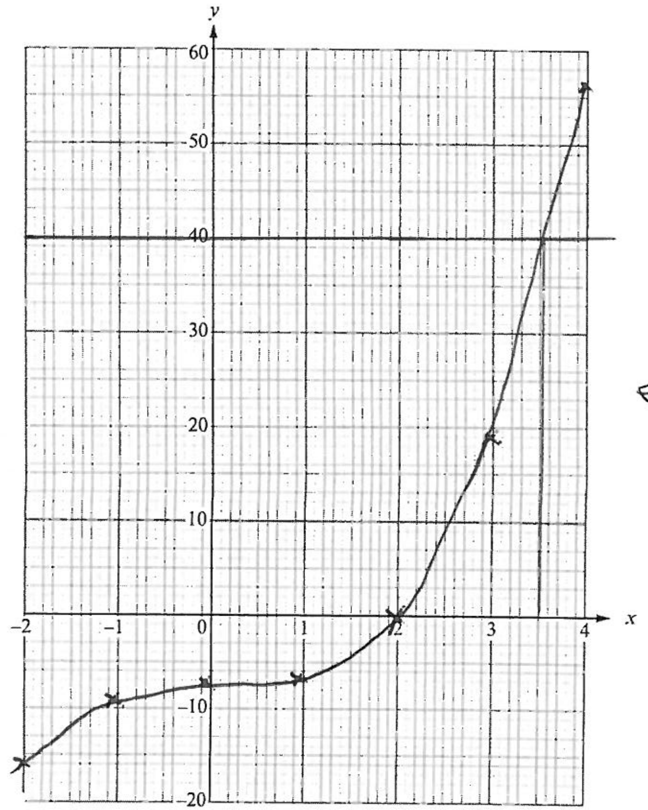
x	-2	-1	0	1	2	3	4
$y = x^3 - 8$	-16	-9	-8	-7	0	19	56

$(-1)^3 - 8 = -1 - 8 = -9$

$(3)^3 - 8 = 27 - 8 = 19$

[2]

(*b*) On the graph paper below, draw the graph of $y = x^3 - 8$ for values of x from -2 to 4. [2]



Shape fits with a cubic graph

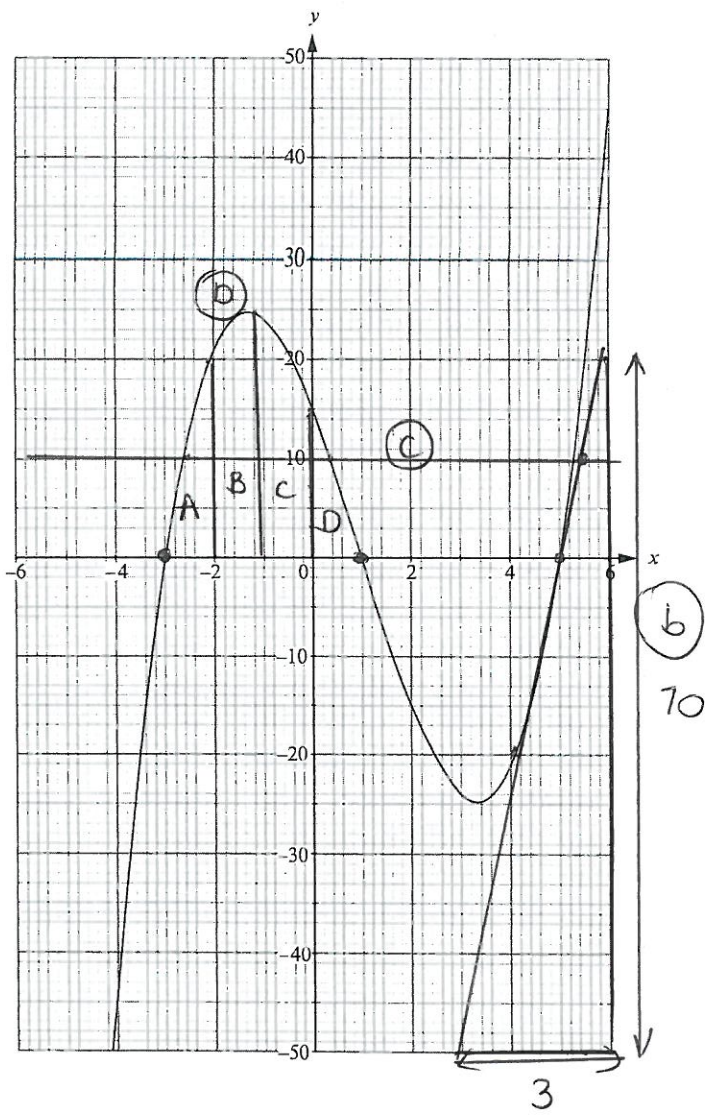
(*c*) Use your graph to solve the equation $x^3 - 8 = 40$.

Draw $y = 40$.

$x = 3.5$

[2]

14. The graph of $y = x^3 - 3x^2 - 13x + 15$, for values of x between $x = -4$ and $x = 6$, has been drawn below.



(a) Use the graph to solve $x^3 - 3x^2 - 13x + 15 = 0$.

Where graph is 0.

$x = -3, 1$ or 5

[1]

(b) Using the graph, estimate the gradient of the curve $y = x^3 - 3x^2 - 13x + 15$ when $x = 4$.

Tangent at $x = 4$.

Gradient $= 70/3 = 23\frac{1}{3}$.

[3]

(c) By drawing an appropriate line on the graph, solve the equation

$$x^3 - 3x^2 - 13x + 5 = 0.$$

Got $y = x^3 - 3x^2 - 13x + 15 = 0$ $-2.3, 0.3$ & 5.7

Want $y = x^3 - 3x^2 - 13x + 5 = 0$

Draw $y = 10$.

[3]

(d) Use the trapezium rule with 4 strips to estimate the area of the region enclosed by the curve $y = x^3 - 3x^2 - 13x + 15$ and the x-axis between $x = -3$ and $x = 1$.

$A = \frac{1 \times 20}{2} = 10$

$B = \frac{20+24}{2} \times 1 = 22$

$C = \frac{24+14}{2} \times 1 = 19$.

$D = \frac{1 \times 14}{2} = 7$.

Total Area $= 10 + 22 + 19 + 7$

$= 58$

[4]

15. *(a)* Factorise the expression $12x^2 + 11x - 15$ and hence solve the equation $12x^2 + 11x - 15 = 0$.

$12x^2 + 20x - 9x - 15$

$4x(3x+5) - 3(3x+5)$

$(4x-3)(3x+5) = 0$

$4x = 3$

$x = 3/4$

or $3x = -5$

$x = -5/3$

X $12x - 15 = -180$

$+$ $+11$

$\boxed{20 \ \& \ -9}$

Harder, double brackets so split middle term

[3]

(b) Factorise the expression $49y^2 - 100$.

$(7y+10)(7y-10)$ Diff of Squares

[2]

(c) Evaluate $16^{-\frac{1}{2}}$.

$\frac{1}{\sqrt{16}} = \frac{1}{4}$

[1]

18. *(a)* Factorise $21x^2 + 4x - 1$. Hence solve $21x^2 + 4x - 1 = 0$.

$21x^2 + 7x - 3x - 1 = 0$

$7x(3x+1) - 1(3x+1) = 0$

$(7x-1)(3x+1) = 0$

$7x - 1 = 0$ or $3x + 1 = 0$

$7x = 1$ $x = 1/7$ $3x = -1$ $x = -1/3$

$\boxed{\begin{matrix} X \ -21 \\ +4 \\ 7 \ \& \ -3 \end{matrix}}$

[3]

(b) (i) Factorise $49x^2 - 64$.

$(7x-8)(7x+8)$ Diff of Squares

[2]

(ii) Hence simplify $\frac{49x^2 - 64}{7x - 8}$.

$= \frac{(7x+8)\cancel{(7x-8)}}{\cancel{(7x-8)}}$ Cancels

$= 7x + 8$

[1]

17. *(a)* Factorise the expression $14w^2 + 23w + 3$ and hence solve the equation $14w^2 + 23w + 3 = 0$.

$14w^2 + 21w + 2w + 3$

$7w(2w+3) + 1(2w+3)$

$(7w+1)(2w+3)$

$\times 42$
$+23$

21 & 2

$(7w+1) = 0$ or $2w+3=0$

$7w = -1$ $\quad$ $2w = -3$

$w = -1/7$ or $w = -3/2$

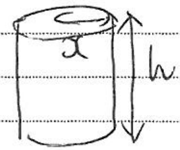
[3]

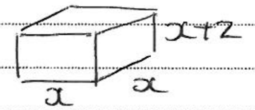
(b) Factorise the expression $9e^2 - 49$.

$(3e-7)(3e+7)$

[2]

22. A cylinder, with base of radius x cm and height h cm, has the same volume as a cuboid with length x cm, width x cm and height $(x + 2)$ cm.
Find an expression for h in terms of x and π, simplifying your answer.





Volume $= x^2(x+2)$

$= x^3 + 2x^2$

Volume $= \pi x^2 h$.

$\pi x^2 h = x^3 + 2x^2$

$x^2(\pi h) = x^2(x+2)$

$\pi h = x + 2$

$h = \dfrac{x+2}{\pi}$

[5]

Simultaneous equations in disguise

11. Sara calculates that five times her age and three times her brother's age gives a total of 100.
The sum of Sara's age and her brother's age is 22.
Find Sara's age and her brother's age.

$5a + 3b = 100$ ①

$a + b = 22$ ②

$5a + 5b = 110$ ② $\times 5$

$-\ 5a + 3b = 100$ ①

$2b = 10$

$b = 5$

Sub into ② $a + 5 = 22$

$a = 17$

[4]

12. There is a positive value of x which satisfies $x^2 = 6{\cdot}5$.
Find this value of x correct to the nearest whole number.
You must justify your answer.

$x^2 = 6.5$

$3^2 = 9$ $\quad$ 2.5 too big

$2^2 = 4$ $\quad$ 2.5 too small.

$2.5^2 = 6.25$ too big.

Between 2.5 & 3

So 3 (whole value)

$$\begin{array}{r} 2.5 \\ \times\ 2.5 \\ \hline 125 \\ 600 \\ \hline 6.25 \end{array}$$

[4]

13. On the graph paper below, draw the region which satisfies **all** of the following inequalities.

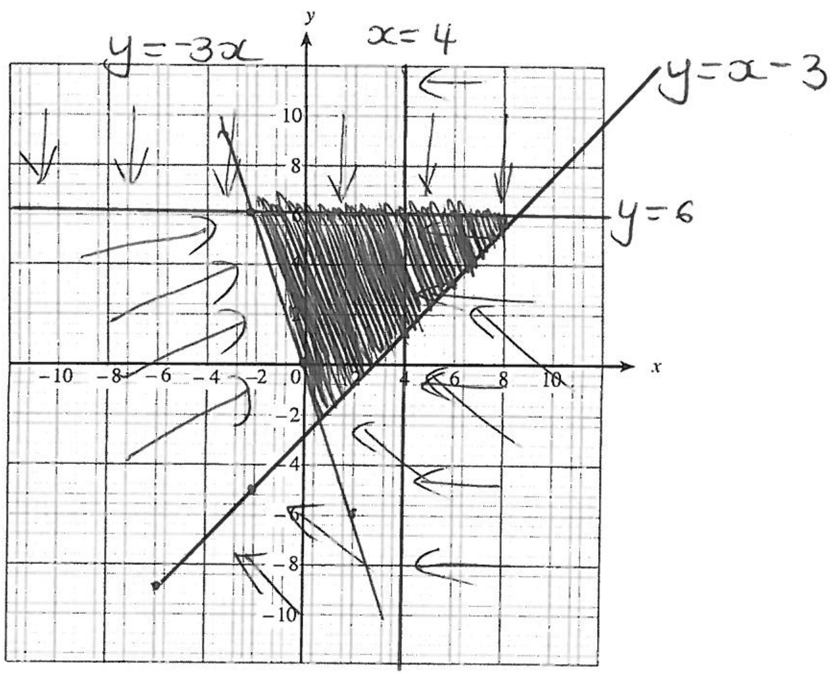
$$y \leqslant 6$$
$$y \geqslant x - 3$$
$$x \leqslant 4$$
$$y \geqslant -3x$$

Make sure that you clearly indicate the region that represents your answer.

$y = x - 3$ (-6, -9) (-2, -5) (6, 3)

$y = -3x$ (-2, +6) (0, 0) (2, -6)

[4]



14. On the graph paper below, draw the region which satisfies **all** of the following inequalities.

$$y \leqslant \frac{x}{2} + 4$$
$$x + y + 6 \geqslant 0$$
$$y \leqslant 4$$
$$x \leqslant 3$$

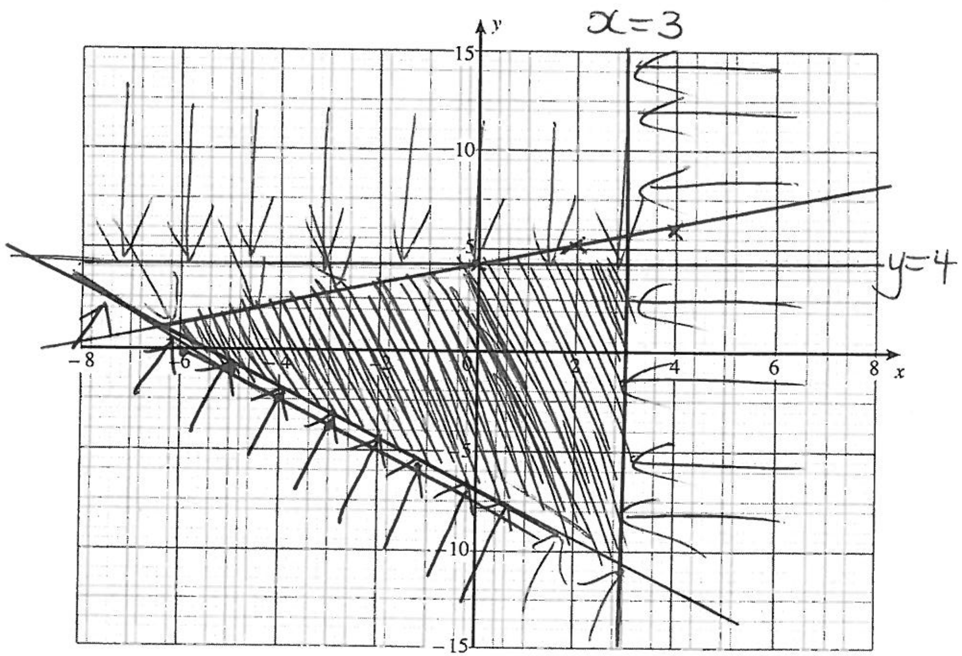
Make sure that you clearly indicate the region that represents your answer.

$x + y + 6 = 0$ (-3, -3) (-4, -2) (-5, -1)

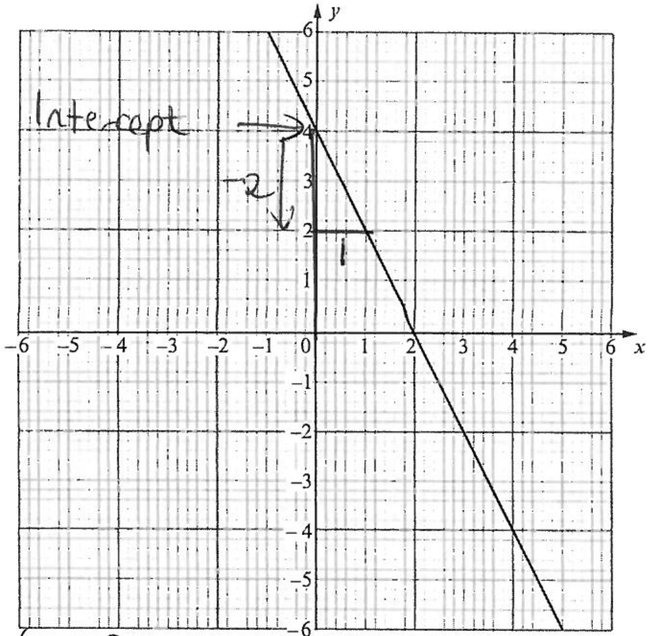
$y = \frac{x}{2} + 4$

(4, 6)
(2, 5)
(0, 4)

[4]



7. *(a)* Find the equation of the straight line shown in the following diagram.
Write your answer in the form $y = mx + c$.



(0, 4) so add 4.
Gradient $\frac{-2}{1} = -2$

Equation of the straight line is $y =$ $-2x+4$

[3]

(b) Write down the equation of a straight line that is parallel to $y = 5x$.

$y = 5x + 3$

[1]

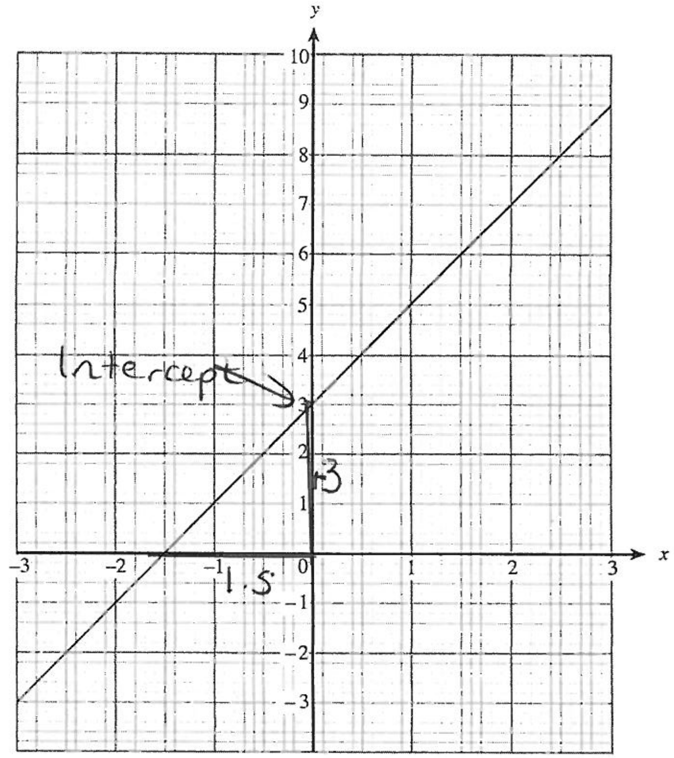
(c) Find the mid-point of the straight line that joins the points with coordinates (2, –7) and (6, 13).

$x = \frac{2+6}{2} = 4$ $y = \frac{7+13}{2}$ 3

Midpt (4, 3)

[3]

11. The diagram shows a straight line graph.



(a) Find the gradient of the straight line.

$\frac{3}{1.5} = 2$

[2]

(b) Write down the equation of the straight line in the form $y = mx + c$.

$y = 2x + 3$

[1]

10. Write down, in terms of n, the nth term of **each** of the following sequences.

(a) 3 7 11 15 19 ....

4 4 4 4

$4n-1$

[2]

(b) 1×3 2×4 3×5 4×6 ....

$n \times (n+2)$

[2]

7. *(a)* Write down the nth term of the sequence 1, 4, 9, 16, 25, ...

3 5 7 9

n^2

not common diff
so try n^2

[1]

6. *(a)* Write down the nth term of the sequence 6, 10, 14, 18, 22, ...

4 4 4 4

$4n+2$

[2]

12. *(a)* Write down the nth term for the sequence 5, 14, 23, 32, 41, ...

9 9 9 9

$9n-4$

[2]

3. *(a)* The diagram shows a number machine.

INPUT → Multiply by 8 → Subtract 5 → OUTPUT

(i) Write down the **OUTPUT** when the **INPUT** is x.

$8x-5$

(ii) Find the **INPUT** when the **OUTPUT** is y.

$\frac{y+5}{8}$

[3]

(b) The nth term of a sequence is $n^2 - 6$. Find the first three terms of the sequence.

$1^2-6 = -5$

$2^2-6 = -2$

$3^2-6 = 3$

$-5, -2, 3$

[2]

19. Make h the subject of the formula

$$10(h - 2e) = 7(h - k).$$

$10h - 20e = 7h - 7k$

$10h - 7h = -7k + 20e$

$3h = -7k + 20e$

$h = \dfrac{-7k + 20e}{3}$ or $\dfrac{20e - 7k}{3}$

[3]

21. Make e the subject of the following formula.

$$10b + 5be = 3e + 7c$$

$10b + 5be = 3e + 7c$

$5be - 3e = 7c - 10b$ ← e's together

$e(5b - 3) = 7c - 10b$ ← Factorise

$e = \dfrac{7c - 10b}{5b - 3}$

[3]

8. *(a)* Solve the following simultaneous equations by an algebraic method.
Show all your working.

$$3x + 4y = 19 \quad (1)$$
$$4x + 5y = 23 \quad (2)$$

$12x + 16y = 76 \quad (1) \times 4$

$12x + 15y = 69 \quad (2) \times 3$

$y = 7$

Sub into (1)

(4×7)

$3x + 28 = 19$

$3x = 19 - 28$

$3x = -9$

$x = -3$

[4]

12. Solve the following simultaneous equations by an algebraic (not graphical) method.

$$2x + 5y = 4 \quad (1)$$
$$3x + 4y = 13 \quad (2)$$

$6x + 15y = 12$ (1) ×3
$- \; 6x + 8y = 26$ (2) ×2
$7y = -14$
$y = -2$

Sub into (1)
$2x - 10 = 4$
$2x = 14$
$x = 7$

[4]

Simultaneous in disguise

11. Three geese and two ducks weigh 32 kg.
Four geese and three ducks weigh 44 kg.
All the geese weigh the same.
All the ducks weigh the same.
What is the total weight of two geese and one duck?

$3g + 2d = 32$ (1)
$4g + 3d = 44$ (2)
$12g + 8d = 128$ (1) × 4
$- \; 12g + 9d = 132$ (2) × 3
$d = 4$

Sub in (1) $3g + 8 = 32$ 2x4
$3g = 24$
$g = 8$

$2g + 1d = 16 + 4 = 20$ kg

[6]

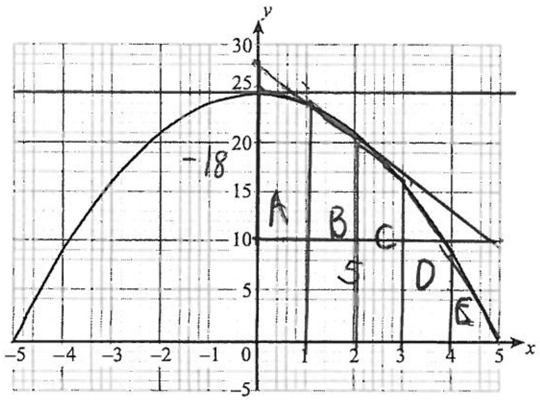
7. *(a)* Three bananas and one apple cost a total of 65p.
Seven bananas and two apples cost a total of £1.49.
How much does one apple cost?

$3b + a = 65$ (1)
$7b + 2a = 149$ (2)
$- \; 6b + 2a = 130$ (1) ×2
$b = 19$ p

Sub in (1) $3b + a = 65$
$57 + a = 65$
$a = 8$ p

[4]

14. The graph of $y = 25 - x^2$ has been drawn below.



(a) Write down the gradient of the curve $y = 25 - x^2$ at $x = 0$.

0.

[1]

(b) Find an estimate for the gradient of the curve $y = 25 - x^2$ at $x = 2$.

$\frac{-18}{5} = -3.6$ $\quad$ -10×2

[3]

(c) Use the trapezium rule, with the ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$, $x = 4$ and $x = 5$, to estimate the area of the region bounded by the curve, the positive x-axis and the positive y-axis.

$A = \frac{25+24}{2} \times 1 = 24.5$

$B = \frac{24+20}{2} \times 1 = 22$

$C = \frac{20+16}{2} \times 1 = 18$

$D = \frac{8+16}{2} \times 1 = 12$

$E = \frac{8 \times 1}{2} = 4$

$\text{Total} = 4+12+18+22+24.5$

$= 80.5$

[4]

9. The table shows some of the values of $y = 4x^3 + 10$ for values of x from -3 to 3.

(a) Complete the table by finding the values of y for $x = 1$ and $x = -2$.

x	-3	-2	-1	0	1	2	3
$y = 4x^3 + 10$	-98	-22	6	10	14	42	118

$4(-2)^3 + 10$

$4 \times -8 + 10 = -22$

$4(1)^3 + 10$

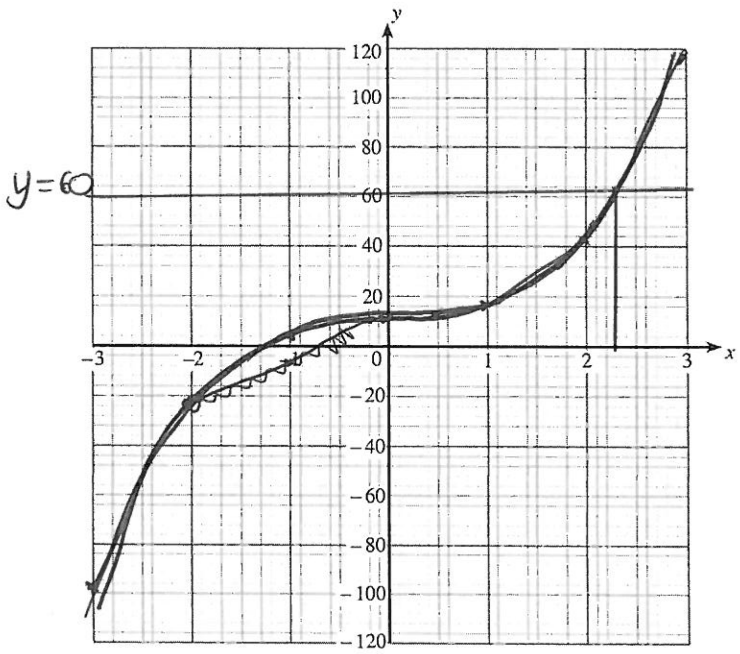
$4 \times 1 + 10$

14

[2]

(b) On the graph paper below, draw the graph of $y = 4x^3 + 10$ for values of x from -3 to 3.

[2]

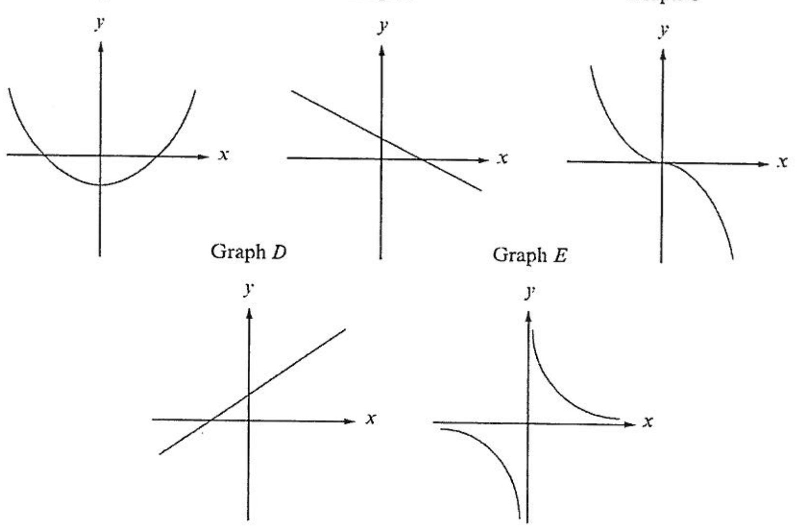


(c) Use your graph of $y = 4x^3 + 10$ to find the value of x for which $4x^3 + 10 = 60$.

$x = 2.3$

[2]

12. The sketches of graphs A, B, C, D and E are shown below.



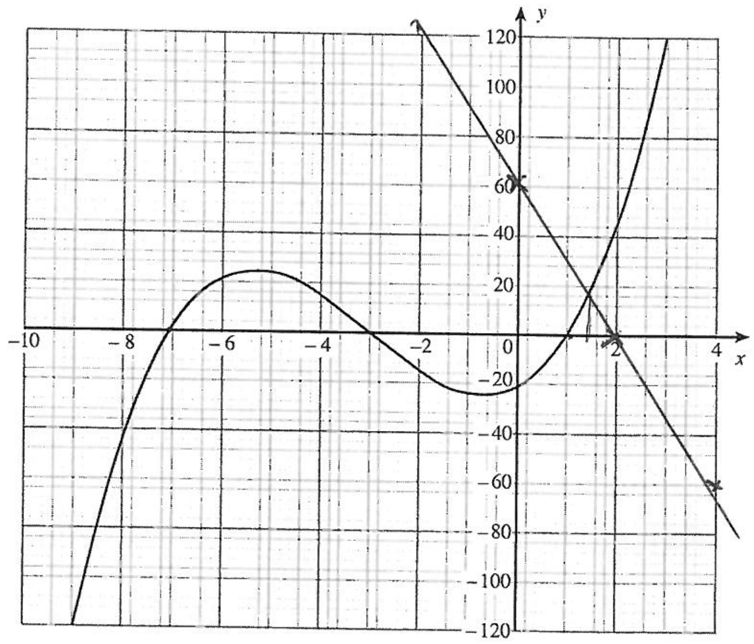
The equations of the graphs A, B, C, D and E are given in the table below.
Complete the table by matching each of the graphs A, B, C, D and E to the equation.

Equation	Graph
$y = \frac{1}{x}$	Graph E
$y = 2x + 3$	Graph D
$y = -x^3$	Graph C
$y = x^2 - 3$	Graph A
$y = 3 - 2x$	Graph B

[4]

negative so downslope gradient -ve

16. The graph of the equation $y = x^3 + 9x^2 + 11x - 21$ is shown on the graph paper below.



Use the graph above to answer the following questions.

(a) Solve $x^3 + 9x^2 + 11x - 21 = 0$.

$-7, -3, 1.$

[2]

(b) By drawing a suitable straight line solve the equation

$$x^3 + 9x^2 + 11x - 21 = 60 - 30x.$$

Draw $60 - 30x$.

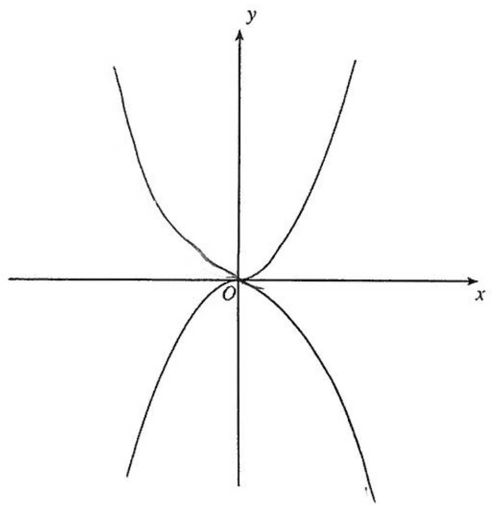
$(0, 60)$
$(2, 0)$
$(4, -60)$

$x = 1.4$

[2]

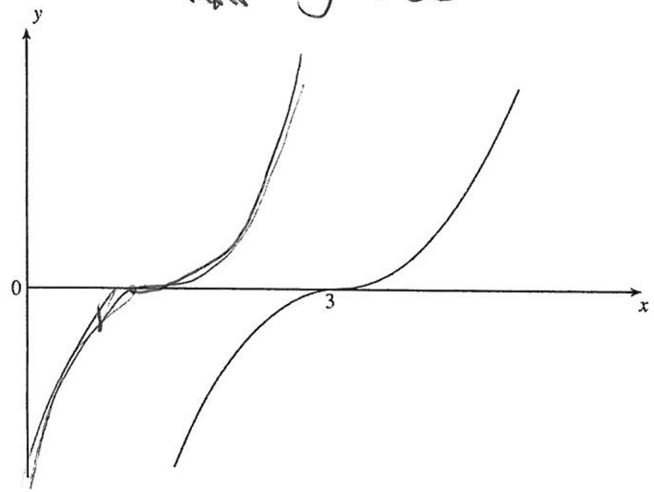
18. *(a)* The diagram shows the sketch of $y = 5x^3$. On the same diagram, sketch the curve $y = -5x^3$. [1]

Reflect in x axis



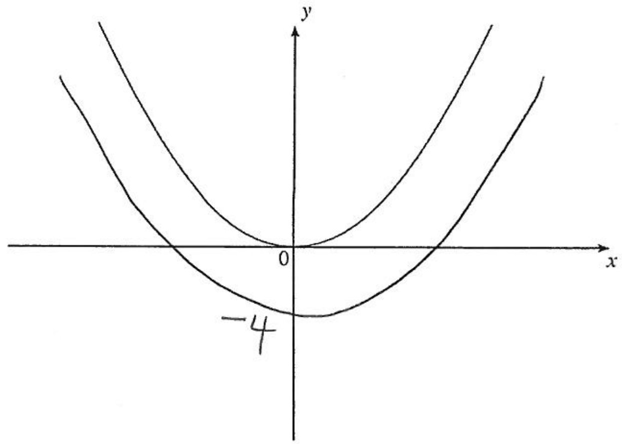
(b) The diagram shows a sketch of $y = f(x)$.
On the same diagram sketch the curve $y = f(x - 2)$.
Mark clearly the coordinates of the point where the curve crosses the x-axis. [2]

Right 2



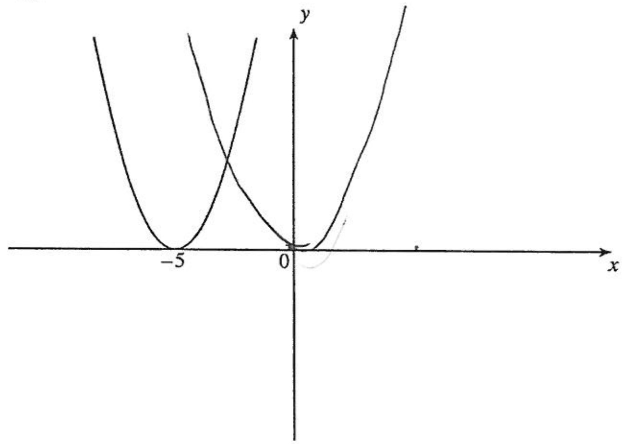
18. *(a)* The diagram shows a sketch of $y = x^2$.
On the same diagram sketch the curve $y = x^2 - 4$.
Mark clearly the coordinates of the point where the curve meets the y-axis. [2]

Down 4.



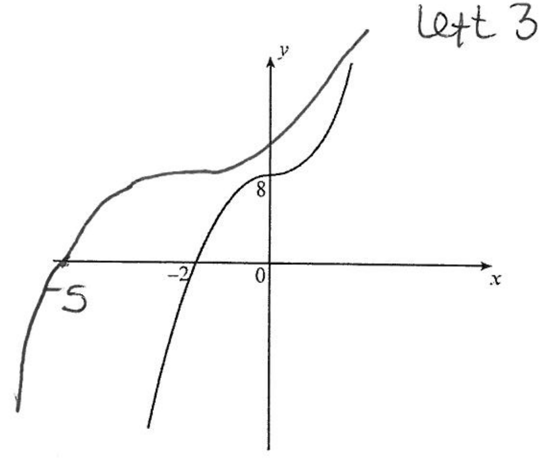
(b) The diagram shows the sketch of $y = f(x)$.
On the same diagram sketch the curve $y = f(x - 5)$.
Mark clearly the point where the curve meets the x-axis. [2]

Right 5



18. *(a)* The diagram shows a sketch of $y = f(x)$.
On the same diagram, sketch the curve $y = f(x + 3)$.
Mark clearly the value of x at the point where this curve crosses the x-axis.

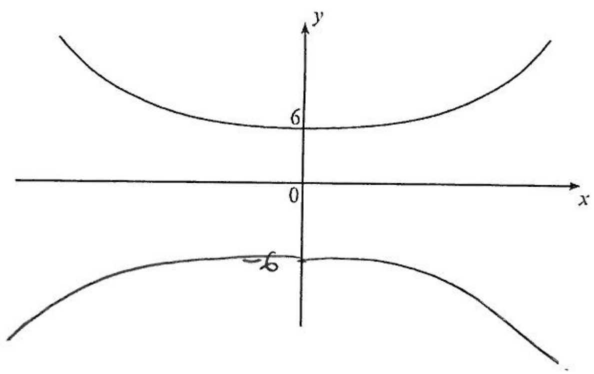
Left 3.



[2]

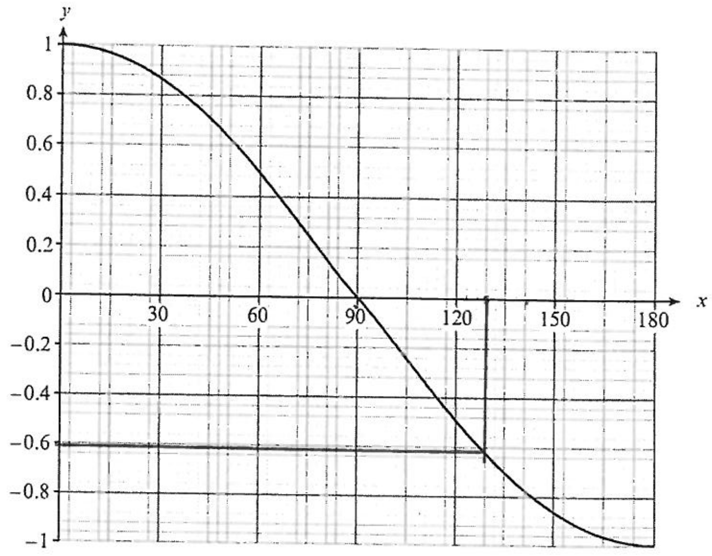
(b) The diagram shows a sketch of $y = g(x)$.
On the same diagram, sketch the curve $y = -g(x)$.
Mark clearly the value of y at the point where this curve crosses the y-axis.

Reflect x axis



[2]

19. The graph of $y = \cos x$ for the values of x between 0° and 180° is given below.



Find all the solutions of the following equations in the range **<u>−180° to 180°</u>**

(a) $\cos x = 0$

90 & −90.

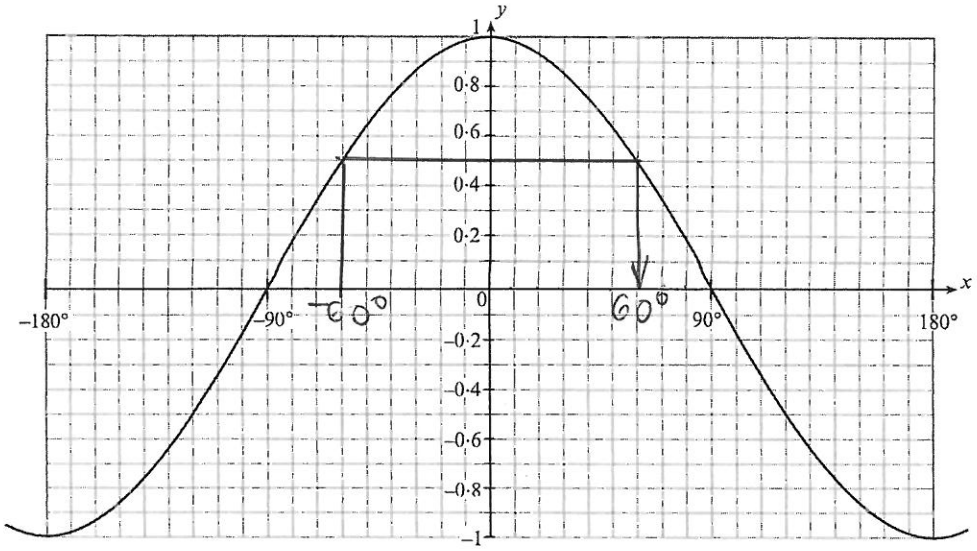
[2]

(b) $\cos x = -0{\cdot}6$

130° & −130°.

[2]

(b) The diagram shows a sketch of $y = \cos x$.



Find the values of x in the range $-180° \leqslant x \leqslant 180°$ which satisfy the equation $\cos x = 0{\cdot}5$.

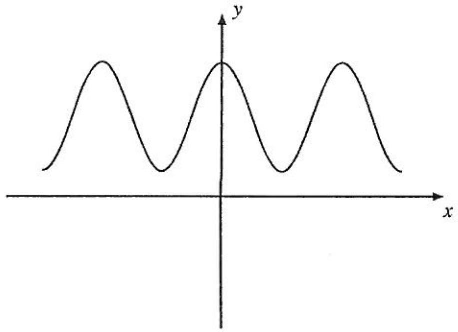
$x = 60°$

$x = -60°$.

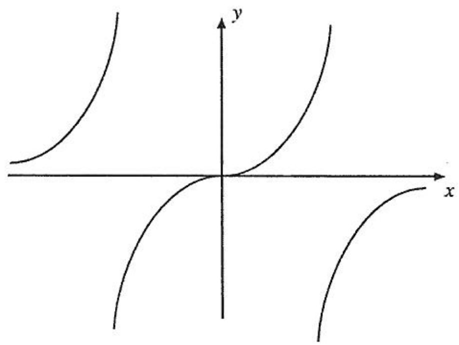
[2]

14. Match each of the following equations to the appropriate sketch by writing the equations in the spaces below.

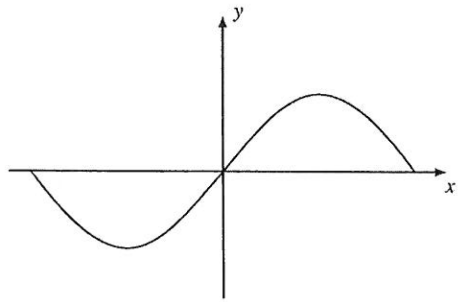
$y = \sin x$ $\quad$ $y = \tan x$ $\quad$ $y = 2 + \cos x$



Equation $y = 2 + \cos x$.



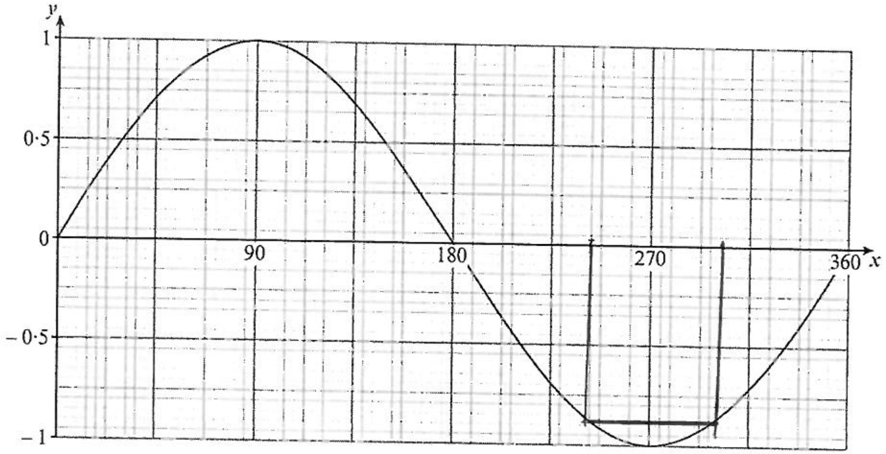
Equation $y = \tan x$



Equation $y = \sin x$

[2]

17. The diagram below shows the graph of $y = \sin x$ for values of x from 0° to 360°.



Find all solutions of the following equation in the range 0° to 360°.

$$\sin x = -0{\cdot}8$$

....

....

....

....

....

.... [2]