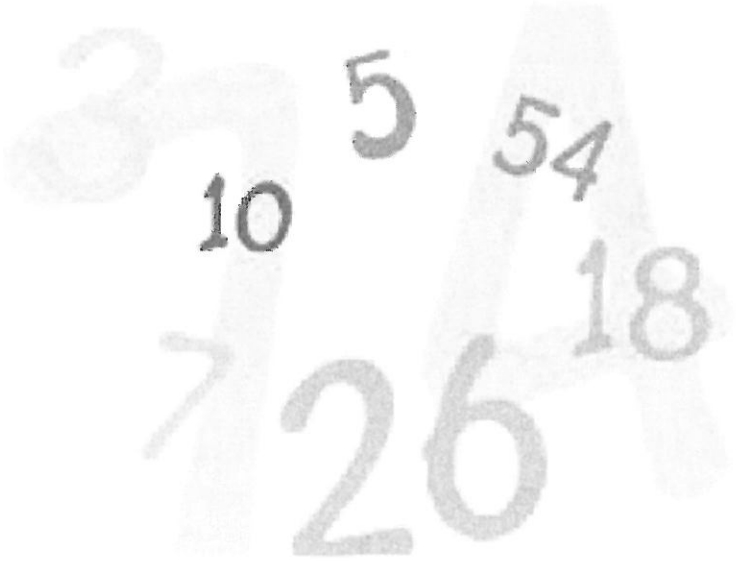
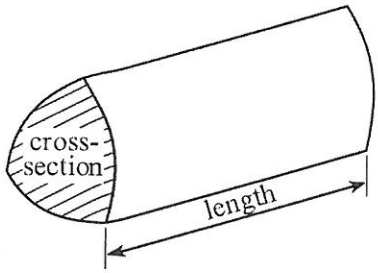
Number Booklet
Non Calculator
Higher



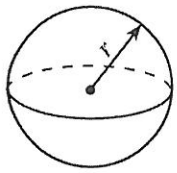
Formula List

Volume of prism = area of cross-section × length



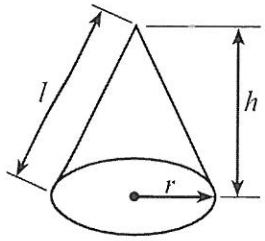
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

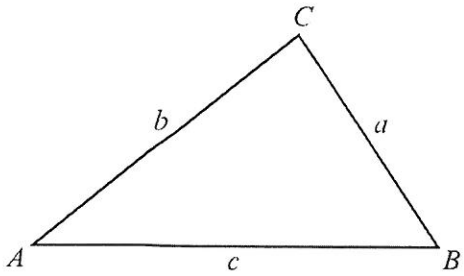


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

8. The length of a table top is 2050 mm, measured to the nearest 10 mm.

(a) Write down the **least** and **greatest** possible values of the length of the table top.

Least value 2045 mm Greatest value 2055 mm

[2]

(b) The width of the table top is 1040 mm, measured to the nearest 10 mm.
Find the least possible perimeter of the table top.

Least perimeter = 1035 + 1035 + 2045 + 2045

= 6160 mm.

[3]

5. *(c)* Estimate the value of $\frac{607 \cdot 45}{0 \cdot 2498}$.

$\approx \frac{600}{0 \cdot 25} = 2400$

$25 \overline{)600}$ 24, 50

[

8. Showing **all** your working, estimate the value of: $\frac{601 \cdot 9 \times 19 \cdot 94}{0 \cdot 305}$

$\approx \frac{600 \times 20}{0 \cdot 3} = \frac{12000}{0 \cdot 3} = 40000$

[3]

6. *(a)* Estimate the value of $\sqrt{902}$.

$\approx \sqrt{900} = 30$

[1]

(b) Write down the square number which is closest to 90.

$9^2 = 81 \quad 10^2 = 100 \quad 81$ closest square number.

[1]

(c) Estimate the value of $\frac{39 \times 403}{79}$.

$\approx \frac{40 \times 400}{80} = \frac{16000}{80} = 200$

[2]

(c) (i) Kevin usually insures his car with Adnil Insurance.
The normal cost of insuring his car is a single payment of £210.
In a special offer, Kevin is offered $\frac{1}{6}$ off his car insurance if he pays one month early.
How much would Kevin save by paying one month early?

35
6 | 2¹10 £35

So he pays £210 - 35 = £175

(ii) Liliput Insurance offers Kevin identical car insurance for an initial payment of £20 followed by 10 monthly payments of £17.50.
Would you advise Kevin to change insurance companies?
You must give reasons for your advice.

20 + 10 × 17.50 = 20 + 175 = £195

Yes because if he can't pay early Liliput will be cheaper but if he can afford to pay early he should stay with Adnil as works out cheaper if he pays early.

[6]

1. *(d)* Find which of the following fractions is closest to $\frac{2}{5}$.

$\frac{7}{20}$ $\frac{3}{8}$ $\frac{3}{10}$

Show how you made your decision.

$\frac{2}{5} = \frac{4}{10} = 40\%$

$\frac{7}{20} = \frac{35}{100} = 35\%$

$\frac{3}{8} = \frac{375}{100} = 37.5\%$

$\frac{3}{10} = 30\%$

$\frac{3}{8}$ is the closest.

[3]

8. *(c)* Write down the value of a half of $\frac{5}{8}$.

$\frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$.

[1]

3. *(d)* Which of the following fractions is closest to $\frac{1}{4}$?

$\frac{13}{20}$ $\frac{2}{5}$ $\frac{3}{10}$

Show how you made your decision.

$\frac{1}{4} = 25\%$

$\frac{13}{20} = \frac{65}{100} = 65\%$

$\frac{2}{5} = \frac{4}{10} = 40\%$

$\frac{3}{10} = 30\%$

So $\frac{3}{10}$ is closest

[3]

(d) Evaluate $2\frac{1}{3} \times 3\frac{3}{7}$. Simplify your answer.

$2\frac{1}{3} = \frac{7}{3} \times \frac{24}{7} = \frac{24}{3} = 8$.

[2]

2. William goes on holiday for two weeks. During his holiday William uses $\frac{3}{4}$ of a large bottle of water every day. What is the least number of large bottles of water William needs to buy to last for the two weeks he is on holiday?

14 days so $\frac{3}{4} \times 14 = 10.5$

$4\overline{)14}$ 3.5

Needs to buy at least 11 bottles.

[3]

3. *(b)* Calculate $6\frac{1}{4}\%$ of £40.

1% = 0.40p.

6% = 2.40

$\frac{1}{4}\%$ = 0.10

$6\frac{1}{4}\%$ = £2.50

[3]

9. *(a)* Evaluate each of the following.

(i) $2^5 - 11^2$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ $\quad 32 - 121 = -89$

$11^2 = 11 \times 11 = 121$

[3]

(ii) 28^0

1

[1]

(iii) $81^{\frac{1}{4}} \times 25^{-\frac{1}{2}}$

$= \sqrt[4]{81} \times \frac{1}{\sqrt{25}}$

$= 3 \times 1/5$

$= 3/5$

[3]

(d) Simplify

(i) $81^{-\frac{1}{2}}$,

$\frac{1}{\sqrt{81}} = \frac{1}{9}$

(ii) $125^{\frac{2}{3}}$.

$\left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

[4]

3. *(c)* William scores 45 out of 50 in an English test. Express this as a percentage.

$\frac{45}{50} = \frac{90}{100} = 90\%$.

[1]

1. *(c)* Sasha scores 45 out of 150 in an English test. Express this as a percentage.

$\frac{45}{150} = \frac{15}{50} = \frac{30}{100} = 30\%$.

[2]

7. *(a)* In an examination, a pupil scores 165 marks out of a total of 300 marks. What percentage is this?

$\frac{165}{300} = \frac{55}{100} = 55\%$.

$3\overline{)165}$ = 55

[2]

5. *(a)* Cheryl scored 60 marks out of 80 in a test.
Express Cheryl's score as a percentage.

$\frac{60}{80} = \frac{15}{20} = \frac{75}{100} = 75\%$

or $\frac{60}{80} = \frac{3}{4} = 75\%$.

[2]

(c) Calculate 75% of £562.80.

75% of 562.80
25% = £140.70
75% = £422.10

$4\overline{)562.80}$ = 140.70

$140.70 \times 3 = 422.10$

[2]

5. *(a)* Freddy goes to buy a ticket for a concert.
A sign by the ticket office states "20% off all original ticket prices".
Freddy comes away having paid a reduced price of £36.80 for his ticket.
What was the original price of Freddy's ticket?

£36.80 is 80% of original price
£4.60 is 10%
£46.00 is 100%.

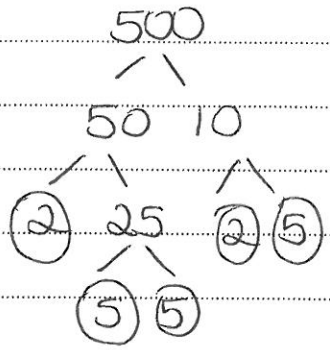
£4.60
8) 36.80

Original price is £46.

Check 20% = £9.20 46 - 9.20 = £36.80 ✓

[3]

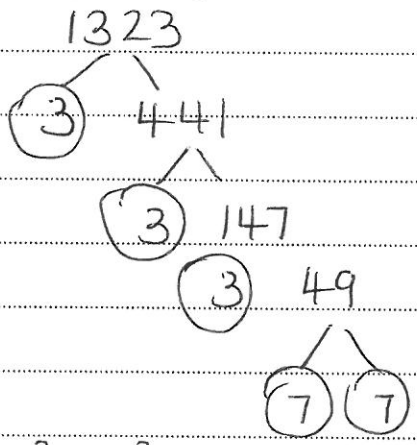
9. Express 500 as a product of prime numbers in index form.



$500 = 2^2 \times 5^3$

[3]

9. *(a)* Express 1323 as a product of prime numbers in index form.



$1323 = 3^3 \times 7^2$

$$\begin{array}{r} 441 \\ 3\overline{)1323} \end{array} \qquad \begin{array}{r} 147 \\ 3\overline{)441} \end{array} \qquad \begin{array}{r} 49 \\ 3\overline{)147} \end{array}$$

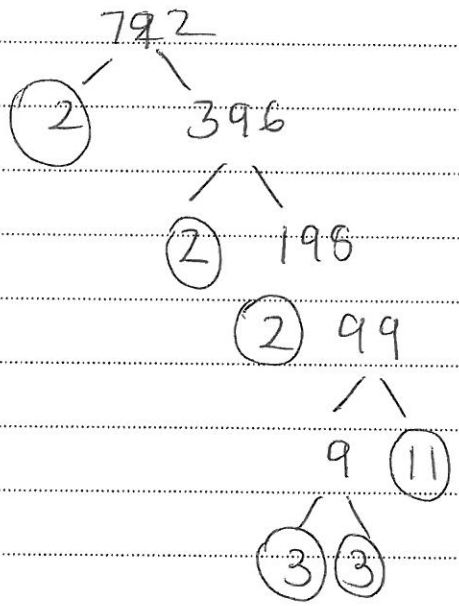
[3]

(b) Write down the least whole number by which 1323 should be multiplied to make the result a perfect square.

7 to even power 3 not so x by 3.

[1]

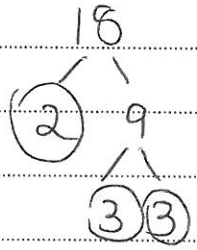
6. *(a)* Express 792 as a product of prime numbers in index form.



$792 = 2^3 \times 3^2 \times 11$

[3]

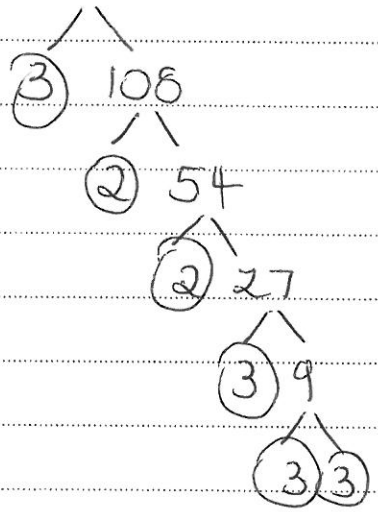
(b) Explain why 18 is not a perfect square.



$18 = 2 \times 3^2$ so not all even powers

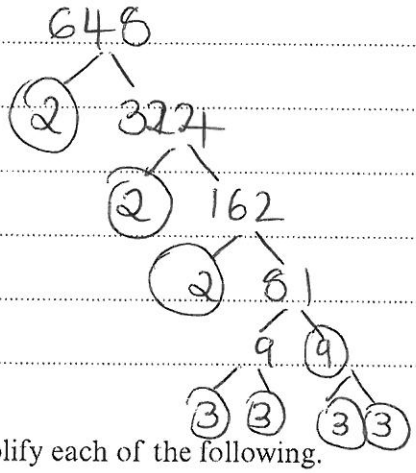
[1]

10. *(a)* (i) Express 324 as a product of prime numbers in index form.



$324 = 2^2 \times 3^4$

(ii) Is 648 a square number? You must explain your answer.



$648 = 2^3 \times 3^4$

Not square not all even powers.

[5]

(b) Simplify each of the following.

(i) $5x^4y^6 \times 4x^5y$

$20x^9y^7$

$5 \times 4 = 20$ $x^4 \times x^5 = x^9$ $y^6 \times y = y^7$

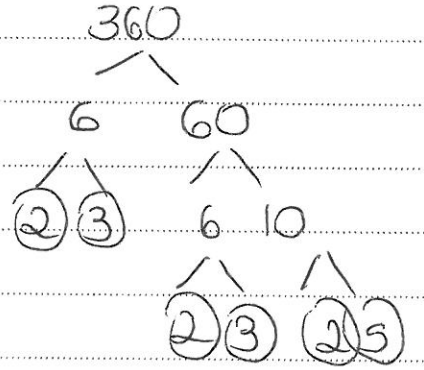
(ii) $\frac{50a^6b}{10a^3b^3}$

$5a^3b^{-2}$

$\frac{50}{10} = 5$ $\frac{a^6}{a^3} = a^3$ $\frac{b}{b^3} = b^{-2}$

[4]

8. *(d)* (i) Express 360 as a product of prime factors using index notation.



$360 = 2^3 \times 3^2 \times 5$

(ii) What is the least number that you must multiply 360 by to make a perfect square?

by 2 & 5 so 10.

[4]

12. Given that y is inversely proportional to x^2, and that $y = 4$ when $x = 5$,

(a) find an expression for y in terms of x,

$y \propto \frac{1}{x^2}$ $\quad 4 = \frac{k}{25}$

$y = \frac{k}{x^2}$ $\quad k = 100$

$y = \frac{100}{x^2}$

[3]

(b) use the expression you found in *(a)* to complete the following table.

x	-1	5	10
y	100	4	1

$y = \frac{100}{x^2}$ $\quad \frac{100}{1} = 100$.

$\frac{100}{x^2} = 1$ $\quad x^2 = 100$ $x = 10$

[2]

14. Given that y is inversely proportional to x^2, and that $y = 2$ when $x = 5$,

(a) find an expression for y in terms of x,

$y \propto \frac{1}{x^2}$ $\quad 2 = \frac{k}{25}$

$y = \frac{k}{x^2}$ $\quad k = 50$

$y = \frac{50}{x^2}$

[3]

(b) calculate

(i) the value of y when $x = 2$,

$y = \frac{50}{2^2} = \frac{50}{4} = 12.5$

(ii) a value of x when $y = 0{\cdot}5$.

$0.5 = \frac{50}{x^2}$

$0.5x^2 = 50$

$x^2 = 100$ $\quad x = 10$.

[3]

15. Given that y is inversely proportional to x, and that $y = 5$ when $x = 2$,

(a) find an expression for y in terms of x,

$y \propto \frac{1}{x}$ | $5 = \frac{k}{2}$

$y = \frac{k}{x}$ | $k = 10$

$y = \frac{10}{x}$.

[3]

(b) complete the following table for values of x and y.

x	-4	2	50
y	-2.5	5	0·2

$y = \frac{10}{x} = \frac{10}{-4} = -2.5$

$0.2 = \frac{10}{x} = 0.2x = 10 \quad x = \frac{10}{0.2} = 50$

[2]

2.

Mixed Berry Yogurt Shake

Serves **8** people

Ingredients:
- 4 cups semi-skimmed milk
- 4 cups low fat natural yogurt
- 16 ounces mixed summer fruits
- 4 tablespoons of honey

The recipe for Mixed Berry Yogurt Shake appears in an old cookery book. Inside the cover of the book the reader is told that 1 cup = 250 ml, 4 ounces is approximately 115 g and 1 tablespoon is 15 ml.

(a) Complete the recipe below for serving 8 people using ml and g.

4 cups = 1000ml.
16 ounces = 4 x 115 = 460g
4 tablespoons = 4 x 15 = 60ml

Mixed Berry Yogurt Shake

Serves **8** people

Ingredients:
- 1000 ml semi-skimmed milk
- 1000 ml low fat natural yogurt
- 460 g mixed summer fruits
- 60 ml of honey

[4]

(b) Jamie has large quantities of natural yogurt, mixed summer fruits and honey but only has 5·5 litres of semi-skimmed milk. Find the largest number of people for whom Jamie can make Mixed Berry Yogurt Shakes.

5.5 L = 5500 ml.
1000 ml → Serves 8 people.
500 ml → Serves 4 people
5.5 L → Serves 44 people.

[3]

5. *(b)* Share £385 in the ratio 2 : 9.

$2+9=11$

$385 \div 11 = 35$

$2 \times 35 = £70$

$9 \times 35 = £315$

Check £385 ✓

$11 \overline{)385}$ = 35

[2]

1. Norah and Janice share £300 in the ratio 9 :1. Calculate the share of the money Norah and Janice will each receive.

£300 ÷ 10 = £30

9 x 30 = £270

1 x 30 = £30

Check £300 ✓

[2]

5. *(b)* Share £300 in the ratio 5 : 7.

£300 ÷ 12 = 25

5 x 25 = £125

7 x 25 = £175

Check = £300 ✓

$12 \overline{)300}$ = 25

[3]

2. *(b)* A recipe for making 12 pancakes includes the following ingredients.

2 large eggs
200ml milk
110g flour

Calculate the quantities of these ingredients needed to make 30 pancakes.

half

x5

6 pancakes = 1 egg 100ml milk 55g flour.

30 pancakes = 5 eggs 500ml milk 275g flour.

[3]

3. *(c)* Write down the reciprocal of 2·5.

$\frac{1}{2.5} = \frac{10}{25} = \frac{2}{5}$

or $\frac{1}{2\frac{1}{2}} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$.

[2]

(e) Find the value of the reciprocal of 0·9.

$\frac{1}{0.9} = \frac{10}{9}$ or $1\frac{1}{9}$.

[2]

13.

(b) Express $0\cdot8\dot{2}\dot{3}$ as a fraction.

$x = 0.8232323\ldots$

$10x = 8.232323\ldots$

$1000x = 823.232323\ldots$

$990x = 815$

$x = 815/990$

[2]

(c) Evaluate $\left(\sqrt{72}-\sqrt{2}\right)^2$.

$\left(\sqrt{72}-\sqrt{2}\right)\left(\sqrt{72}-\sqrt{2}\right)$

$= 72 - \sqrt{144} - \sqrt{144} + 2$

$= 72 - 12 - 12 + 2$

$= 50$

[3]

15. *(a)* Express $0\cdot6\dot{5}\dot{2}$ as a fraction.

$x = 0\cdot652525252$

$10x = 6\cdot525252$

$1000x = 652\cdot5252$

$990x = 646$

$x = \frac{646}{990}$.

[2]

(b) Given that $p = \sqrt{5}$, $q = \sqrt{13}$ and $r = \sqrt{325}$, simplify pqr.

$pqr = \sqrt{5}\sqrt{13}\sqrt{325}$

$= \sqrt{5}\sqrt{13}\sqrt{13}\times5$

$= 5\sqrt{5}\times13$

$= 65\sqrt{5}$.

$\sqrt{325} = \sqrt{25}\sqrt{13}$

[2]

18. *(a)* Find the value of $\left(\sqrt{12}-\sqrt{3}\right)^2$.

$= \left(\sqrt{12}-\sqrt{3}\right)\left(\sqrt{12}-\sqrt{3}\right)$

$= 12 - \sqrt{36} - \sqrt{36} + 3$

$= 12 - 6 - 6 + 3 \quad = 3.$

[3]

(b) Given that $p = \sqrt{5}$, $q = \sqrt{13}$ and $r = \sqrt{65}$, simplify pqr.

$pqr = \sqrt{5}\sqrt{13}\sqrt{65}$

$= \sqrt{65}\sqrt{65}$

$= 65.$

[2]

(c) Express $0{\cdot}8\dot{4}7\dot{}$ as a fraction.

$x = 0.8\cdot474747$

$10x = 8\cdot474747$

$1000x = 847\cdot4747$

$990x = 839.$

$x = 839/990.$

[2]

9. *(a)* Write each of the following numbers in standard form.

(i) 5800

5.8×10^3

(ii) 0·004

4×10^{-3}

[2]

(b) Find, in standard form, the value of $\frac{5{\cdot}6 \times 10^6}{2 \times 10^{-3}}$.

2.8×10^9

$\frac{5.6}{2} = 2.8$

$\frac{10^6}{10^{-3}} = 10^9$

[2]

(c) Find the value of $(8 \times 10^3) - (2 \times 10^3)$.

$8000 - 2000$

6000

[1]

9. (iv) $3{\cdot}4 \times 10^3 + 1{\cdot}2 \times 10^2$

3400 + 120

3520.

[2]

9. *(b)* Evaluate the following. Express your answer in standard form.

$$\frac{2^8 \times 5^2}{2^2}$$

$\frac{256 \times 25}{4}$ $\quad 4\overline{)256}$ (64)

64×25

$16 \times 100 = 1600$

1.6×10^3

$\frac{2^8}{2^2} \times 5^2$

$2^6 \times 5^2$

$2^2 \times 5^2 \div 2^4 = 10^2 \times$ 16

$= 1.6 \times 10^3$

[3]

9. *(a)* Write **each** of the following numbers in standard form.

(i) 0·0047

4.7×10^{-3}

(ii) 32000

3.2×10^4

[2]

(b) Find, in standard form, the value of

$$(2{\cdot}1 \times 10^{-5}) \times (3 \times 10^8).$$

6.3×10^3

[2]