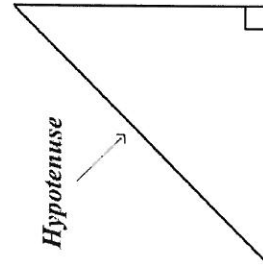
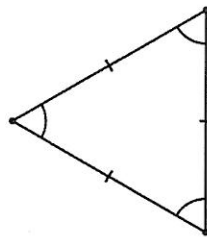
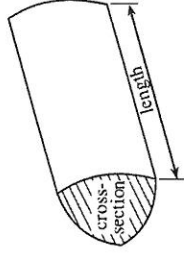


Shape Booklet Non Calculator Higher

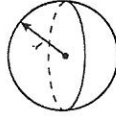


Formula List

Volume of prism = area of cross-section $\times$ length

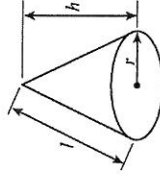


Volume of sphere = $\frac{4}{3} \pi r^3$



Surface area of sphere = $4\pi r^2$

Volume of cone = $\frac{1}{3} \pi r^2 h$



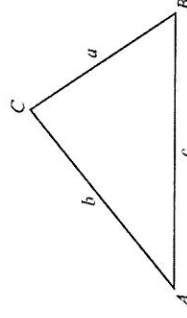
Curved surface area of cone = $\pi r l$

In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



3. (a)

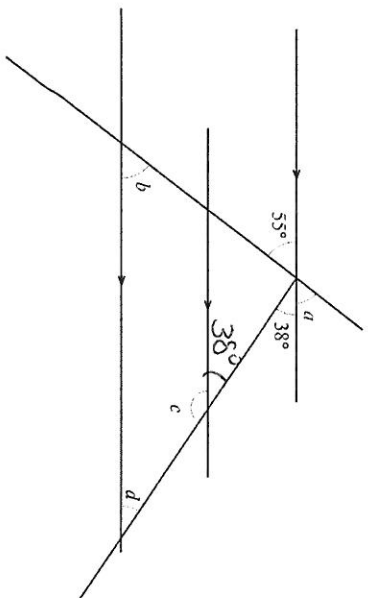


Diagram not drawn to scale

Find the size of each of the angles marked a , b , c and d .

$$a = 55^\circ \quad b = 55^\circ \quad c = 142^\circ \quad d = 38^\circ$$

[4]

4. (a) Calculate the size of each of the angles marked x , y and z in the diagram below.

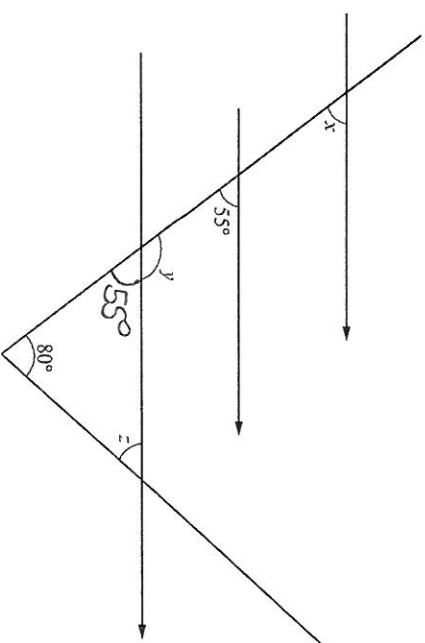


Diagram not drawn to scale

$$\begin{array}{r} 80 \\ 55 \\ \hline 135 \end{array}$$

$$x = 55^\circ \quad y = 125^\circ \quad z = 45^\circ$$

[4]

(b) Calculate the size of each of the exterior angles of a regular pentagon.

$$360 \div 5 = 72^\circ$$

5 sides
5 angles

[3]

(b)

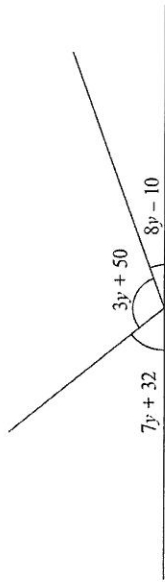


Diagram not drawn to scale

All of the angles are measured in degrees.

Find the size of each of the three angles.

$$7y + 32 + 8y - 10 + 3y + 50 = 180$$

$$18y + 72 = 180$$

$$18y = 108$$

$$y = 6$$

$$7 \times 6 + 32$$

$$3 \times 6 + 50$$

$$8 \times 6 - 10$$

$$7y + 32 = 74^\circ$$

$$3y + 50 = 68^\circ$$

$$8y - 10 = 38^\circ$$

[5]

2. (a) The diagram shows a regular hexagon.

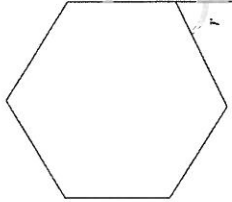


Diagram not drawn to scale

Calculate the size of the angle marked r .

$$r = 360 \div 6 = 60^\circ$$

$$r = 60^\circ$$

[2]

(b) Find the size of the angles marked p and q .

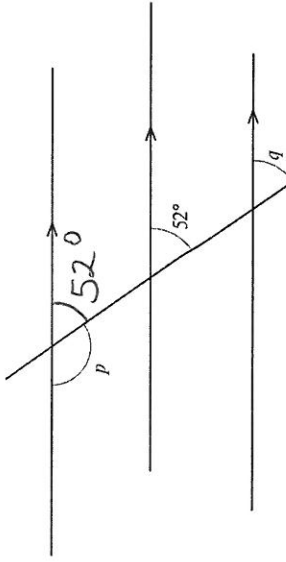


Diagram not drawn to scale

$$p = 128^\circ$$

$$q = 52^\circ$$

[2]

5. (a) The angles of a quadrilateral are x° , 49° , $3x^\circ$ and 111° .
Form an equation in x , and use your equation to find the value of x .

$$x + 49 + 3x + 111 = 360$$

$$4x + 160 = 360$$

$$4x = 200 \quad x = 50^\circ$$

[4]

- (b) Find the size of each of the angles marked x and y in the following diagram.

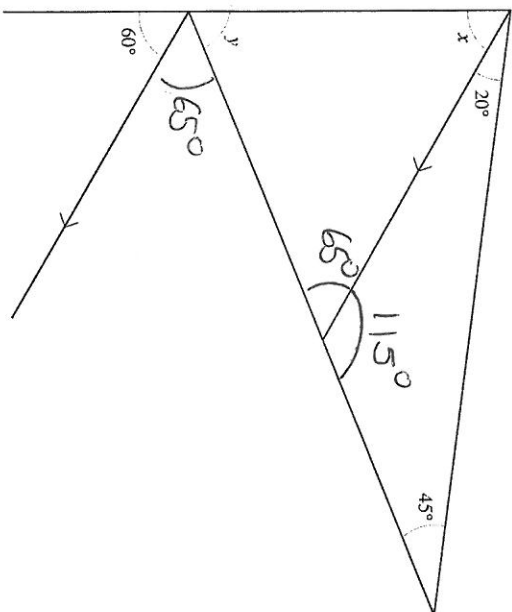


Diagram not drawn to scale.

$$x = 60^\circ \quad y = 55^\circ$$

[3]

5. In the diagram below, $ABCD$ and $DCEF$ are parallelograms with $\hat{BCE} = 135^\circ$ and $\hat{ABC} = 80^\circ$.
Find the size of the angle marked x .

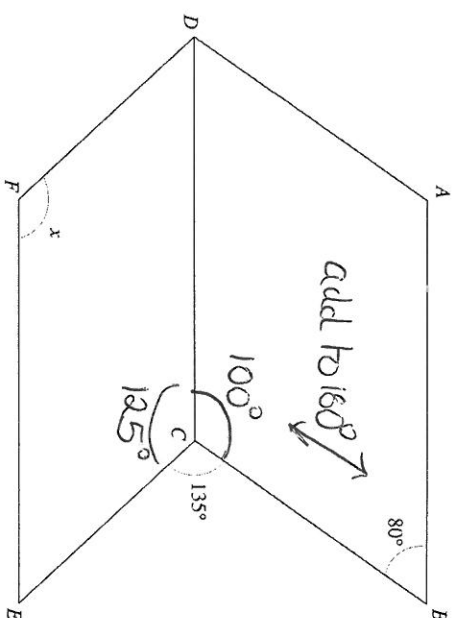


Diagram not drawn to scale.

$$\begin{array}{r} 100 \\ 135 \\ \hline 235 \end{array}$$

$$x = 125^\circ$$

[4]

3. The diagram shows a regular hexagon. Showing all your working, calculate the size of the angle marked x .

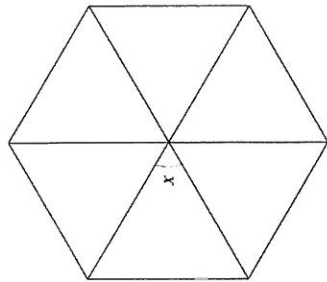
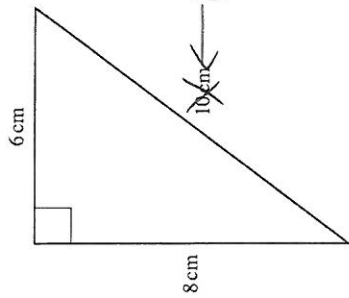


Diagram not drawn to scale.

$$360 \div 6 = 60^\circ$$

[2]

2. (a) Calculate the area of the following triangle, stating the units of your answer.



~~10 cm~~ ← Not needed.

Diagram not drawn to scale.

$$\frac{\text{Base} \times \text{height}}{2} = \frac{8 \times 6}{2} = 24 \text{ cm}^2$$

[3]

- (b) Calculate the perimeter of the shape shown in the diagram below.

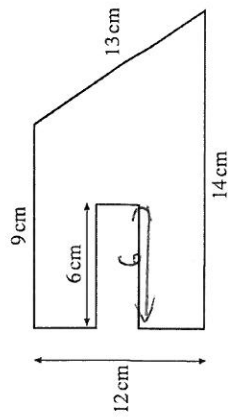


Diagram not drawn to scale.

$$P = 12 + 14 + 13 + 6 + 6 + 12 = 60 \text{ cm}$$

[2]

3. (a) The diagram shows a triangle DEF with $EF = 12\text{ cm}$, $\hat{DXF} = 90^\circ$ and $DX = 5\text{ cm}$.

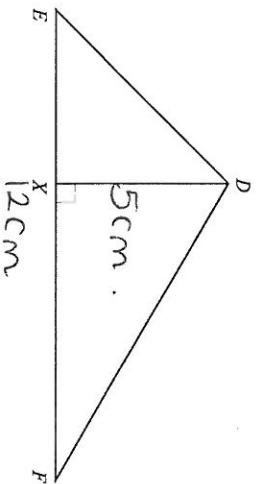


Diagram not drawn to scale

Find the area of the triangle DEF . State appropriate units for your answer.

$$\text{Area} = \frac{5 \times 12}{2} = 30\text{ cm}^2$$

[3]

(b)

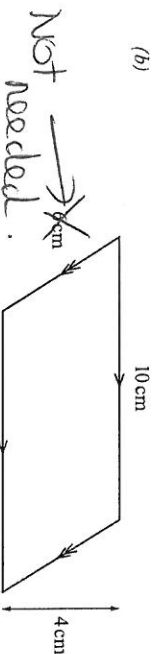


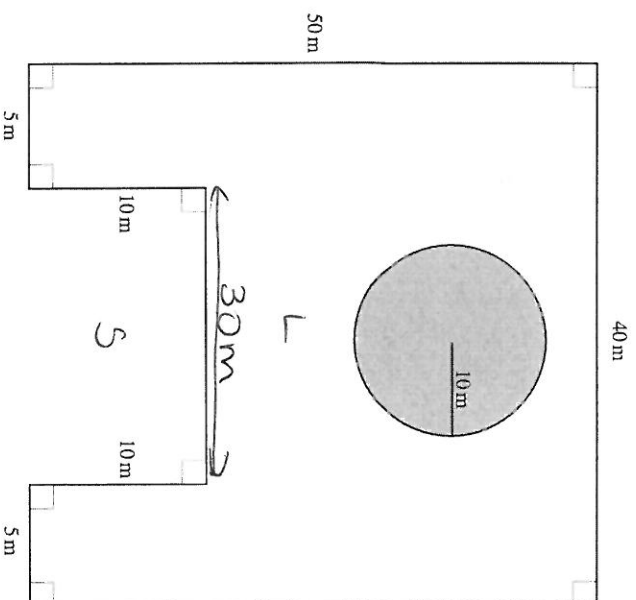
Diagram not drawn to scale

Calculate the area of the parallelogram.

$$\begin{aligned} \text{Area} &= \text{base} \times \text{height} \\ \text{Area} &= 10 \times 4 \\ &= 40\text{ cm}^2 \end{aligned}$$

[2]

7. The following diagram shows a paved area with a circular pond of radius 10 m .



Using the value of π as 3.14, calculate the area of the paved surface clearly indicating the units of your answer.

$$\begin{aligned} \text{Large Rectangle} &= 50 \times 40 = 2000\text{ m}^2 \\ \text{Small Rectangle} &= 30 \times 10 = 300\text{ m}^2 \\ \text{Circle Pond} &= \pi \times 10^2 \\ &= 3.14 \times 100 \\ &= 314\text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Paved Surface} &= 2000 - 300 - 314 \\ &= 2000 - 614 \\ &= 1386\text{ m}^2 \end{aligned}$$

[6]

(c)

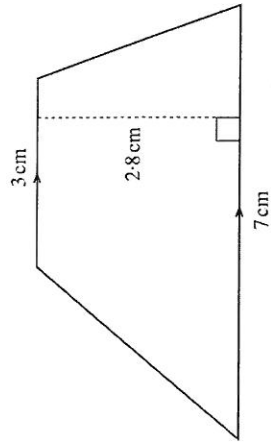


Diagram not drawn to scale.

Calculate the area of the trapezium shown in the diagram.

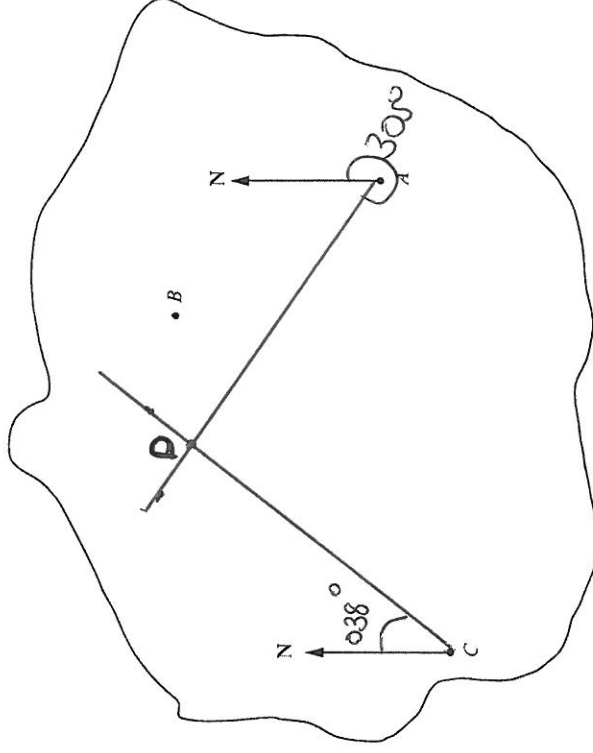
$$\text{Area} = \frac{a+b}{2} \times h$$

$$= \frac{7+3}{2} \times 2.8$$

$$= 5 \times 2.8 = 14 \text{ cm}^2$$

[2]

(c)



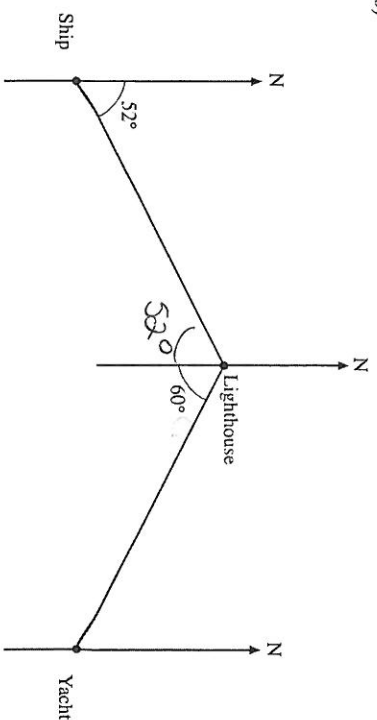
(i) Write down the bearing of point B from point A.

350 (±1)

(ii) A point D is to be plotted on the above plan.
The bearing of D from C is 038°, and the bearing of D from A is 305°. Find and mark the position of D on the above plan.

[4]

(b)



Write down the bearing of:

(i) the yacht from the lighthouse.

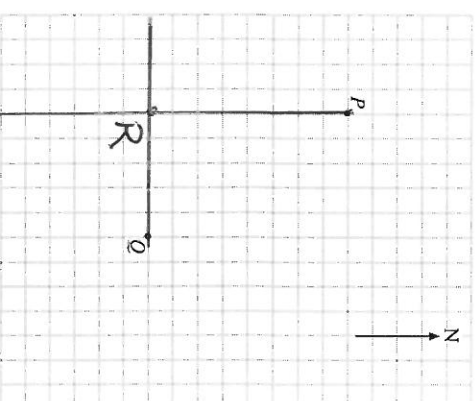
120°

(ii) the ship from the lighthouse.

232°

[3]

5. (a) The points P and Q on the grid represent two towns.



(i) Find the bearing of P from Q .

330°

(ii) Another town, R , is on a bearing 270° from Q and 180° from P . Mark R on the grid.

[3]

13. The points A , B and C lie on the circumference of a circle.
The straight line PBT is a tangent to the circle and $\widehat{CBP} = x$, where x is measured in degrees.

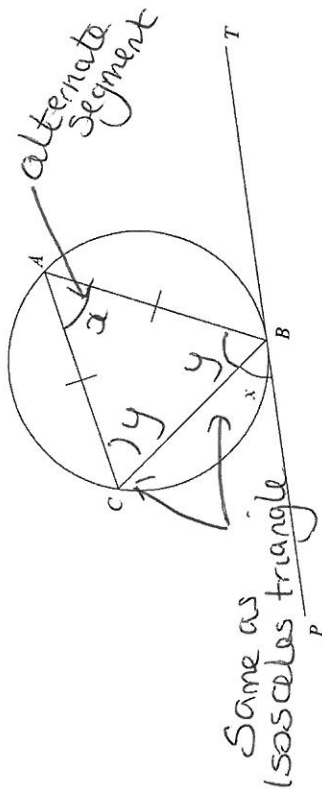


Diagram not drawn to scale

Show, giving reasons in your answer, that the size of $\widehat{ABC}$ in degrees is $90 - \frac{1}{2}x$.

$$\begin{aligned} 2y + x &= 180 \\ 2y &= 180 - x \\ y &= 90 - \frac{1}{2}x \\ \widehat{ABC} &= 90 - \frac{1}{2}x \end{aligned}$$

[2]

16.

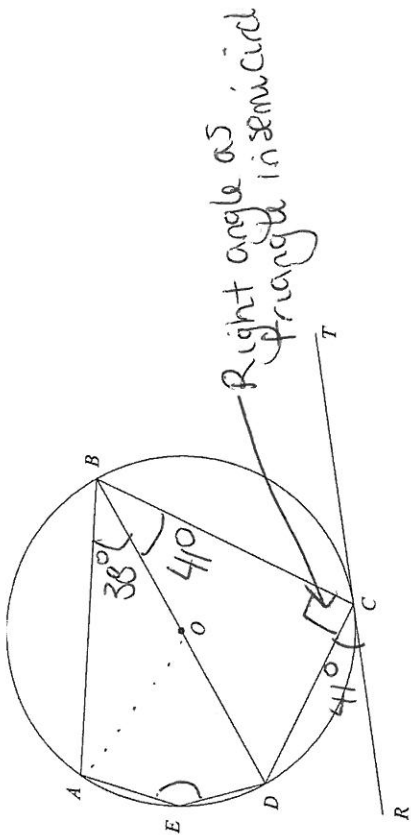


Diagram not drawn to scale

Five points A , B , C , D and E lie on the circumference of the circle centre O with BOD a straight line.
The tangent RT touches the circle at C .

Given that $\widehat{ABD} = 38^\circ$ and $\widehat{DCR} = 41^\circ$, find each of the following angles, giving reasons for your answers.

(a) $\widehat{AED}$

$$142^\circ \quad (\text{opposite in cyclic quadrilateral})$$

(b) $\widehat{AOD}$

$$\begin{aligned} \widehat{AOD} &= 2 \times 38^\circ \\ &= 76^\circ \end{aligned} \quad \begin{array}{l} \text{(angle at centre} \\ \text{twice angle at} \\ \text{circumference)} \end{array}$$

(c) $\widehat{BDC}$

$$\begin{aligned} \widehat{CBD} &= 41^\circ \quad \text{alternate segment} \\ \widehat{BDC} &= 180 - 41 - 90 \\ &= 49^\circ \end{aligned}$$

[6]

(d)

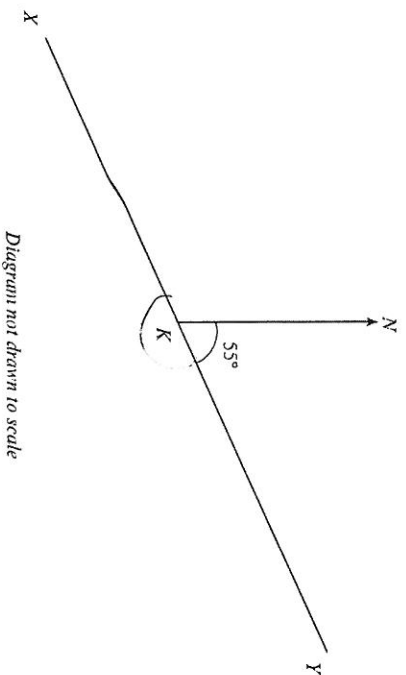


Diagram not drawn to scale

The above diagram shows three points X, K and Y which lie on a straight line.
The bearing of Y from K is 055° . Find the bearing of X from K.

$$55 + 180^\circ = 235^\circ$$

[2]

11. Triangle in Semi-circle

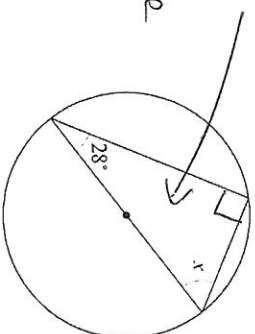


Diagram not drawn to scale

Find the size of the angle marked x.

$$= 180 - 90 - 28^\circ$$

$$= 62^\circ$$

[2]

20.

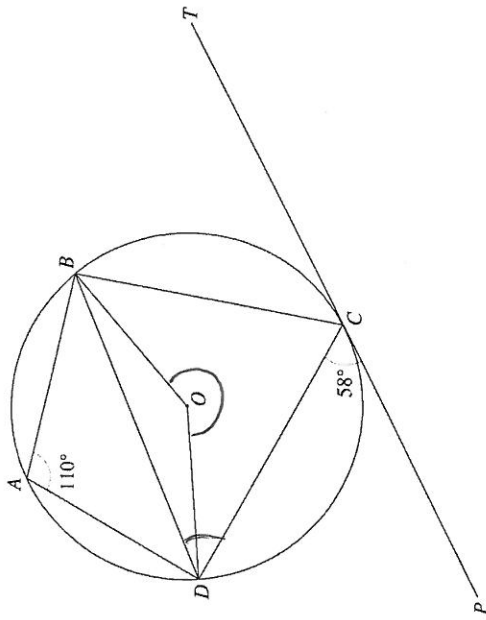


Diagram not drawn to scale.

Four points A, B, C and D lie on the circumference of the circle with centre O. The tangent TP touches the circle at C. Given that $\angle AOB = 110^\circ$ and $\angle BCP = 58^\circ$, find each of the following angles, giving reasons for your answers.

- (a) Reflex $\angle DOB$ 220° angle at centre
twice angle at edge
(Circumference)

[1]

- (b) $\angle BDC$ 58°
alternate segment

[2]

13. The three points A, B and C lie on the circumference of a circle centre O. The tangent XBY touches the circle at B.

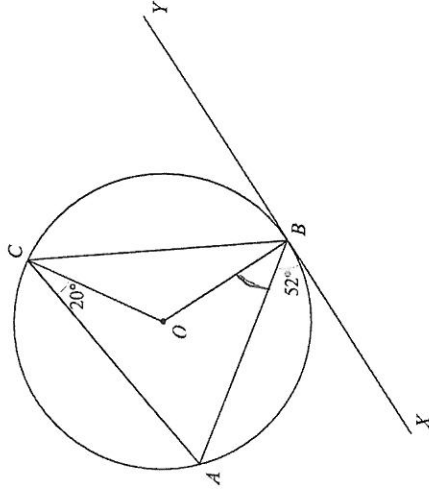


Diagram not drawn to scale

Find each of the following angles. Give reasons for your answers.

- (a) $\angle OBA$

[2]

- (b) $\angle ACB$

[2]

- (c) $\angle CAB$

[2]

4. (a) Calculate the circumference of a circle with a radius of 5 cm, using 3.14 as the value of π .

$$A = \pi r^2$$

$$= \pi \times 5^2$$

$$= 3.14 \times 25 = 3.14 \times 100 = 78.5 \text{ cm}^2$$

(b)

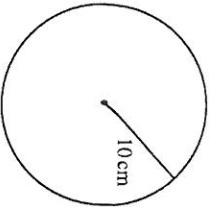


Diagram not drawn to scale.

Calculate the area of the circle shown in the diagram above, using 3.14 as the value of π .

$$A = \pi r^2$$

$$= \pi \times 10^2$$

$$= 3.14 \times 100$$

$$= 314 \text{ cm}^2$$

[2]

4. (a) Complete the following conversion table $\times \frac{5}{8}$

Conversion	
Miles	Kilometres
15	24
75	120
100	160

$\times \frac{5}{8}$

- (b) Kevin's car manual states that the fuel consumption for his car is 20 km per litre.

Kevin's car gives distance travelled in miles.
What is the fuel consumption of Kevin's car in miles per litre?

$$20 \text{ km} = 20 \times \frac{5}{8} = 12.5 \text{ miles per litre}$$

[2]

10. A newspaper reporter finds the information below on a page on the Internet.
The table gives information about 5 different ships.

Date	Name	Owned by	Weight (tons)	Decks	People	approx. feet	feet	knots
1912	Titanic	White Star Line	46328	9	3000	952.5	92	21
1934	Queen Mary	Cunard	81237	12	3131	1019	—	28.5
1962	Carrier Enterprise	US Navy	89600	—	5830	1101	133	30
2002	Brilliance of the Seas	Royal Caribbean	90090	13	2501	962	105.6	25
2004	Queen Mary 2	Cunard	150000	23	3873	1132	147.5	30

A newspaper report is to be written using some of the above data.

The reporter decides to include a table in his report, in which he expresses for each ship:

- the weight, in tons, in standard form correct to 2 significant figures;
- the length, in feet, correct to 1 significant figure.

Complete the reporter's table.

Date	Name	Owned by	Weight (tons)	Length (feet)
1912	Titanic	White Star Line	4.6×10^4	1000 ft
1934	Queen Mary	Cunard	8.1×10^4	1000 ft
1962	Carrier Enterprise	US Navy	9.0×10^4	1000 ft
2002	Brilliance of the Seas	Royal Caribbean	9.0×10^4	1000 ft
2004	Queen Mary 2	Cunard	1.5×10^5	1000 ft

[4]

6. (a) A solid metal ingot weighs 1.5 kg and has a volume of 300 cm^3 . Find the density of the metal in g/cm^3 .

$$1.5 \text{ kg} = 1500 \text{ g}$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{1500}{300} = 5 \text{ g/cm}^3$$

[3]

- (b) It takes 50 seconds to fill a 4 litre bucket with water from a hose pipe. It takes 10 minutes to fill a tank with water using the same hose pipe with the same rate of flow. Find the volume of the tank.

$$4 \text{ L} = 50 \text{ seconds}$$

$$4000 \text{ mL} = 50 \text{ seconds}$$

$$400 \text{ mL} = 10 \text{ seconds}$$

$$2400 \text{ mL} = 60 \text{ seconds } 1 \text{ minute}$$

$$24000 \text{ mL} = 10 \text{ minutes} = 24 \text{ L}$$

[3]

- (c) Two boxes have heights of 134 mm and 23 mm, each measured to the nearest mm. Find the maximum height when the boxes are placed one on top of the other.

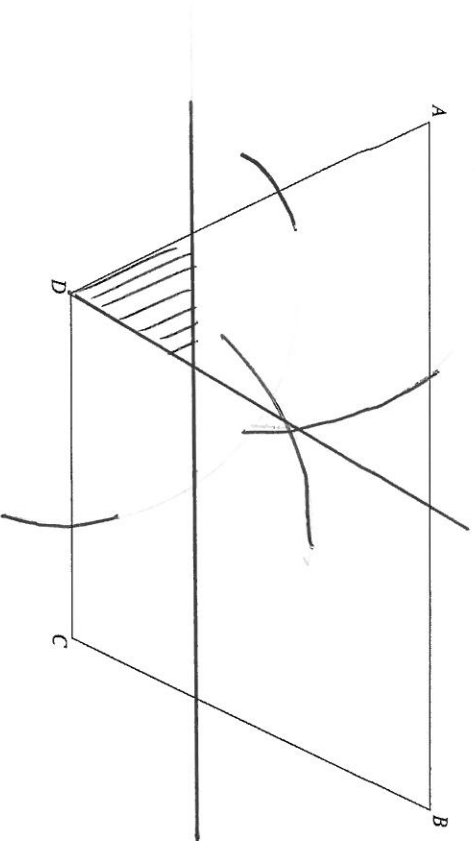
$$134.5 + 23.5 = 158 \text{ mm}$$

[1]

11. Find and shade the region inside the trapezium $ABCD$ that satisfy **both** of the following conditions.

- (i) The points are nearer to AD than to DC .
- (ii) The points are further than 3 cm from the line AB .

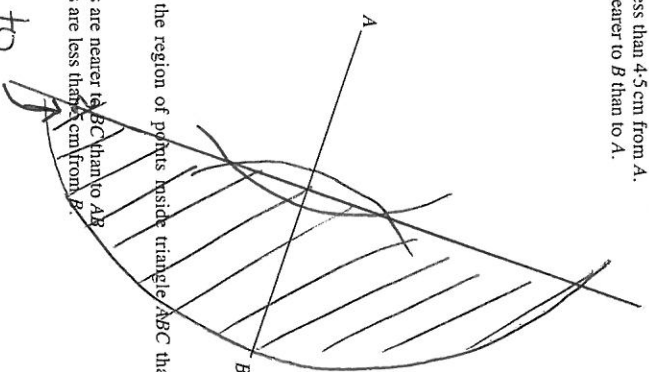
[3]



7. Shade the region that satisfies both of the following conditions.

- (i) The points are less than 4.5 cm from A .
- (ii) The points are nearer to B than to A .

[3]

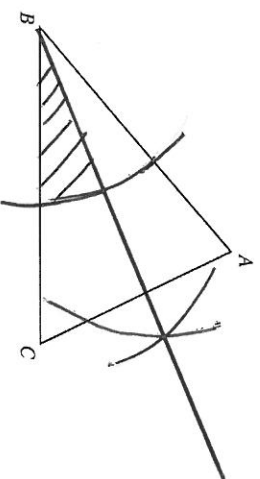


5. (b) Find and shade the region of points inside triangle ABC that satisfy both the following conditions.

- (i) The points are nearer to BC than to AB .
- (ii) The points are less than 2.5 cm from B .

[3]

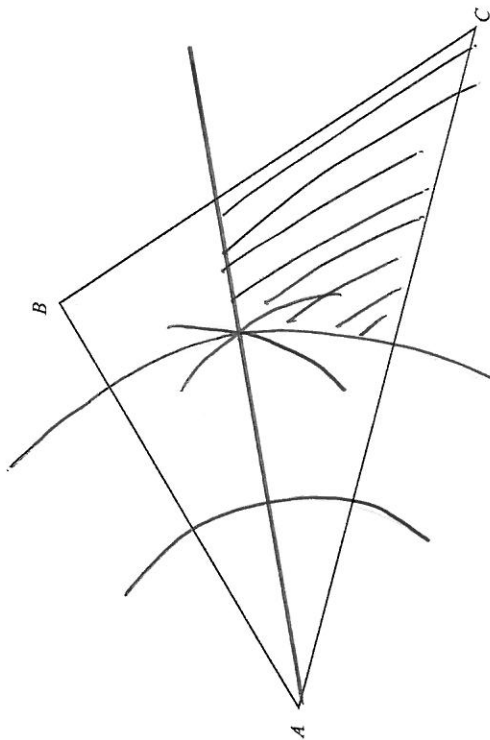
Changed to 2.5



7. Find and shade the region of points inside triangle ABC that satisfy both of the following conditions.

- (i) The points are nearer to AC than to AB .
- (ii) The points are greater than 5 cm from A .

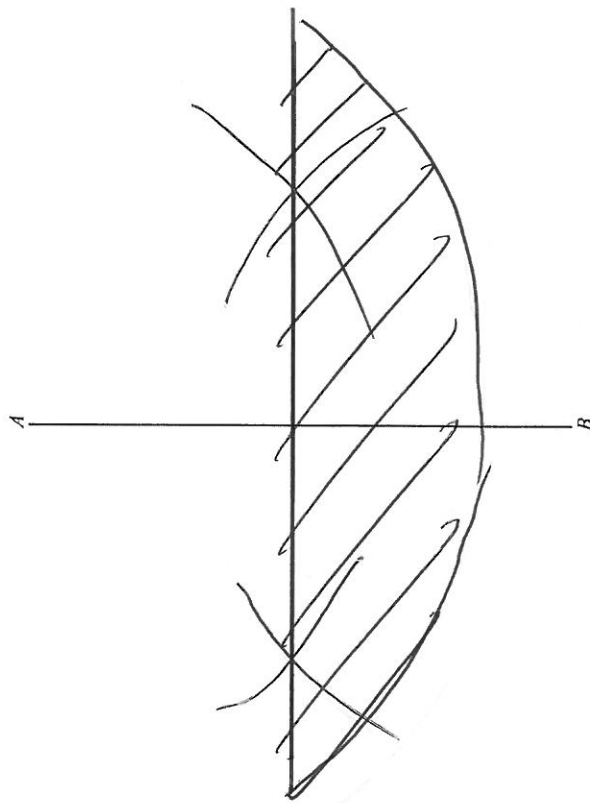
[3]



6. Find and shade the region of points that satisfy both of the following conditions.

- The points are nearer to B than to A .
- The points are less than 6 cm from A .

[3]



6. (a)

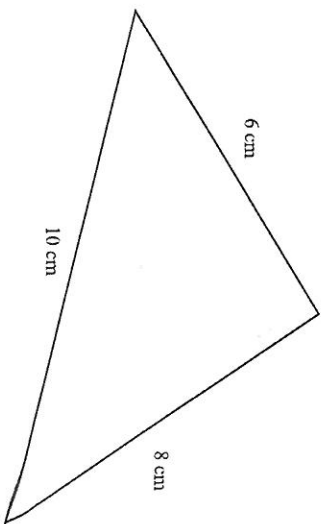


Diagram not drawn to scale

Show, by calculation, that the triangle drawn above is a right-angled triangle.

$$6^2 + 8^2 = 36 + 64 = 100$$

$$\sqrt{100} = 10$$

So pythagoras works. So right angled triangle

[2]

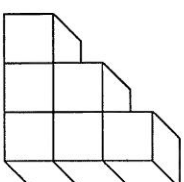
(b) Calculate the area of a semicircle with a radius of 20 cm using $\pi = 3.14$.

$$\text{Circle area} = \pi \times 20^2 = 3.14 \times 400$$

$$\text{Semi Circle} = 3.14 \times 200 = 628 \text{ cm}^2$$

[2]

1. This stairway is made with 6 cubes. It has 3 steps.



How many cubes would be needed to make a similar stairway with 9 steps?

$$3 \text{ steps} = 1 + 2 + 3$$

$$9 \text{ steps} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 \text{ steps}$$

[3]

5. (a) Complete the following table by placing a tick (✓) in any box where the given statement is true.

Statement	Square	Parallelogram	Trapezium
The diagonals are equal in length	✓		
Opposite angles are equal	✓	✓	
Only one pair of opposite sides are parallel			✓
The diagonals are lines of symmetry	✓		

[3]

- (b) Explain why three lines of lengths 3 cm, 5 cm and 10 cm cannot be used to form a triangle.

$3 + 5 < 10$
So sides wouldn't meet.

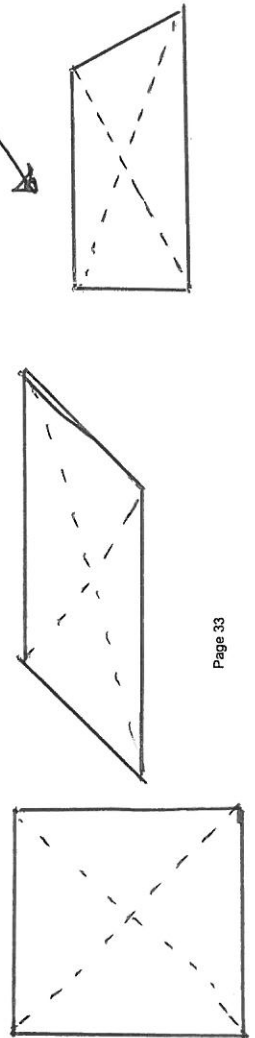
[1]

- (c) Two exterior angles of a triangle are 150° and 110° . Calculate the size of the third exterior angle of the triangle.

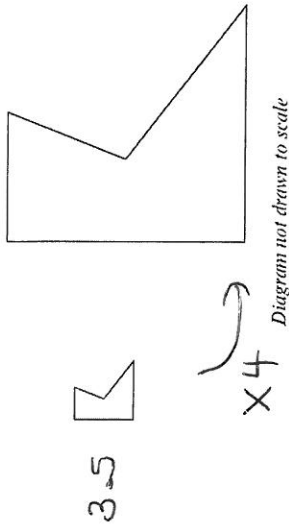
$$150 + 110 = 260^\circ$$

$$3rd = 100^\circ \quad (\text{sum} = 360^\circ)$$

[3]



- (b) The diagram shows two similar shapes.



Each length on the larger shape is four times the corresponding length on the smaller shape. The area of the smaller shape is 3.5 cm^2 . Find the area of the larger shape.

$$Sf = 4$$

$$\text{Area } Sf = 4^2 = 16$$

$$3.5 \times 16 =$$

Area of larger shape 56 cm^2

[3]

16. In the diagram, AB is parallel to DE .

(a) Show that triangles ABC and EDC are similar.

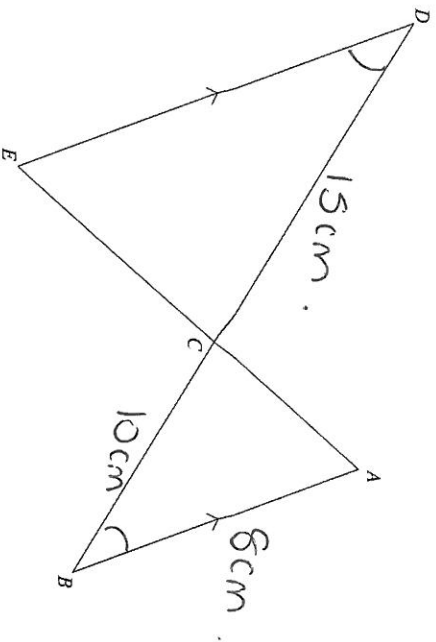


Diagram not drawn to scale.

$\hat{CDE} = \hat{ABC}$
 $\hat{CED} = \hat{ACB}$ all angles equal.
 $\hat{BAC} = \hat{CED}$

[2]

(b) The lengths $AB = 8\text{ cm}$, $BC = 10\text{ cm}$ and $DC = 15\text{ cm}$.
 Calculate the length of DE .

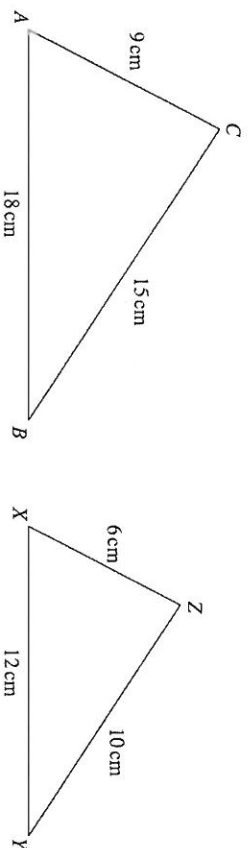
$$\frac{8}{10} = \frac{15}{DE} - 1.5$$

$$DE = 8 \times 1.5$$

$$= 12\text{ cm}$$

[2]

16. (a) Explain clearly why triangles ABC and XYZ are similar.



Diagrams not drawn to scale.

$$\frac{18}{12} = 1.5$$

$$\frac{9}{6} = 1.5$$

$$\frac{15}{10} = 1.5$$

all sides sf 1.5

[2]

(b) Triangle PQR , in which $PQ = 15\text{ cm}$, is similar to both triangles ABC and XYZ . Calculate the length of QR .

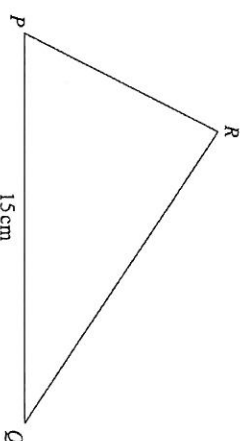


Diagram not drawn to scale.

$$\frac{15}{12} = 1.25$$

$$RQ = 10 \times 1.25 = 12.5\text{ cm}$$

[2]

12. The diagram shows two similar triangles, ABC and PQR .

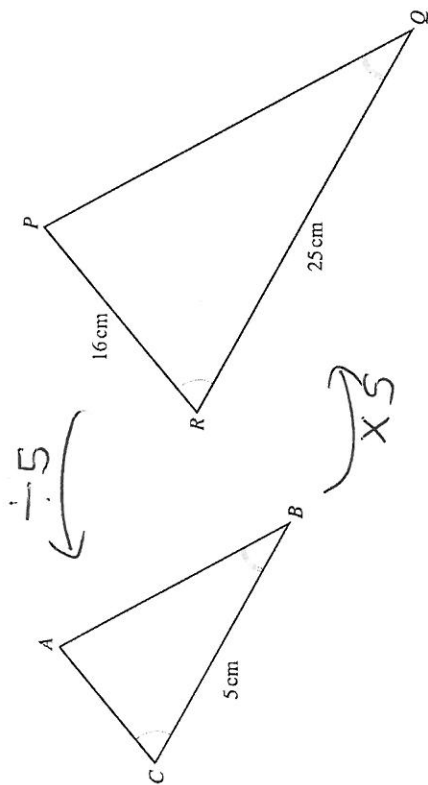


Diagram not drawn to scale.

Given that $CB = 5$ cm, $RQ = 25$ cm and $PR = 16$ cm, find the length of AC .

$$AC = \frac{16}{5} = 3\frac{1}{5} = 3.2 \text{ cm}$$

[2]

11. (a) The diagram shows two similar triangles.

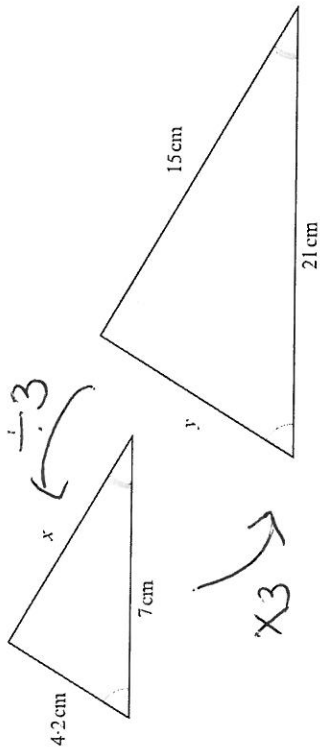


Diagram not drawn to scale

Calculate the lengths of the sides marked x and y .

$$5x = \frac{21}{3} = 7$$

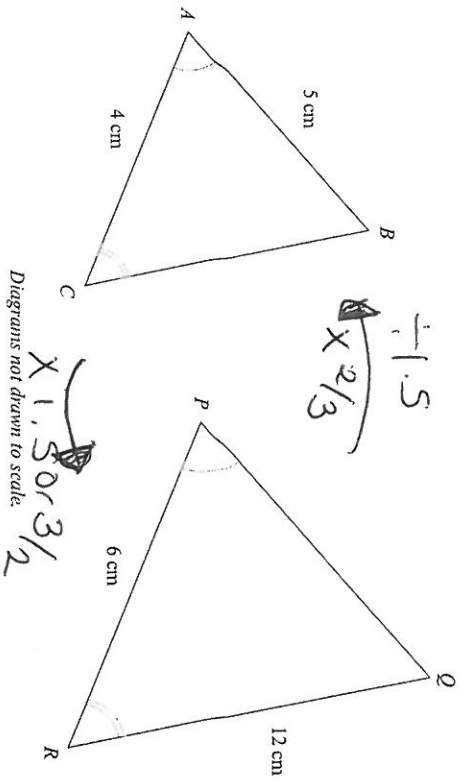
$$x = \frac{7}{5} = 1.4$$

$$y = 4.2 \times 3 = 12.6$$

$$x = 1.4 \text{ cm} \quad y = 12.6 \text{ cm}$$

[4]

14. Triangles ABC and PQR are similar, with $\hat{BAC} = \hat{QPR}$, $\hat{BCA} = \hat{QRP}$, $AB = 5$ cm, $AC = 4$ cm, $PR = 6$ cm and $RQ = 12$ cm.



Showing all your working, find

- (a) the length of QP ,

$$\frac{6}{4} = 1.5$$

$$QP = 5 \times 1.5 = 7.5 \text{ cm.}$$

- (b) the length of BC .

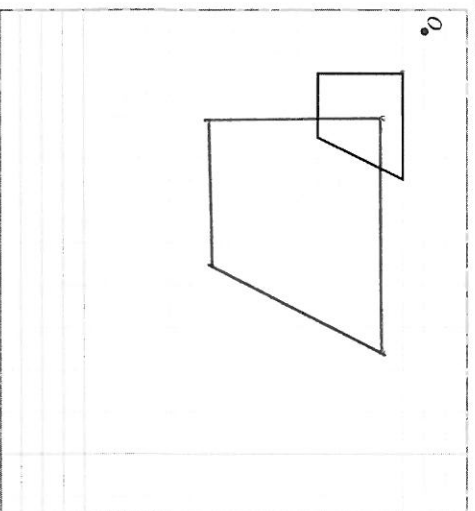
$$12 \div 1.5 = 8$$

$$8 \times \frac{2}{3} = 5.33$$

[1]

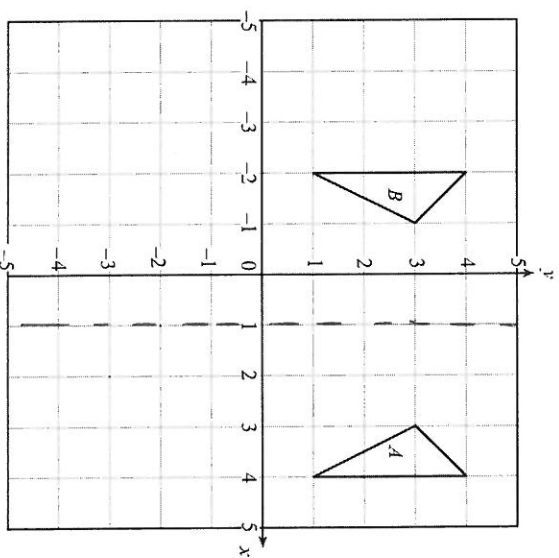
[2]

2. (a) On the grid below, draw the enlargement of the given shape using a scale factor of 2 and centre O .



[3]

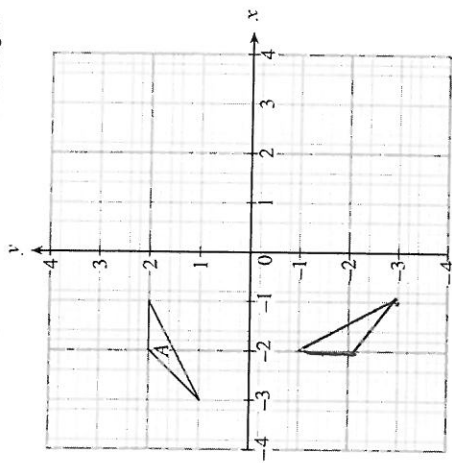
- (b) Describe fully the transformation that transforms triangle A into triangle B .



Reflect in $x = 1$.

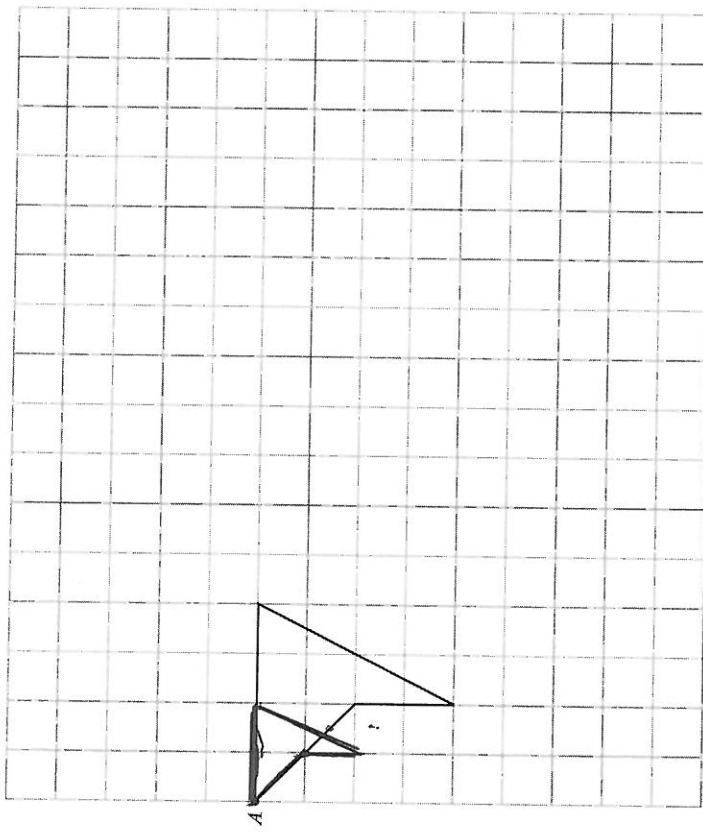
[2]

- (c) Rotate the triangle A through 90° anticlockwise about the origin.



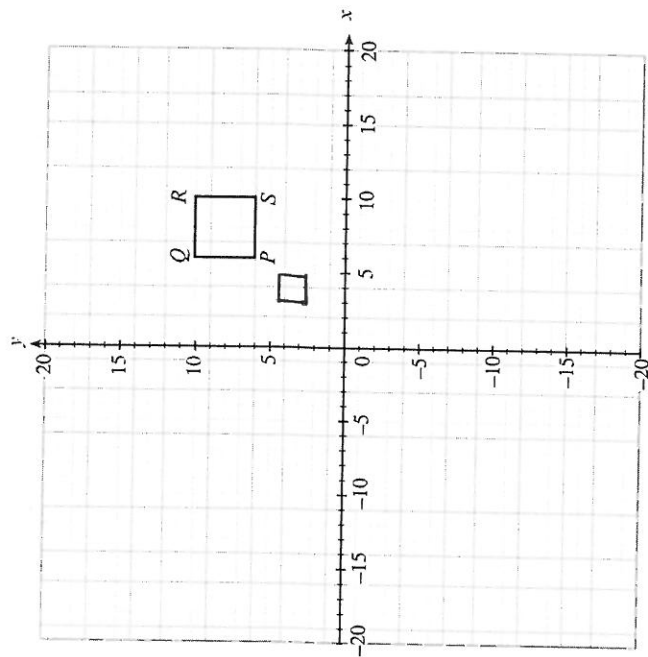
[2]

2. (a) Enlarge the shape shown on the grid by a scale factor of 2 using A as the centre of the enlargement.

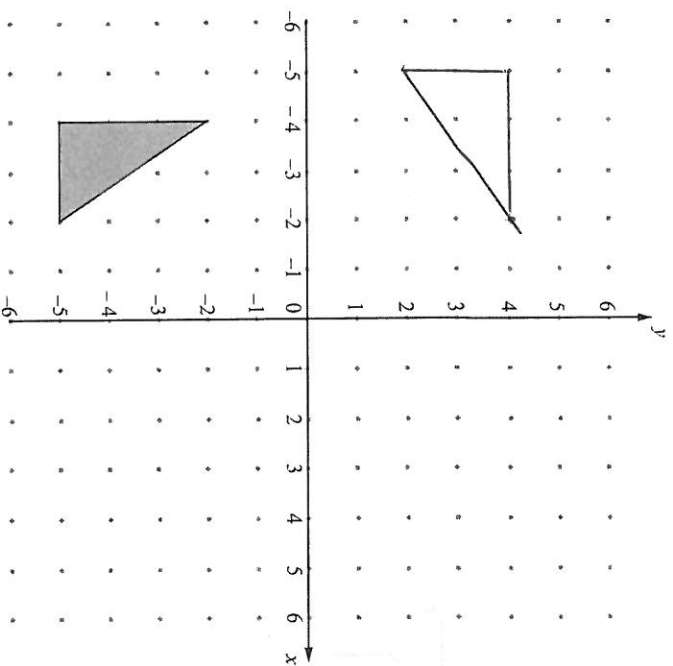


[3]

- (d) Enlarge the square $PQRS$ using centre $(0, 0)$ by a scale factor of $\frac{1}{2}$.

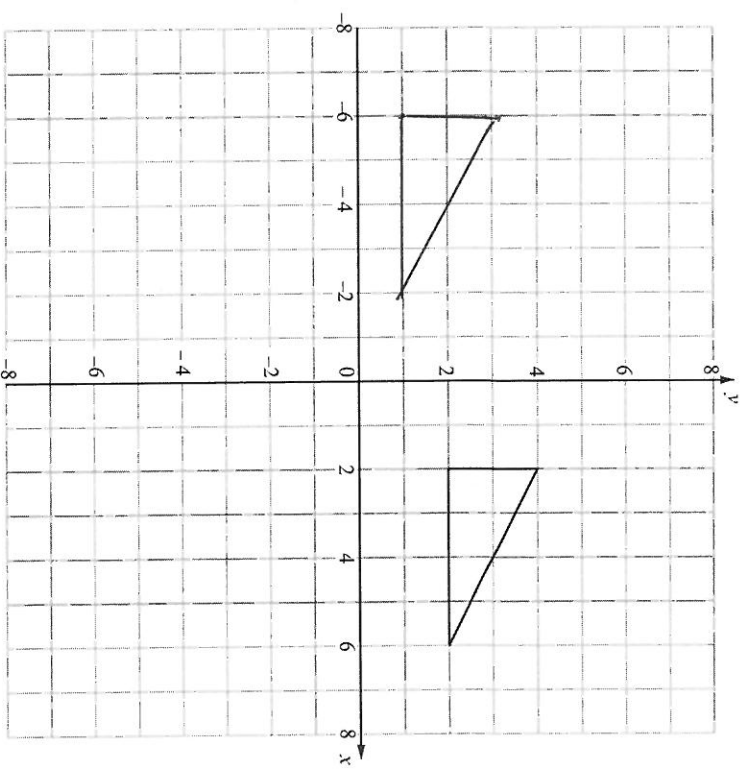


(b) Rotate the triangle shown through 90° clockwise about the origin.



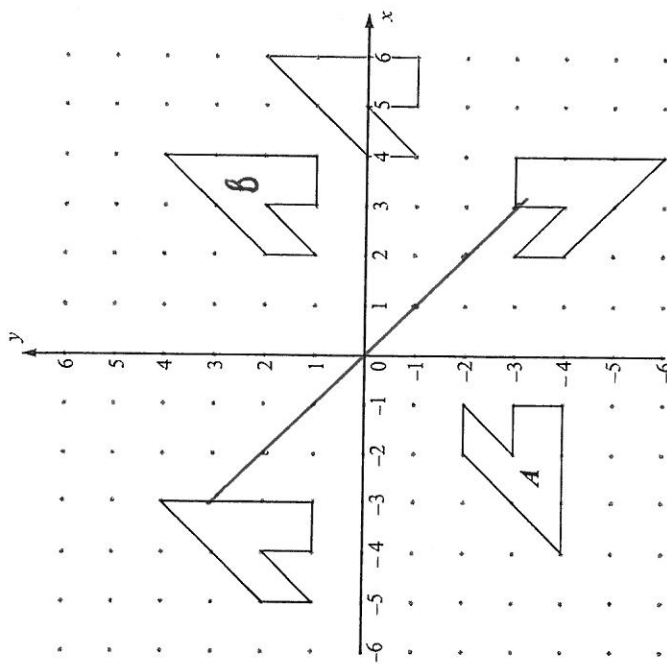
[2]

(c) Translate the triangle shown by 8 units to the left and 1 unit down.



[1]

(d)



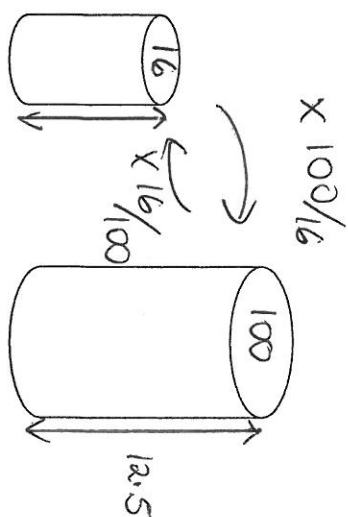
Indicate with the letter *B* on the diagram which one of the shapes shown may be obtained by reflecting shape *A* in the line $y = -x$.

[1]

10. A cuboid with a volume of 912 cm^3 has dimensions 4 cm , $(x + 2) \text{ cm}$ and $(x + 9) \text{ cm}$. Write down an equation in terms of x . Hence, solve the equation to find the dimensions of the cuboid.

$$\begin{aligned} \text{Volume} &\Rightarrow 4(x+2)(x+9) = 912 \\ (x+2)(x+9) &= 228 \\ x^2 + 2x + 9x + 18 &= 228 \\ x^2 + 11x + 18 &= 228 \\ -x^2 + 11x - 210 &= 0 \\ (x+21)(x-10) &= 0 \\ \underline{x = -21} \quad \text{as } x \neq -21 \\ \hline \begin{array}{r} 228 \\ 4 \overline{) 912} \\ \underline{88} \\ 32 \\ \underline{32} \\ 0 \end{array} & \quad \begin{array}{|l} x-210 \\ +11 \\ \hline x-10 \end{array} \end{aligned}$$

19. The diagram shows two similar cylinders.



Diagrams not drawn to scale.

The areas of the ends of the smaller and larger cylinders are 16 cm^2 and 100 cm^2 respectively. Given that the height of the larger cylinder is 12.5 cm , find the height of the smaller cylinder.

Area of Circle increased $100 = 16$

Small circle $16/100$ of large

Area scale factor $= 16/100$

Scale factor $= \sqrt{16/100} = 4/10$

$$12.5 \times 4/10 = 1.25 \times 4 = 5 \text{ cm} \quad [3]$$

17. A cone, with base of radius $x \text{ cm}$ and height $h \text{ cm}$, has the same volume as a cuboid with length $x \text{ cm}$, width $(x-3) \text{ cm}$ and height $(x+3) \text{ cm}$. Find an expression for h in terms of x and π in its simplified form.



$$\text{Volume} = \frac{1}{3} \pi x^2 h$$

$$\text{Volume} = (x+3)(x-3)x = (x^2-9)x = x^3-9x^2$$

$$\frac{1}{3} \pi x^2 h = x^3 - 9x^2$$

$$\frac{1}{3} \pi x^2 h = x^2(x-9)$$

$$\frac{1}{3} \pi h = x-9$$

$$\pi h = 3x-27$$

$$h = \frac{3x-27}{\pi}$$

[5]