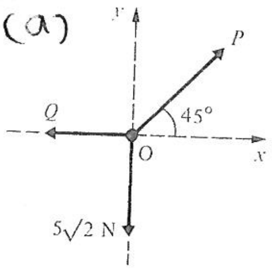
Statics

We have already seen how a system of forces acting on a particle in be resolved into the sums of vertical and horizontal (or parallel and perpendicular) forces. If the particle is to move, then these forces are resolved into one equivalent (resultant) force.

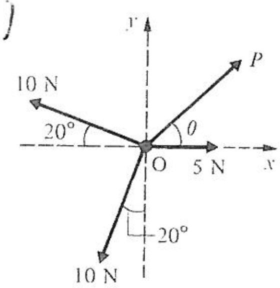
We will now consider what happens if the sum of these forces is zero. This will mean that the resultant force is zero and the particle remains at rest. In such a state, the particle is said to be in ***equilibrium***.

Eg1 Each of the diagrams below show a particle in equilibrium under the forces shown. In each case, by resolving in the directions Ox and Oy, find the unknown forces and angles:

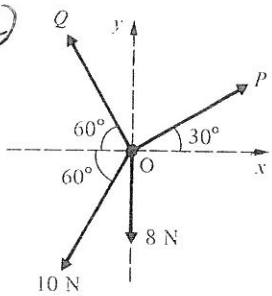
(a)



(b)



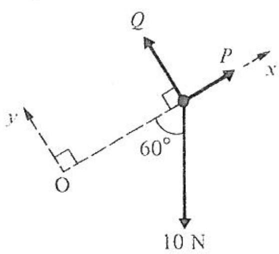
(c)



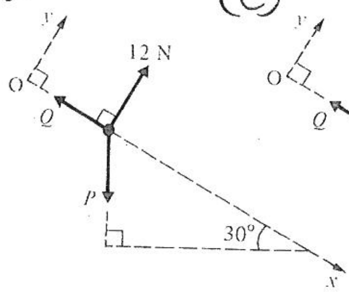
Exercise 4A Pg 95 Q's 1 to 9 Odds

Eg2 Each of the diagrams below show a particle in equilibrium under the forces shown. In each case, by resolving in the directions Ox and Oy, find the unknown forces and angles:

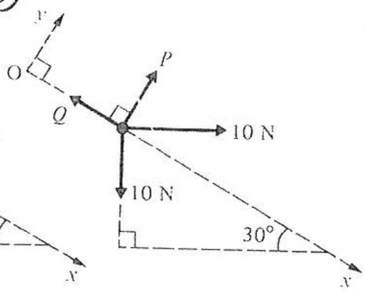
(a)



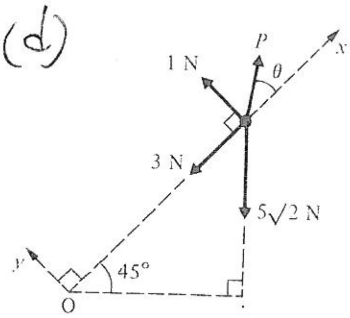
(b)



(c)

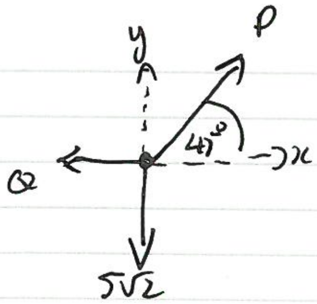


(d)



Exercise 4A Pg 96 Q's 11 to 15

Eg 1 (a)



$\Sigma F_x: \quad P\cos 45 - Q = 0 \quad —①$

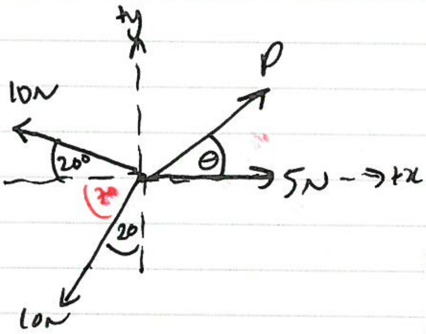
$\Sigma F_y: \quad P\sin 45 - 5\sqrt{2} = 0 \quad —②$

$\frac{P}{\sqrt{2}} = 5\sqrt{2}$

$P = 10\ N$

in ① $\quad Q = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}\ N$

(b)



$\Sigma F_x: \quad P\cos\theta + 5 - 10\cos 20 - 10\cos 70 = 0 \quad —①$

$\Sigma F_y: \quad P\sin\theta + 10\sin 20 - 10\sin 70 = 0 \quad —②$

From ① $\quad P\cos\theta = 10\cos 20 + 10\cos 70 - 5 \quad —③$

From ② $\quad P\sin\theta = 10\sin 70 - 10\sin 20 \quad —④$

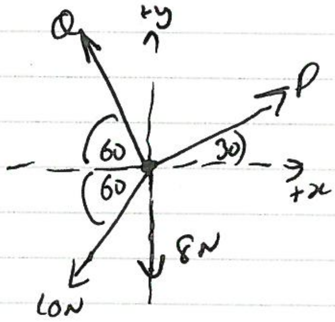
④ ÷ ③ $\quad \frac{P\sin\theta}{P\cos\theta} = \tan\theta = \frac{10\sin 70 - 10\sin 20}{10\cos 20 + 10\cos 70 - 5}$

$\tan\theta = 0.764\ldots$

$\theta = 37.4°$

in ④ $\quad P = \frac{10\sin 70 - 10\sin 20}{\sin 37.4} = 9.84N$

(c)



$\Sigma F_x: \quad P\cos 30 - Q\cos 60 - 10\cos 60 = 0 \quad —①$

$\Sigma F_y: \quad P\sin 30 + Q\sin 60 - 10\sin 60 - 8 = 0 \quad —②$

From ① $\quad \frac{P\sqrt{3}}{2} - \frac{Q}{2} - \frac{10}{2} = 0$

$Q = P\sqrt{3} - 10 \quad —③$

in ② $\quad \frac{P}{2} + (P\sqrt{3} - 10)\cdot\frac{\sqrt{3}}{2} - \frac{10\cdot\sqrt{3}}{2} - 8 = 0$

$\times 2 \quad P + 3P - 10\sqrt{3} - 10\sqrt{3} = 16$

$4P = 16 + 20\sqrt{3}$

$P = 4 + 5\sqrt{3} = 12.66N$

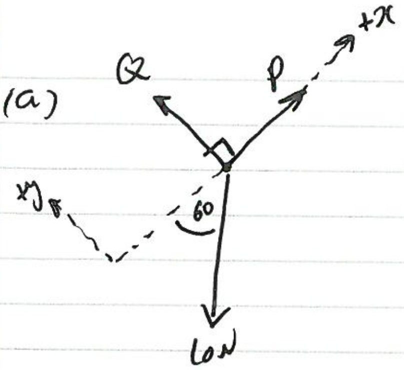
in ③ $\quad Q = (4 + 5\sqrt{3})\sqrt{3} - 10$

$Q = 4\sqrt{3} + 15 - 10$

$Q = 4\sqrt{3} + 5\ N$

$= 11.9N$

Q2 (a)



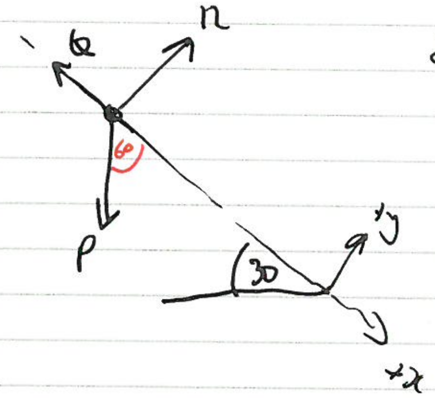
$\Sigma F_x: \quad P - 10\cos 60 = 0$

$P = 5N$

$\Sigma F_y: \quad Q - 10\sin 60 = 0$

$Q = 5\sqrt{3}\ N$

(b)



$\Sigma F_x: \quad P\cos 60 - Q = 0$

$Q = \frac{P}{2}$ — (1)

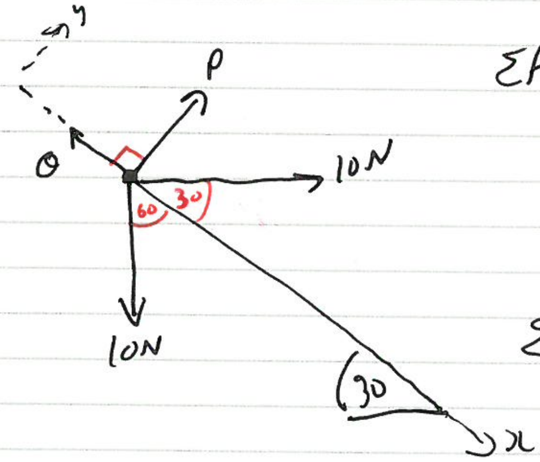
$\Sigma F_y: \quad 12 - P\sin 60 = 0$

$12 = \frac{P\sqrt{3}}{2}$

$P = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3}\ N$

in (1) $\quad Q = 4\sqrt{3}\ N$

(c)



$\Sigma F_x: \quad 10\cos 30 + 10\cos 60 - Q = 0$

$Q = \frac{10\sqrt{3}}{2} + \frac{10}{2} = 5\sqrt{3} + 5\ N$

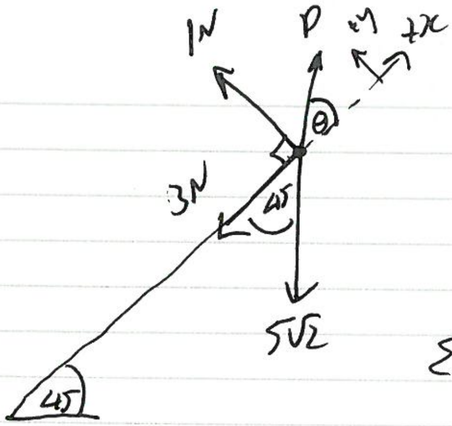
$= 13.66\ N$

$\Sigma F_y: \quad P + 10\sin 30 - 10\sin 60 = 0$

$P = \frac{10\sqrt{3}}{2} - \frac{10}{2} = 5\sqrt{3} - 5\ N$

$= 3.66\ N$

Q2 (d)



ΣF_x: $P\cos\theta - 3 - 5\sqrt{2}\cos 45 = 0$

$P\cos\theta = 3 + 5\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 8$ — ①

ΣF_y: $P\sin\theta + 1 - 5\sqrt{2}\sin 45 = 0$

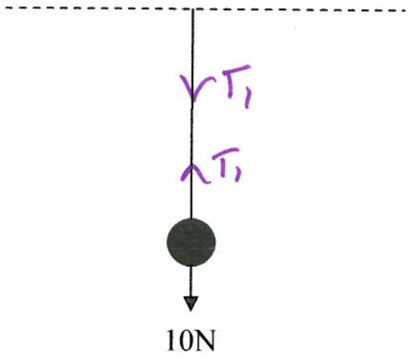
$P\sin\theta = 4$ — ②

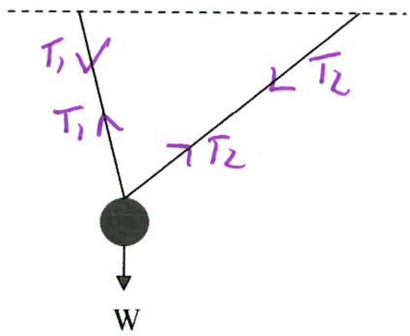
② ÷ ① $\tan\theta = \frac{1}{2}$

$\theta = \underline{26.6^\circ}$

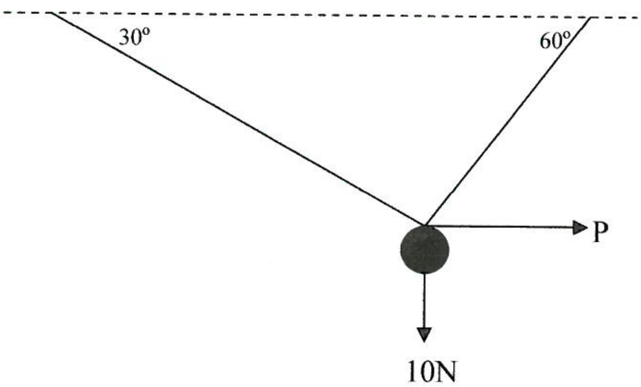
in ② $P = \frac{4}{\sin 26.6} = \underline{8.94}\,N$

Each part of a system in equilibrium, is in equilibrium, ie

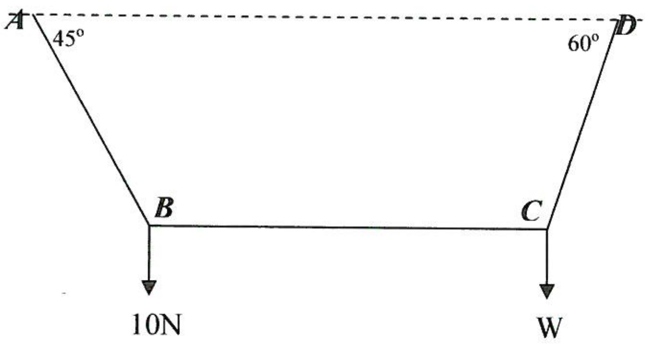




Eg3 A string is tied to two points on the same level and a ***smooth ring*** of weight 10N which can slide freely along the string is pulled by a horizontal force, P. For the position of equilibrium shown in the diagram, find P and the tension in the string.

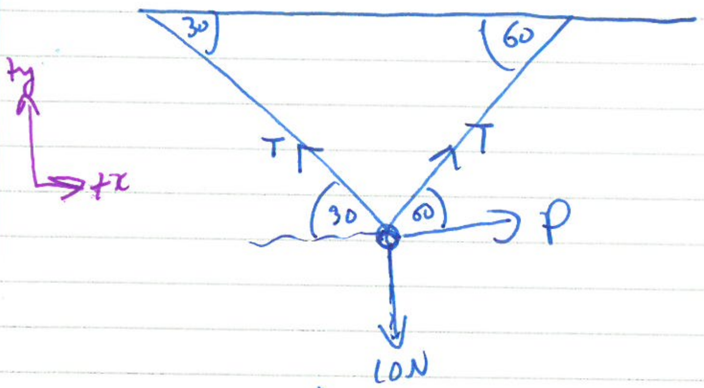


Eg4 $ABCD$ is a string knotted at B and C. Find W and the tensions in AB, BC and CD.



Exercise 4B Pg 101 Evens

Eg3,



Smooth ring
∴ tensions equal
throughout string.

System in equilibrium ∴ $\Sigma F_x = 0$
$\Sigma F_y = 0$

ΣF_x: $P + T\cos 60 - T\cos 30 = 0$ — (1)

ΣF_y: $T\sin 60 + T\sin 30 - 10 = 0$ — (2)

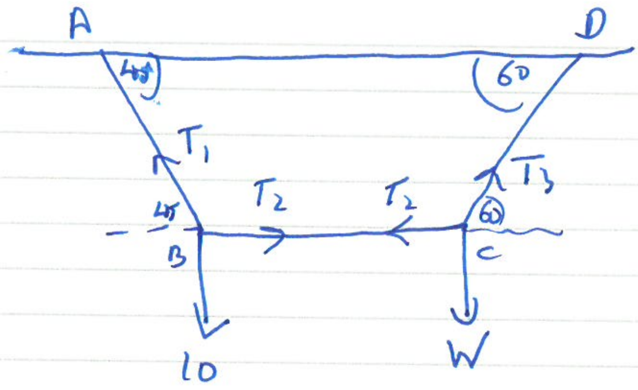
From (2) $\frac{T\sqrt{3}}{2} + \frac{T}{2} = 10$

$T(\sqrt{3}+1) = 20$

$T = \frac{20}{\sqrt{3}+1}$ N (7.32N)

In (1) $P = \frac{T\sqrt{3}}{2} - \frac{T}{2} = T\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right] = 2.68\text{N}$.

Eg4



Forces @ B : Σf_x: $T_2 - T_1 \cos 45 = 0$ — (1)

Σf_y: $T_1 \sin 45 - 10 = 0$ — (2)

$$\frac{T_1}{\sqrt{2}} = 10$$

$$T_1 = 10\sqrt{2} \ N$$

u (1) $\quad T_2 = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10 N$

Forces @ C : Σf_x : $T_3 \cos 60 - T_2 = 0$

$$T_3 . \frac{1}{2} = 10$$

$$T_3 = 20 \ N$$

Σf_y: $T_3 \sin 60 - W = 0$

$$20 . \frac{\sqrt{3}}{2} = W$$

$$W = 10\sqrt{3} \ N$$

Limiting Equilibrium

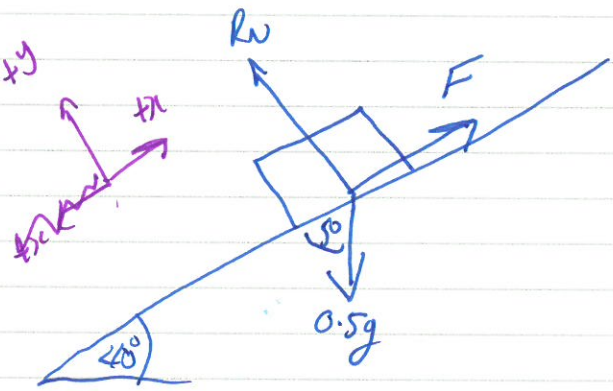
We can use the relationship $F_{\max} = \mu N_R$ to determine whether or not an object will remain in equilibrium when acted on by a system of forces whilst in contact with a rough surface.

Eg5 A body of mass 500g is placed on a rough plane which is inclined at 40^{o} to the horizontal. If the coefficient of friction between the body and the plane is 0.6, find the frictional force acting and state whether motion will occur.

Eg6 A body of mass 2kg lies on a rough plane which is inclined at 30^{o} to the horizontal. When a horizontal force of 20N is applied to the body in an attempt to push it up the plane, the body is found to be on the point of moving up the plane, ie in limiting equilibrium. Find the coefficient of friction between the body and the plane.

Exercise 4C Pg 109 Q's 6 to 12 evens

Eg5



If body moves, it will be <u>down</u> plane $\therefore$ F acts up the plane

~~Consider body~~

Body will move if $0.5g\cos 50 > F_{max}$ — ①

$$F_{max} = 0.6N_R$$

$$N_R - 0.5g\sin 50 = 0$$

$$N_R = 0.5g\sin 50$$

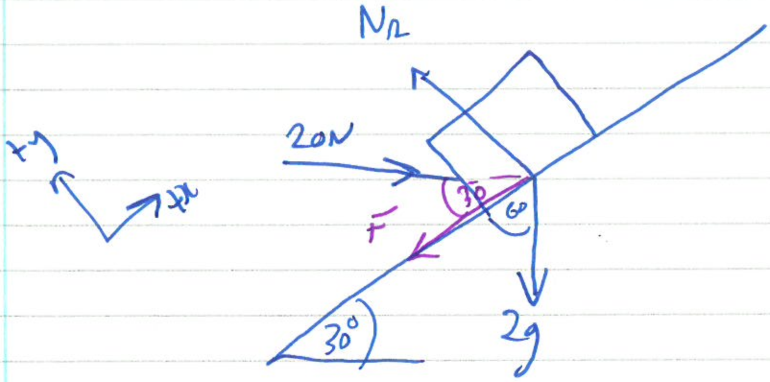
$$\therefore \quad F_{max} = 0.6(0.5g\sin 50) = \cancel{2.9}\ 0.3g\sin 50 = 2.25N$$

So ~~sub~~ in ①, body will slide if $0.5g\cos 50 > 0.3g\sin 50$

$$3.15 > 2.25$$

$\therefore$ yes the body will slide

Eg6



body on point of moving up plane $\therefore F_{MAX}$ down plane

Σf_x: $20\cos 30 - 2g\cos 60 - F_{MAX} = 0$

$F_{MAX} = \cancel{g - 10\sqrt{3}}\ 10\sqrt{3} - g$

Σf_y: $N_R - 20\sin 30 - 2g\sin 60 = 0$

$N_R = 10 + g\sqrt{3}$

$F_{MAX} = \mu R_N$

$\mu = \cancel{\dfrac{g - 10\sqrt{3}}{10 + g\sqrt{3}}} = \dfrac{10\sqrt{3} - g}{10 + g\sqrt{3}} = 0.279$