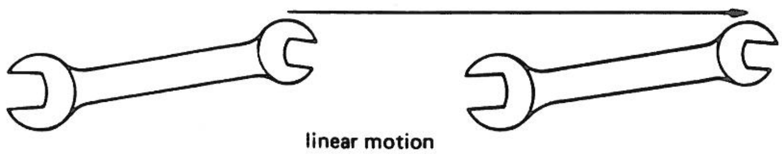
Mechanics - an introduction

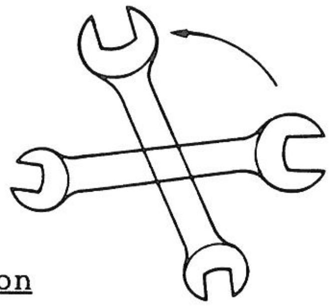
Mechanics is concerned with the motion of objects. There are three different kinds of motion:

1. Translation or linear motion, ie motion from one place to another, while keeping the same shape and orientation. (Note that linear motion doesn't necessarily mean motion in a straight line.)



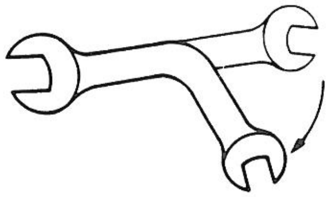
linear motion

2. Rotation or Angular motion



angular motion

3. Change of shape or elastic motion



elastic motion

Most objects in the real world move in a complicated way which is a combination of all these three types. We want to be able to analyse this motion mathematically. Since it is so complicated, we need to make some simplifications before we can start. We first simplify the objects we are going to consider.

Rigid Body: If an object is not very elastic, we can think of it as being a rigid body. A rigid body is an object which never changes its shape.

There is no such thing as a rigid body in the real world; every object in the real world is to some extent elastic, even a block of concrete. A rigid body is a mathematical simplification, or mathematical model, of a fairly inelastic object.

A rigid body is capable of only two different kinds of motion: translation and rotation.

Particle: If an object is not very elastic and not very big, compared to the distances being considered, we can think of it as being a particle. For example, if we consider a football in flight across a pitch we can sensibly think of the ball as a particle, but if we consider the motion of a spider crawling over the football, then we need to think of the ball as an object of finite size. A particle is an infinitely small object, a point body.

A particle is capable of only one kind of motion: translation.

Mechanics can be divided into three fields:

1. <u>Statics</u> - The study of forces which keep objects at rest (in equilibrium).

2. <u>Dynamics</u> - The study of moving objects resulting from the forces applied to it.

3. <u>Kinematics</u> - The study of the relationships between displacement, velocity, acceleration and time of a moving object.

Constant Acceleration Formulae

When the motion of a body is being considered, the letters u, v, a, t and s usually have the following meanings:

u = initial velocity
a = acceleration
s = displacement
v = final velocity
t = time interval or time taken

Consider a car travelling in a straight line. If initially its velocity is $5ms^{-1}$ and 3s later its velocity is $11ms^{-1}$, the car is said to be accelerating.

Acceleration is a measure of the rate at which velocity is changing. In this example, the velocity increases by $6ms^{-1}$ in 3s. If the acceleration is assumed to be uniform, or constant, then it is $6ms^{-1}$ in 3 seconds, or $2ms^{-1}$ each second which is written $2ms^{-2}$.

Derivation of the Constant Acceleration Formulae

In general, $$\text{acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

If the acceleration is uniform, then the average velocity is the average of the initial and final velocities,

By eliminating firstly v, and then t from these equations, a further two formulae can be derived:

These four formulae are important and need to be **memorised**. Remember also, that they can only be applied to situations involving ***constant*** acceleration.

Distance and Displacement

In the above formulae, s represents displacement. In practice s is also used to denote distance because distance and displacement are often equal. There need be no confusion provided that care is taken in any particular question.

When the direction of motion of a body remains unchanged, then the distance travelled and the displacement are equal.

If the direction of motion changes part way through the motion, then the distance travelled and the displacement will not be equal.

eg1 A body moves along a straight line from A to B with uniform acceleration $^2/_3 ms^{-2}$. The time taken is 12s and the velocity at B is $25ms^{-1}$. Find
(a) the velocity at A,
(b) the distance AB.

eg2 A cyclist travelling downhill accelerates uniformly at $1.5ms^{-2}$. If his initial velocity at the top of the hill is $3ms^{-1}$, find
(a) how far he travels in 8 seconds,
(b) how far he travels before reaching a velocity of $7ms^{-1}$.

eg3 A stone slides in a straight line across a horizontal sheet of ice. It passes a point A with a velocity of $14ms^{-1}$, and a point B 2.5 seconds later. Assuming the retardation is uniform and that AB = 30m, find
(a) the retardation,
(b) the velocity at B,
(c) how long after passing A the stone comes to rest.

Exercise 2A Page 9 Prime's

Exercise 2B Page 15 Odd's

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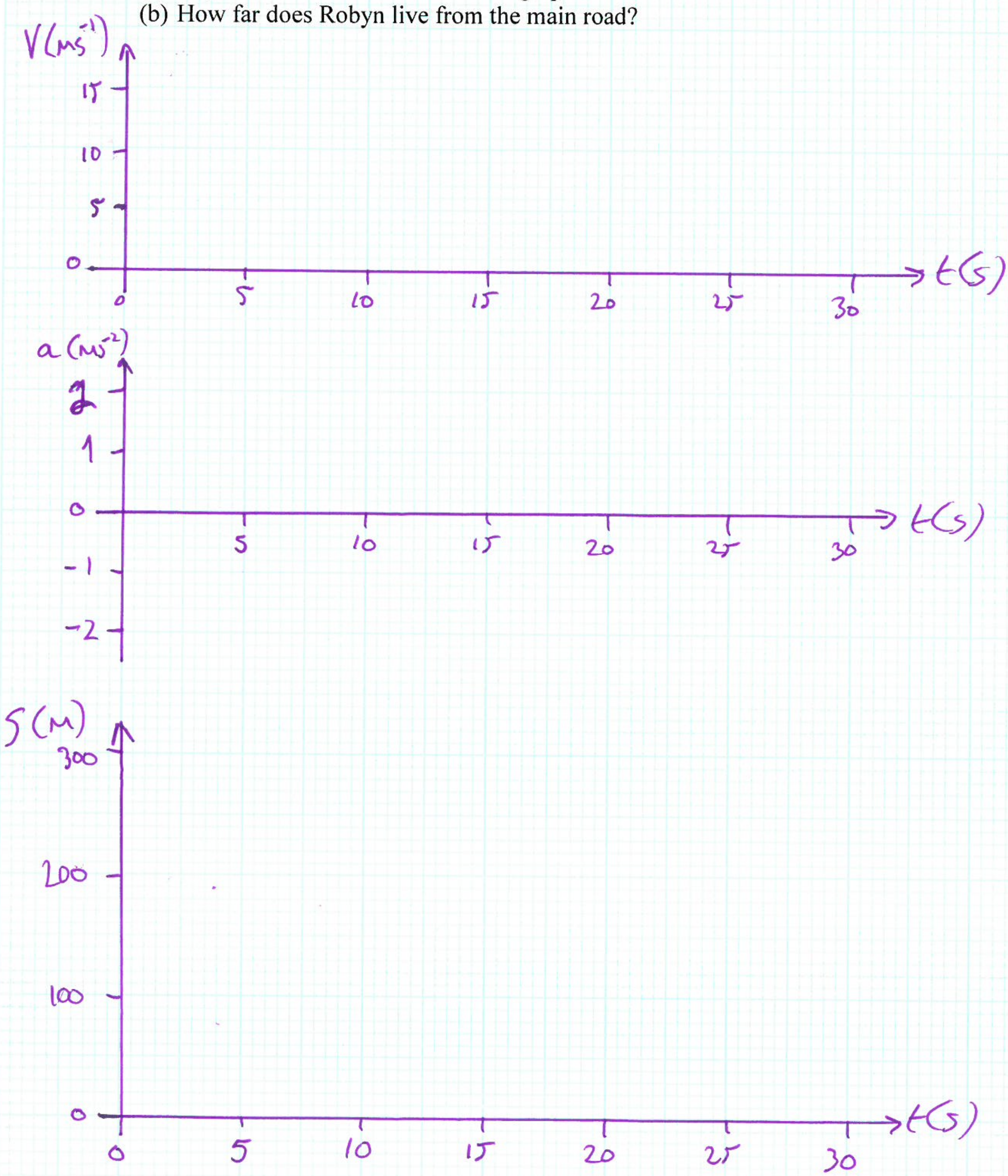
Representing Motion Graphically

Eg1 Robin is cycling home. He turns off the main road and $4ms^{-1}$ and accelerates uniformly to $10ms^{-1}$ over the next 6 seconds. He maintains this speed for 20 seconds and then slows uniformly for 4 seconds.
(a) Represent this information on the graphs below
(b) How far does Robyn live from the main road?



Exercise 2D Pg 30

Constant Acceleration Formulae

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Derivation of the Constant Acceleration Formulae

In general, $\text{acceleration} = \dfrac{\text{change in velocity}}{\text{time interval}}$

$$a = \frac{v-u}{t}$$

$$ta = v - u$$

$$v = u + at \quad \text{— (1)}$$

If the acceleration is uniform, then the average velocity is the average of the initial and final velocities,

average vel = $\dfrac{u+v}{2}$

but average vel = $\dfrac{\text{displacement}}{\text{time}}$

$= \dfrac{s}{t}$

equating $\dfrac{s}{t} = \dfrac{u+v}{2}$

$$s = \frac{1}{2}(u+v)t \quad \text{— (2)}$$

By eliminating firstly v, and then t from these equations, a further two formulae can be derived:

Sub (1) in (2) $s = \frac{1}{2}(u+u+at)t$

$s = \frac{1}{2}(2u+at)t$

$s = ut + \frac{1}{2}at^2 \quad \text{— (3)}$

From (1) $t = \dfrac{v-u}{a}$

in (2) $s = \frac{1}{2}(u+v)\left(\dfrac{v-u}{a}\right)$

$2as = (u+v)(v-u)$

$2as = uv - u^2 + v^2 - uv$

$2as = v^2 - u^2$

$v^2 = u^2 + 2as \quad \text{— (4)}$

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Exercise 3A Page 40 Odd's

$u = ?$ $V = 25$ $a = \frac{2}{3}$ $t = 12$ $s = ?$

(a) Using $V = u + at$

$25 = u + \frac{2}{3} \times 12$

$u = 25 - 8 = 17 \, ms^{-1}$

(b) $S = ut + \frac{1}{2}at^2$

$S = (17 \times 12) + \frac{1}{2} \times \frac{2}{3} \times (12)^2 = 204 + 48 = 252$ metres.

Q2 (a) $u = 3$ $t = 8$ $a = 1.5$ $V = ?$ $s = ?$

$S = ut + \frac{1}{2}at^2$

$S = (3 \times 8) + \frac{1}{2} \times 1.5 \times 8^2 = 24 + 48 = 72$ metres

(b) $V = 7$ $t = ?$

$V^2 = u^2 + 2as$

$7^2 = 3^2 + 2 \times 1.5 \, S$

$49 = 9 + 3S$

$S = 13\frac{1}{3}$ metres

Q3 (a) $u = 14$ $S = 30$ $t = 2.5$ $V = ?$ $a = ?$

$S = ut + \frac{1}{2}at^2$

$30 = (14 \times 2.5) + \frac{1}{2} \times a \times 2.5^2$

$30 - 35 = 3.125a$

$a = -1.6 \, ms^{-2}$

(b) $V = 14 + -1.6 \times 2.5$

$V = 10 \, ms^{-1}$

(c) $V = 0$ $0 = 14 + -1.6t$

$t = \frac{14}{1.6} = 8.75$ secs.

Ex2A

1) $a=3 \quad u=2 \quad t=6 \quad V=?$

$V = 2+3\times6 = 20 ms^{-1}$

2) $u=1.2 \quad V=7.6 \quad t=4 \quad a=?$

$7.6 = 1.2+4a$

$a = 1.6 ms^{-1}$

3) $u=10 \quad V=0 \quad a=? \quad t=16$

$0 = 10+16a$

$a = -\frac{5}{8} ms^{-2}$ ie decel of $5/8 ms^{-2}$

5) $s=360 \quad t=15 \quad V=28 \quad u=?$

$360 = \frac{(u+28)}{2}15$

$720 = 15u+420$

$u = 20 ms^{-1}$

4) $u=2.4 \quad V=8 \quad t=5 \quad s=?$

$s = \frac{(2.4+8)}{2}\times5 = 26m$

6) $s=120 \quad u=18 \quad t=10 \quad V=?$

$120 = \frac{(18+V)}{2}10$

$V = 24-18 = +6 ms^{-1}$

7) $a=0.5 \quad u=3 \quad t=12$

(a) $V = 3+0.5\times12 = 9 ms^{-1}$

(b) $s = \frac{(3+9)}{2}\times12 = 72m$

(8) $s=24$ $t=6$ $v=5$ $u=?$ $a=?$

(a) $24 = \frac{(5+u)}{2} \times 6$

$u = 8-5 = 3ms^{-1}$

(b) $5 = 3 + 6a$

$a = \frac{2}{6} = \frac{1}{3}\ ms^{-2}$

(9) $a=-1.2$ $t=6$ $v=2$ $u=?$ $s=?$

(a) $2 = u + (-1.2 \times 6)$

$u = 9.2ms^{-1}$

(b) $s = \frac{(9.2+2)}{2} \times 6 = 33.6m$

(10) $a=0.6$ $u = 72kmh^{-1} = \frac{72 \times 1000}{60 \times 60} = 20ms^{-1}$ $t=25$ $v=?$

(a) $v = 20 + (-0.6 \times 25) = 5ms^{-1} \times \frac{60 \times 60}{1000} = 18kmh^{-1}$

(b) $s = \frac{(20+5)}{2} \times 25 = 312.5m$

(11) $a=-4$ $u=32$ $v=0$ $t=?$ $s=?$

(a) $0 = 32 - 4t$

$t = 8$ sec

(b) $s = \frac{(32+0)}{2} \times 8 = 128m$

(12) $u=16$ $t=40$ $v=0$ $a=?$ $s=?$

(a) $0 = 16 + 40a$

$a = -0.4ms^{-2}$ ie decel $0.4ms^{-2}$

(b) $s = \frac{(16+0)}{2} \times 40 = 320m$

(13) From $A \to B$: $u=2$ $V=7$ $t=20$

(a) $7=2+20a$

$a=0.25\,ms^{-2}$

(b) From $B \to C$: $u=7$ $V=11$ $a=0.25$ $t=?$

$11=7+0.25t$

$t=16$ sec

(c) From $A \to C$ $u=2$ $V=11$ $a=0.25$ $t=36$ sec

$$S=\left(\frac{2+11}{2}\right)36 = 234\,m.$$

(14) From $A \to B$ $a=1.5$ $u=1$ $t=12$ $V=?$

(a) $V=1+(1.5\times 12) = 19\,ms^{-1}$

(b) From $B \to C$ $a=?$ $u=19$ $V=43$ $t=10$

$43=19+10a$

$a=2.4\,ms^{-2}$

(c) $S_{A \to C} = S_{AB}+S_{BC}$

$$S_{AB} = \left(\frac{1+19}{2}\right)\times 12 = 120\,m$$

$$S_{BC} = \left(\frac{19+43}{2}\right)\times 10 = 310$$

$\therefore$ $S_{AC} = 430\,m$

(15) (a) $a=x$ $u=0$ $V=5$ $t=20$

$5=0+20x$

$x=0.25\,ms^{-2}$

(b) 2nd part of journey ~~s~~ t $u=5$ $a=-\frac{1}{2}x=-0.125$ $V=0$ $t=?$

$0=5+-0.125\times t$ $t=40$ sec.

$\therefore$ total dist = dist 1st part + dist 2nd part

$$=\left(\frac{0+5}{2}\right)\times 20 + \left(\frac{5+0}{2}\right)\times 40 = 50+100 = 150\,m$$

(16) a) from $A \to B$ $u = 20$ $V = 30$ $a = a$ $t = t_1$

$$30 = 20 + at_1$$
$$\therefore at_1 = 10 \quad -(1)$$

from $B \to C$ $u = 30$ $V = 45$ $a = a$ $t = t_2$

$$45 = 30 + at_2$$
$$at_2 = 15 \quad -(2)$$

$(1) \div (2)$ $\frac{\cancel{a}t_1}{\cancel{a}t_2} = \frac{10}{15} \Rightarrow \frac{t_1}{t_2} = \frac{2}{3}$

(b) from $A \to C$ $u = 20$ $V = 45$ ~~$t = t_1 + t_2$~~ $t = 50$ $s = ?$

$$s = \frac{(20 + 45)}{2} \times 50 = 1625\,m$$

Ex2B

(1) $a=2.5 \quad u=3 \quad s=8 \quad v=?$

$v^2 = u^2 + 2as$
$v^2 = 3^2 + 2\times 2.5 \times 8$
$v^2 = 49$
$v = 7\,ms^{-1}$

(2) $a=? \quad u=8 \quad t=6 \quad s=60$

$60 = (8\times 6) + \frac{1}{2}a6^2$

$60 = 48 + 18a$
$a = \frac{2}{3}\,ms^{-2}$

(3) $u=12 \quad v=0 \quad s=36 \quad a=?$

$0^2 = 12^2 + 2\times 36 \times -a$

$72a = 144$
$a = 2\,ms^{-2}$

(4) $a=1.5 \quad t=4 \quad s=22 \quad v=?$

$22 = 4v - \frac{1}{2}(1.5)4^2$

$22 = 4v - 12$
$v = 8.5\,ms^{-1}$

(5) $a=2 \quad u=5.5 \quad s=20 \quad t=?$

$20 = 5.5t + \frac{1}{2}\times 2 \times t^2$

$20 = 5.5t + t^2$

$t^2 + 5.5t - 20 = 0$
$t=-8$
$t = 2.5$

$\therefore t = 2.5\,sec$

(6) $u = 54\,\text{kmh}^{-1} \times \frac{1000}{60\times 60} = 15\,\text{ms}^{-1}$

$V = 72\,\text{kmh}^{-1} \times \frac{1000}{60\times 60} = 20\,\text{ms}^{-1}$

$s = 500$

$a = ?$ $\quad V^2 = u^2 + 2as$

$20^2 = 15^2 + 2\times a \times 500$

$a = 0.175\,\text{ms}^{-2}$

(7) $u=4 \quad V=16 \quad s=48 \quad a=? \quad t=?$

(a) $16^2 = 4^2 + 2\times a \times 48$

$a = 2.5\,\text{ms}^{-2}$

(b) $16 = 4 + 2.5t$

$t = 4.8\,\text{s}.$

(8) $a=3 \quad s=38 \quad t=4 \quad u=? \quad V=?$

(a) $s = ut + \frac{1}{2}at^2$

$38 = 4u + \frac{1}{2}\times 3 \times 4^2$

$38 = 4u + 24$

$u = 3.5\,\text{ms}^{-1}$

(b) $V = 3.5 + (3\times 4) = 15.5\,\text{ms}^{-1}$

(9) $u = 18$ ~~$s = ?$~~ $a = -3$ $V = 0$ ~~a~~ $s = ?$ $t = ?$

(a) $\cancel{s = \frac{18+0}{2}}$ $V = u + at$

$0 = 18 - 3t$

$t = 6s$

$0^2 = 18^2 + 2 \times -3 \times s$

$6s = 18^2$

$s = 54m$

(10) $u = 12$ $a = -0.8$ $V = 0$ $s = ?$

(a) $0^2 = 12^2 + 2 \times -0.8 \times s$

$1.6s = 144$

$s = 90m$

(b) $s = 45$. $v = ?$

$V^2 = 12^2 + 2 \times -0.8 \times 45$

$V^2 = 144 - 72$

$V = \sqrt{72} = 8.5 ms^{-1}$

(11) $a = 2.5$ $u = 8$ $s = 40$ $t = ?$ $V = ?$

a) $40 = 8t + \frac{1}{2} \times 2.5 \times t^2$

$40 = 8t + 1.25t^2$

$1.25t^2 + 8t - 40 = 0$

$t = 3.3$ sec

(b) $V = 8 + (2.5 \times 3.3) = 16.2 ms^{-1}$

(12) (a) $a = -2 \quad s = 32 \quad u = 12 \quad t = ?$

$$32 = 12t + \frac{1}{2} \times -2t^2$$

$$t^2 - 12t + 32 = 0$$

$$t = 4 \text{ or } t = 8$$

(b) when $t = 4$ $\quad V = 12 + (-2 \times 4) = +4\,ms^{-1}$ ie $4\,ms^{-1}$ in direction $A \to B$

when $t = 8$: $\quad V = 12 + (-2 \times 8) = -4\,ms^{-1}$ ie $4\,ms^{-1}$ in direction $\overrightarrow{BA}$

(13) (a) $a = -5 \quad u = 12 \quad s = 8 \quad t = ?$

$$8 = 12t + \frac{1}{2} \times -5t^2$$

$$16 = 24t - 5t^2$$

$$5t^2 - 24t + 16 = 0$$

$$t = 0.8,\ 4$$

(b) $s = -8 \qquad V^2 = u^2 + 2as$

$$V^2 = 12^2 + 2 \times -5 \times -8$$

$$V^2 = 144 + 80$$

$$V = \sqrt{224} = 15.0\ ms^{-1}$$

(14) $a = -4 \quad u = 14 \quad s = 22.5 \quad t = ?$

$$22.5 = 14t + \frac{1}{2} \times -4t^2$$

$$2t^2 - 14t + 22.5 = 0$$

(a) $t_1 = 2.5 \quad t_2 = 4.5 \quad \therefore$ diff in time $= 2$ sec

(b) time at direction change $V = 0$

$$0 = 14 - 4t \quad t = 3.5\text{ sec}$$

Speed @ A $\quad V = 14 + (-4 \times 2.5) = 4\,ms^{-1}$

$\therefore$ Dist travelled between A & rest $\qquad 0^2 = 4^2 + 2 \times -4 \times S$

$S = 2m$

$\therefore$ Total Dist $= 2 \times 2 = 4M.$

(15) From A→B: $a=?$ $s=100$ $V=14$ $u=?$

From B→C: $a=?$ $s=300$ $u=14$ $V=20$

$$V^2 = u^2 + 2as$$
$$20^2 = 14^2 + 2\times a \times 300$$
$$a = 0.34\,ms^{-2}$$

From A→C: $s=400$, $V=20$, $a=0.34$, $t=?$

$$s = vt - \frac{1}{2}at^2$$

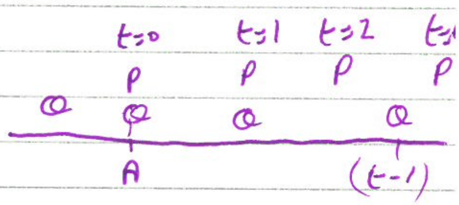
$$400 = 20t - \frac{1}{2}\times 0.34t^2$$
$$0.17t^2 - 20t + 400 = 0$$

either $t = 25.5$ or 92.1

$\therefore$ $t = 25.5$ sec

(16) P: $a_P = 2$ $u_P = 4$ $t=t$ $s=?$

Q: $a_Q = 3.6$ $u_Q = 3$ $t = t-1$ $s=?$



(a) $S_P = 4t + \frac{1}{2}\times 2\times t^2 = 4t + t^2$ — (1)

$$S_Q = 3(t-1) + \frac{1}{2}\times 3.6(t-1)^2 = 3(t-1) + 1.8(t-1)^2$$

(b) when they meet, $S_P = S_Q$

$$4t + t^2 = 3t - 3 + 1.8t^2 - 3.6t + 1.8$$

$$0.8t^2 - 4.6t - 1.2 = 0$$

$t = 6$ sec

(c) in (1) $S = 4\times 6 + 6^2 = 60$ Metres

Free Fall Motion Under Gravity

Galileo found experimentally that, if air resistance is ignored, any freely falling object, whatever its mass, has the same constant acceleration towards the centre of the Earth. This acceleration is only approximately constant – it increases slightly as the object gets nearer the centre of the Earth. (Thus it varies also from one part of the Earths surface to another since the Earth is not precisely spherical.)

Arrow Convention

In any particular example, care is needed to ensure that the directions of the vectors involved are all the same.

$u = 25ms^{-1} \uparrow$, implies that the initial velocity is $25ms^{-1}$ upwards

$a = 9.8ms^{-2} \downarrow$ implies a downward acceleration of $9.8ms^{-2}$
$= -9.8ms^{-2} \uparrow$

Before substituting numerical values into the constant acceleration formulae, the arrows of the variables involved must all be in the same direction.

eg4 A brick is thrown vertically downwards from the top of a building and has an initial velocity of $1.5ms^{-1}$. If the height of the building is $19^{2}/_{7}$m, find
(a) the velocity with which the brick hits the ground
(b) the time taken for the brick to fall.

eg5 A ball is thrown vertically upwards with a velocity of $14.7ms^{-1}$ from a platform 19.6m above ground level. Find
(a) the time taken for the ball to hit the ground,
(b) the velocity of the ball when it hits the ground.

eg6 A particle is projected vertically upwards with a velocity of $34.3ms^{-1}$. Find how long after projection the particle is at a height of 49m above the point of projection for (a) the first time, (b) the second time.

Exercise 2C Page 23

Eg4 (a) $u = 1.5 \downarrow$
$s = 19\frac{2}{7} \downarrow$
$a = 9.8 \downarrow$
$v = ?$

$v^2 = 1.5^2 + 2 \times 9.8 \times 19\frac{2}{7}$

$v^2 = 380.25$

$v = 19.5 \text{ ms}^{-1} \downarrow$. 20 ms^{-1} to 2 s.f.

(b) $19.5 = 1.5 + 9.8t$

$t = 1.8$ sec

Eg5 $u = 14.7 \uparrow = -14.7 \downarrow$
$a = 9.8 \downarrow$
$s = 19.6 \downarrow$
$t = ?$

(a) $19.6 = -14.7t + \frac{1}{2} \times 9.8t^2$

$4.9t^2 - 14.7t - 19.6 = 0$

$t = -1, 4$ $\therefore$ impact after 4 sec

(b) $v = -14.7 + (9.8 \times 4) = 24.5 \text{ ms}^{-1} \downarrow$.

Eg6 $u = 34.3 \uparrow$
$a = 9.8 \downarrow = -9.8 \uparrow$
$s = 49 \uparrow$
$t = ?$

$49 = 34.3t + \frac{1}{2} \times -9.8t^2$

$49 = 34.3t - 4.9t^2$

$4.9t^2 - 34.3t + 49 = 0$

$t = 2, 5$

$\therefore$ Above 49m for 3 sec

Ex2C

(1) $u = 14\uparrow$ $\quad$ $v^2 = u^2 + 2as$

$v = 0\uparrow$ $\quad$ $0 = 14^2 + 2\times -9.8s$

$a = 9.8\downarrow = -9.8\uparrow$ $\quad$ $19.6s = 14^2$

$s = ?$ $\quad$ $s = 10$ metres

(2) $u = 0\downarrow$
$s = 50\downarrow$
$a = 9.8\downarrow$
$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$50 = 0 + \frac{1}{2}\times 9.8t^2$$

$$t^2 = \frac{50}{4.9}$$

$$t = 3.2 \text{ sec}$$

(3) $u = 0\downarrow$
$t = 0.6$
$a = 9.8\downarrow$
$s = ?$

$$s = 0 + \frac{1}{2}(9.8)0.6^2 = 1.8m$$

(4) $u = 20\uparrow$
$s = 0$
$a = 9.8\downarrow = -9.8\uparrow$
$t = ?$

$$0 = 20t - \frac{1}{2}9.8t^2$$

$$0 = 20t - 4.9t^2$$

$$4.9t^2 - 20t = 0$$

$$t(4.9t - 20) = 0$$

either $t = 0$ or $t = \frac{20}{4.9} = 4.1$ sec

(5) $u = 18\downarrow$
$t = 1.6$
$a = 9.8\downarrow$
$s = ?$

$$s = (18\times 1.6) + \frac{1}{2}\times 9.8\times (1.6)^2$$

$$= 28.8 + 12.544$$

$$= 41.3M$$

6 (a) $u = 24\uparrow$ @ Max ht $V = 0\uparrow$

$a = 9.8\downarrow = -9.8\uparrow$

$0^2 = 24^2 + 2\times -9.8 \times S$

$19.6S = 24^2$

$S = 29.4M$

(b) $0 = 24 + -9.8t$

$t = 2.4 sec$

7 (a) $u = 18\uparrow$

$S = 15\uparrow$

$a = 9.8\downarrow = -9.8\uparrow$

$V = ?\uparrow$

$V^2 = 18^2 - (19.6\times 15)$

$V = \pm 5.5 ms^{-1}$ Speed $= 5.5 ms^{-1}$

(b) $u = 18\uparrow = -18\downarrow$

$S = 4\downarrow$ ~~-4↑~~

$a = 9.8\downarrow$

$V = ?\downarrow$

$V^2 = (-18)^2 + (2\times 9.8\times 4)$

$V = \pm 20 ms^{-1}$ speed $= 20$.

8 $u = 4\downarrow$

$S = 80\downarrow$

$a = 9.8\downarrow$

$V = ?\downarrow$

(a) $V^2 = 4^2 + 2\times 9.8\times 80$

$V = 40 ms^{-1}$

(b) $40 = 4 + 9.8t$

$t = 3.7 sec$

9 $u = ?\uparrow = -u\downarrow$

$V = 10\downarrow$

$a = 9.8\downarrow$

$t = 5$

(a) $10 = -u + 9.8\times 5$

$u = 39 ms^{-1}$

(b) @ Max ht, $V = 0$

$u = 39\uparrow$

$a = 9.8\downarrow = -9.8\uparrow$

$S = ?\uparrow$

$\cancel{S = \frac{(39+0)}{2}\times}$

$V^2 = u^2 + 2as$

$0 = 39^2 - 19.6S$

$S = \frac{39^2}{19.6} = 77.6 M.$

(10) $u = 21\uparrow \;\; = -21\downarrow$
$s = ?\downarrow$
$a = 9.8\downarrow$
$t = 4.5$

$$s = (-21 \times 4.5) + \frac{1}{2} \times 9.8 \times 4.5^2$$

$$s = 4.7\text{m}.$$

(11) $u = 16\uparrow \;\; = -16\downarrow$
$s = 3\downarrow$
$a = 9.8\downarrow$
$t = ?$

(a) $3 = -16t + \frac{1}{2}9.8t^2$

$$3 = -16t + 4.9t^2$$

$$4.9t^2 - 16t - 3 = 0$$

$$t = 3.4 \text{ sec}$$

(b) Dist travelled upwards

$u = 16\uparrow \;\; = \cancel{-16\downarrow}$
$a = 9.8\downarrow = -9.8\uparrow$
$v = 0$
$s = ?$

$$0 = (+16)^2 + 2 \times 9.8 \times s$$

$$s = \frac{16^2}{19.6} = 13.1\text{m}$$

Dist travelled downwards $= 13.1 + 3\text{m} = 16.1\text{m}$

$\therefore$ total dist $= 29.1$ metres

(12) $u = 24.5\uparrow$
$s = 21\uparrow$
$a = 9.8\downarrow = -9.8\uparrow$
$t = ?$

$$21 = 24.5t - 4.9t^2$$

$$4.9t^2 - 24.5t + 21 = 0$$

$$t = 1.1, 3.9$$

$\therefore$ particle above 21m for $3.9 - 1.1$
$= 2.8$ sec

(13) (a) $u = u\uparrow$
$V = u/3 \uparrow$
$a = -9.8\uparrow$
$t = 2$

$\frac{u}{3} = u - 9.8 \times 2$

$\frac{2u}{3} = 19.6$

$u = \frac{19.6 \times 3}{2} = 29.4 ms^{-1}$

(b) $S = 0$
$u = 29.4\uparrow$
$a = 9.8\uparrow$
$t = ?$

$0 = 29.4t - 4.9t^2$

$t(4.9t - 29.4) = 0$

either $t = 0$ or $t = \frac{29.4}{4.9} = 6$ sec

(14) (A) $u = 5\downarrow$
$S = x\downarrow$
$a = 9.8\downarrow$
$t = t$

(B) $u = 18\uparrow$
$S = (46 - x)\uparrow$
$a = -9.8\uparrow$
$t = t.$

$x = 5t + 4.9t^2$ — (1) $46 - x = 18t - 4.9t^2$ — (1)

~~Sub~~ (1) + (2)

$46 = 23t$
$t = 2$

∴ impact after 2 sec

in (1) $x = 5 \times 2 + 4.9 \times 2^2 = 29.6 m$

(15) Speed of 1st impact: $u = 0\downarrow$
$a = 9.8\downarrow$
$V = ?$
$S = 10\downarrow$

$V^2 = 0^2 + 2 \times 9.8 \times 10$
$V = \sqrt{196}$

(a) ∴ for 1st rebound: $u = 3\sqrt{196}/4 \uparrow$
$V = 0$
$a = -9.8\uparrow$
$S = ?$

$0 = 110.25 - 19.6S$
$S = 5.6m.$

(b) Speed of 2nd impact: $u = 0\downarrow$
$a = 9.8\downarrow$
$V = ?$
$S = 5.6\downarrow$

$V^2 = 0^2 + 2 \times 9.8 \times 5.6$
$V = \sqrt{110.25}$

(15) cont'd $\therefore$ for 2nd rebound $u = \frac{3\sqrt{110.25}}{4} \uparrow$

$v = 0$

$a = -9.8 \uparrow$

$s = ?$

$$0 = \left(\frac{3\sqrt{110.25}}{4}\right)^2 - 19.6s$$

$s = 3.2m.$

(16) (P) $u = 12 \uparrow$

$a = 9.8 \downarrow = -9.8 \uparrow$

$t = t$

$s = s$

(Q) $u = 20 \uparrow$

$a = -9.8 \uparrow$

$t = (t-1)$

$s = s.$

$$s = 12t - 4.9t^2 \quad \text{(1)} \qquad s = 20(t-1) - 4.9(t-1)^2 \quad \text{(2)}$$

equating $12t - \cancel{4.9t^2} = 20t - 20 - \cancel{4.9t^2} + 9.8t - 4.9$

$$17.8t = 24.9$$

$$t = 1.4 \text{ sec}$$

in (1) $s = 12(1.4) - 4.9(1.4)^2 = 7.2m.$

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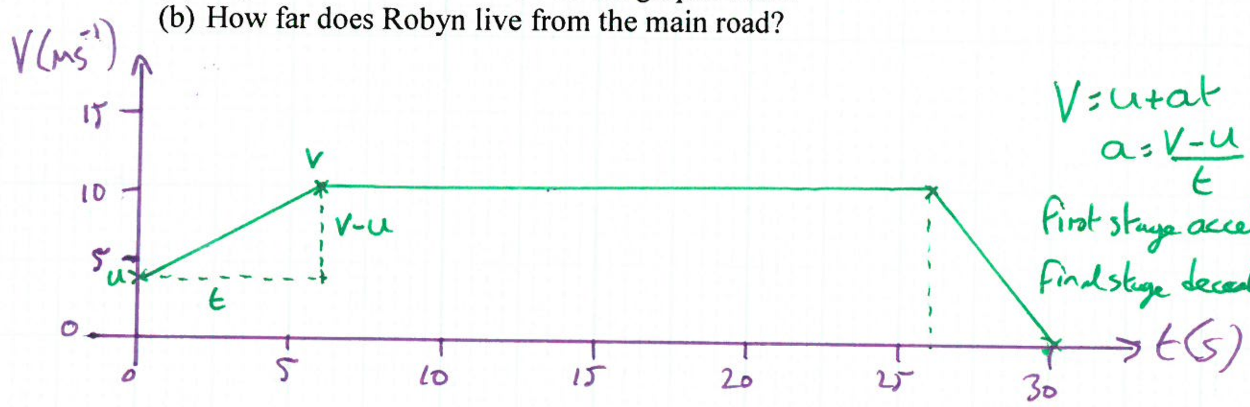
Representing Motion Graphically

Eg1 Robin is cycling home. He turns off the main road and $4ms^{-1}$ and accelerates uniformly to $10ms^{-1}$ over the next 6 seconds. He maintains this speed for 20 seconds and then slows uniformly for 4 seconds.

(a) Represent this information on the graphs below

(b) How far does Robyn live from the main road?

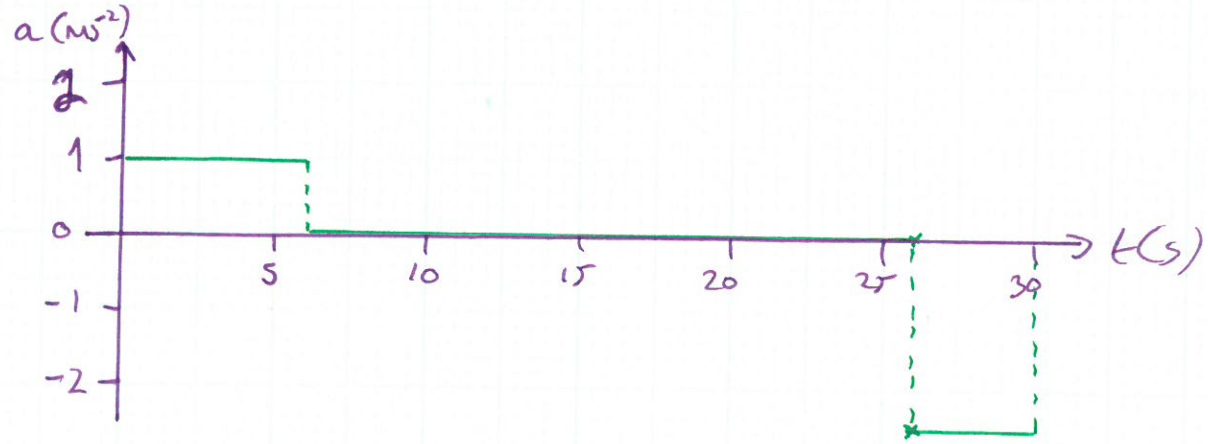


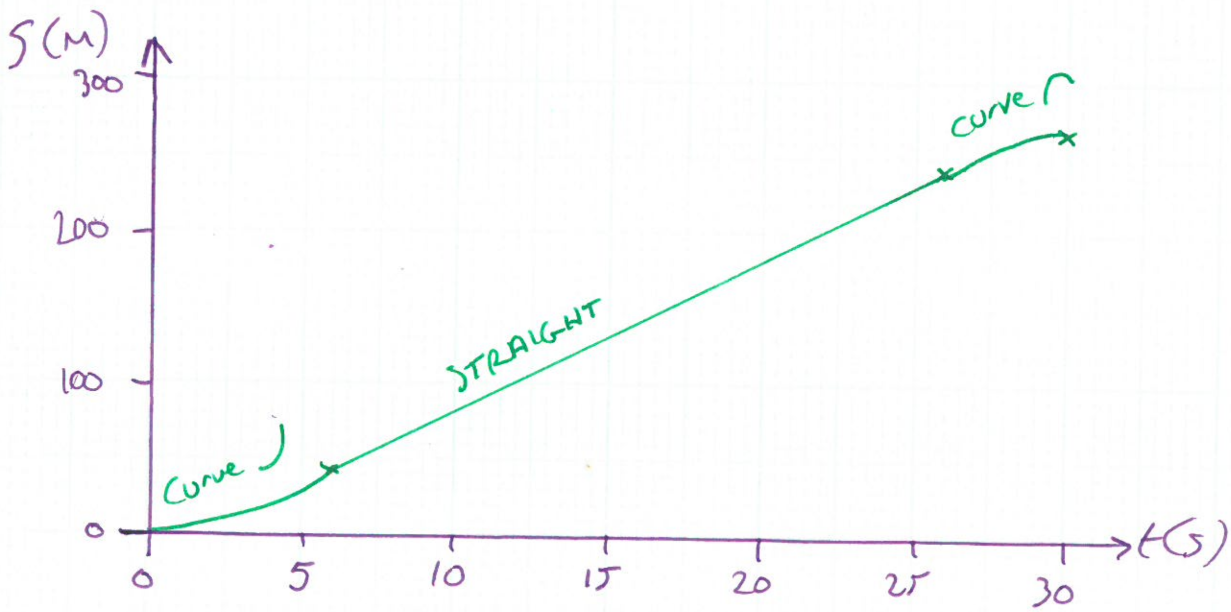
$V = u + at$

$a = \frac{V-u}{t}$ = gradie

First stage accel $= \frac{6}{6} = 1ms$

Final stage decel $= \frac{10}{4} = -2.$ ms





Exercise 2D Pg 30 Q's 2, 4, 5, 6, 8, 10, 11, 12

From v-t graph

$v = u + at$

$a = \frac{v-u}{t} = \text{gradient}$

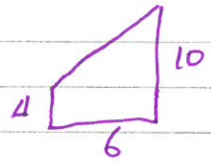
First stage accel $= \frac{10-4}{6} = 1\,ms^{-2}$

Final stage accel $= \frac{10-0}{4} = -2.5\,ms^{-2}$

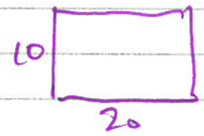
Middle stage accel $= 0$

Area under v-t graph = distance.

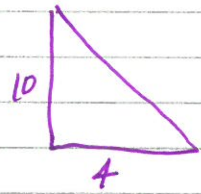
First stage distance $= \frac{1}{2}(4+10) \times 6 = 42m$



Second stage distance $= 20 \times 10 = 200$



Final stage distance $= \frac{1}{2} \times 4 \times 10 = 20m$

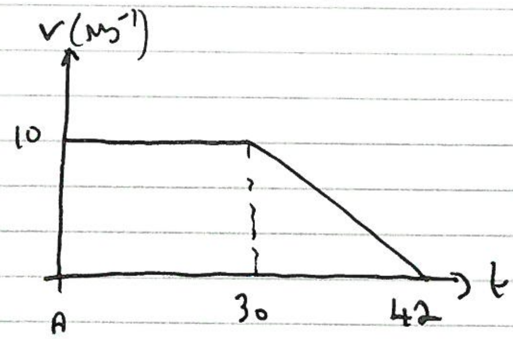


Ex 2D

① (a) accel $= \frac{9}{4} = 2.25\,ms^{-2}$

(b) dist $= \frac{(12+8)}{2} \times 9 = 90m$

② (a)



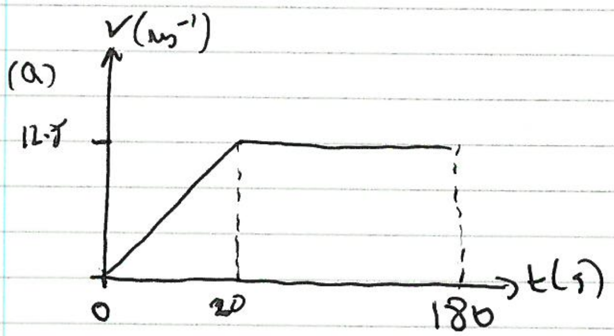
(b) $S = \frac{(30+42)}{2} \times 10 = 360m$

③ (a) $a = \frac{8}{20} = 0.4\,ms^{-2}$

(b) $a = -\frac{8}{15} =$

(c) $S = \frac{(40+75)}{2} \times 8 = 460m$

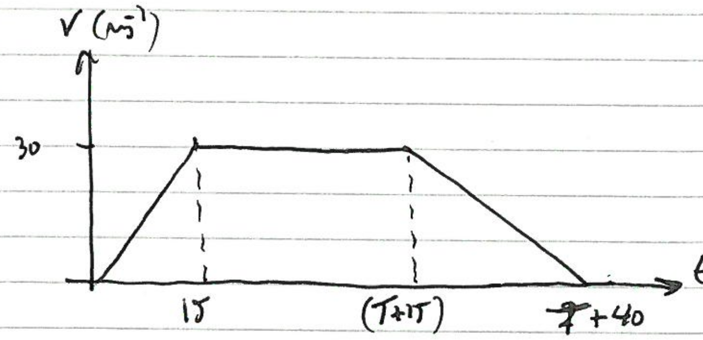
④ (a)



$45\,kmh^{-1} = \frac{45\,000}{60 \times 60} = 12.5\,ms^{-1}$

(b) $S = \frac{(180+160)}{2} \times 12.5 = 2125m.$

⑤



$2400 = \left(\frac{T + T+40}{2}\right) \times 30$

$\frac{2400}{15} = 2T + 40$

$T = \frac{160-40}{2} = 60$

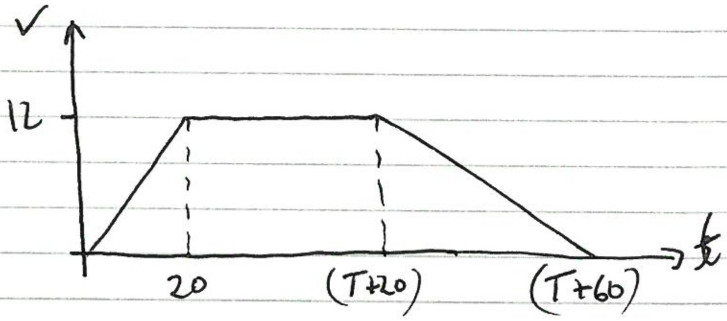
$\therefore$ time to travel from S to F $= 60 + 40 = 100$ sec

(6) (a) $a = -\frac{24}{30} = -0.8\,ms^{-2}$

(b) $S = \left(\frac{40+16}{2}\right)\times 30 + (16\times 70) = 1960\,m$

(7) $u=0 \quad a=0.6 \quad t=20 \quad V=?$ $\quad V = 0 + 0.6\times 20 = 12\,ms^{-1}$

(a)



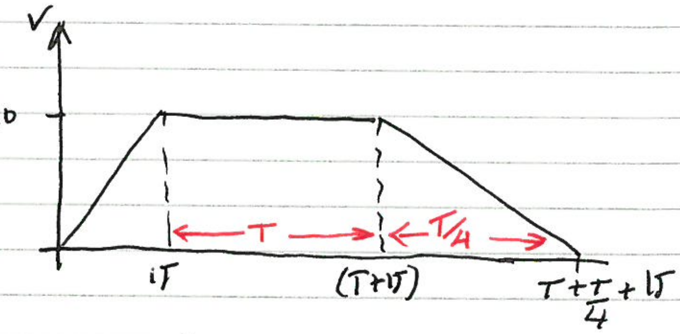
(b) $4200 = \left(\frac{T + T + 60}{2}\right)\times 12$

$\frac{4200}{6} = 2T + 60$

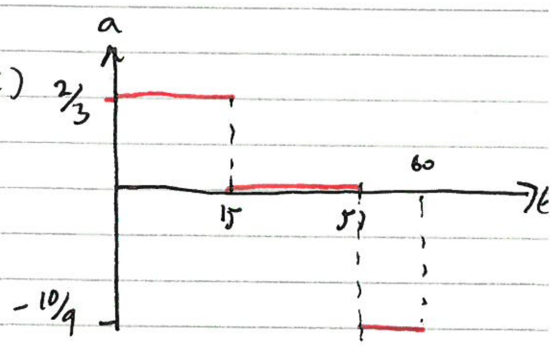
$T = \frac{700-60}{2} = 320$

(C) $12\times 320 = 3840\,m.$

(8) (a)



(c)



(b) $480 = \frac{\left(T + \frac{T}{4} + 15\right) + T}{2} \times 10$

$\frac{480}{5} = \frac{9T}{4} + 15$

$96 - 15 = \frac{9T}{4}$

$T = 36$

Total Time $= 36 + \frac{36}{4} + 15 = 60$ sec

Init accel $= \frac{2}{3}\,ms^{-2}$

decel $= -\frac{10}{9}$

9 (a) $100 = \frac{(u+10)}{2} \times 3 + \frac{(9+7)}{2} \times 10$

$200 = 3u + 30 + 160$

$3u = 10$

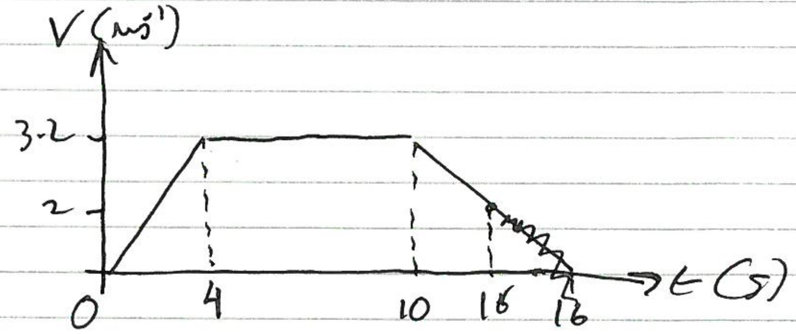
$u = \frac{10}{3}\ ms^{-1}$

(b) $10 = \frac{10}{3} + 3a$

$30 = 10 + 9a$

$a = \frac{20}{9}\ ms^{-2}$

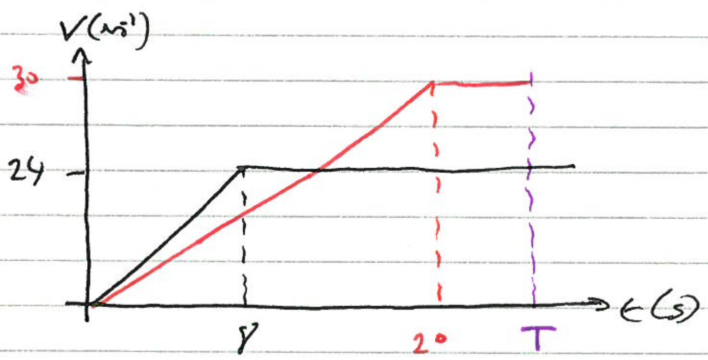
10



$u=0\ \ V=?\ \ a=0.8\ \ t=4 \qquad V = 0 + 0.8 \times 4 = 3.2$

$u=3.2\ \ t=6\ \ a=-0.2\ \ V=? \qquad V = 3.2 - 0.2 \times 6 =$

11 (a) M: $u=0\ \ v=?\ \ a=3,\ t=8 \qquad V = 24ms^{-1}$



(b) They meet when they've covered same distance

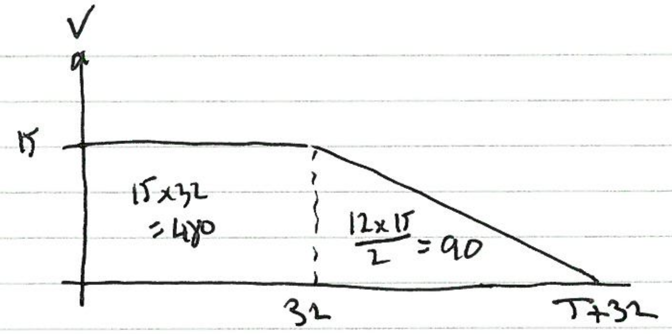
M: $S = \frac{T + (T-8)}{2} \times 24 = 24T - 96$ — 1

C: $S = \frac{T + T - 20}{2} \times 30 = 30T - 300$

equating $30T - 300 = 24T - 96$

$6T = 204 \quad T = 34$ sec in 1 $24(34) - 96 = 720m$

12 (a)



(b) $$570 = \frac{(T+32+32)}{2} \times 15$$

$$T = 76 - 64 = 12$$

(c)