M1 – JAN 03

More Nasty Vectors!

3. A particle P of mass 0.4 kg is moving under the action of a constant force $\mathbf{F}$ newtons. Initially the velocity of P is $(6\mathbf{i} - 27\mathbf{j})$ m s^{-1} and 4 s later the velocity of P is $(-14\mathbf{i} + 21\mathbf{j})$ m s^{-1}.

(*a*) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the acceleration of P.

(3)

(*b*) Calculate the magnitude of $\mathbf{F}$.

(3)

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4. Two ships P and Q are moving along straight lines with constant velocities. Initially P is at a point O and the position vector of Q relative to O is $(6\mathbf{i} + 12\mathbf{j})$ km, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors directed due east and due north respectively. The ship P is moving with velocity $10\mathbf{j}$ km h^{-1} and Q is moving with velocity $(-8\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time t hours the position vectors of P and Q relative to O are $\mathbf{p}$ km and $\mathbf{q}$ km respectively.

(*a*) Find $\mathbf{p}$ and $\mathbf{q}$ in terms of t.

(3)

(*b*) Calculate the distance of Q from P when $t = 3$.

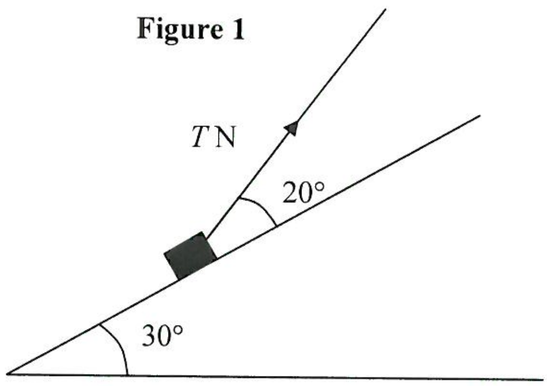
(3)

(*c*) Calculate the value of t when Q is due north of P.

(2)

5.

Figure 1



A box of mass 1.5 kg is placed on a plane which is inclined at an angle of 30° to the horizontal. The coefficient of friction between the box and plane is $\frac{1}{3}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in Fig. 2. The box is in limiting equilibrium and is about to move up the plane. The tension in the string is T newtons. The box is modelled as a particle.

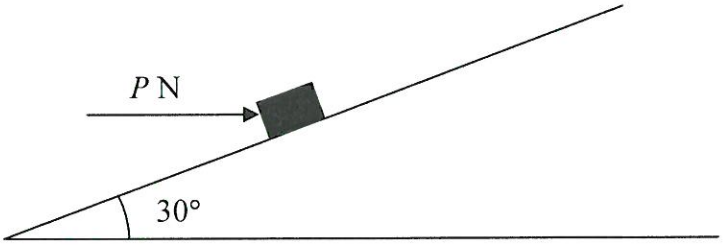
Find the value of T.

(10)

MI – JAN 08

5.

Figure 2



A parcel of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown in Figure 2. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18 N. The coefficient of friction between the parcel and the plane is μ. Find

(*a*) the value of P,

(4)

(*b*) the value of μ.

(5)

The horizontal force is removed.

(*c*) Determine whether or not the parcel moves.

(5)

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6. [*In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(*a*) the speed of A,

(2)

(*b*) the direction in which A is moving, giving your answer as a bearing.

(3)

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(*c*) show that A and B will collide at a point P and find the position vector of P.

(5)

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(*d*) find the distance of B from P when $t = 7$ s.

(6)

M1 – JAN 05

6. A stone S is sliding on ice. The stone is moving along a straight line ABC, where $AB = 24$ m and $AC = 30$ m. The stone is subject to a constant resistance to motion of magnitude 0.3 N. At A the speed of S is 20 m s^{-1}, and at B the speed of S is 16 m s^{-1}. Calculate

(*a*) the deceleration of S,

(2)

(*b*) the speed of S at C.

(3)

(*c*) Show that the mass of S is 0.1 kg.

(2)

At C, the stone S hits a vertical wall, rebounds from the wall and then slides back along the line CA. The magnitude of the impulse of the wall on S is 2.4 N s and the stone continues to move against a constant resistance of 0.3 N.

(*d*) Calculate the time between the instant that S rebounds from the wall and the instant that S comes to rest.

(6)

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7. Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\mathbf{i} + 10\mathbf{j})$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(14\mathbf{i} - 6\mathbf{j})$ km. Three hours later, P is at the point with position vector $(29\mathbf{i} + 34\mathbf{j})$ km. The ship Q travels with velocity $12\mathbf{j}$ km h^{-1}. At time t hours after midnight, the position vectors of P and Q are $\mathbf{p}$ km and $\mathbf{q}$ km respectively. Find

(*a*) the velocity of P, in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(*b*) expressions for $\mathbf{p}$ and $\mathbf{q}$, in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(4)

At time t hours after midnight, the distance between P and Q is d km.

(*c*) By finding an expression for $\overrightarrow{PQ}$, show that

$$d^2 = 25t^2 - 92t + 292.$$

(5)

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into sight of the observer,

(*d*) find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

(5)

TOTAL FOR PAPER: 75 MARKS

END

M1 – JUNE 06

7. [*In this question the unit vectors* **i** *and* **j** *are due east and north respectively*]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j})$ km. Find

(*a*) the speed of S,

(2)

(*b*) the bearing on which S is moving.

(2)

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(*c*) Find the position vector of R.

(2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1}. Assuming that S continues to move with this new constant velocity, find

(*d*) an expression for the position vector of the ship t hours after 1400,

(4)

(*e*) the time when S will be due east of R,

(2)

(*f*) the distance of S from R at the time 1600.

(3)

TOTAL FOR PAPER: 75 MARKS

END

M1 – JAN 03

Q3 $u = \begin{pmatrix} 6 \\ -27 \end{pmatrix}$ $v = \begin{pmatrix} -14 \\ 21 \end{pmatrix}$ $t = 4$

(a) $v = u + at$

$$\begin{pmatrix} -14 \\ 21 \end{pmatrix} = \begin{pmatrix} 6 \\ -27 \end{pmatrix} + 4a$$

$$\begin{pmatrix} -14 \\ 21 \end{pmatrix} - \begin{pmatrix} 6 \\ -27 \end{pmatrix} = 4a$$

$$4a = \begin{pmatrix} -20 \\ 48 \end{pmatrix} \qquad a = -5\underline{i} + 12\underline{j}$$

(b) $F = ma$

$$F = 0.4 \begin{pmatrix} -5 \\ 12 \end{pmatrix} = \begin{pmatrix} -2 \\ 4.8 \end{pmatrix}$$

$$|F| = \sqrt{4 + 23.04}$$
$$= 5.2N.$$

M1 – JAN 03

Q4 (a) Ship P: $r_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $V = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$

$r_p = r_0 + Vt$

$p = r_p = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 10 \end{pmatrix} t = \begin{pmatrix} 0 \\ 10t \end{pmatrix}$

Ship Q $r_0 = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$ $V = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$

$q = r_Q = \begin{pmatrix} 6 \\ 12 \end{pmatrix} + \begin{pmatrix} -8 \\ 6 \end{pmatrix} t = \begin{pmatrix} 6-8t \\ 12+6t \end{pmatrix}$

(b) Need relative position of Q from P $r_{PQ} = -r_p + r_q$

$= -\begin{pmatrix} 0 \\ 10t \end{pmatrix} + \begin{pmatrix} 6-8t \\ 12+6t \end{pmatrix}$

$= \begin{pmatrix} 6-8t \\ 12-4t \end{pmatrix}$

when $t=3$ $r_{PQ} = \begin{pmatrix} 6-24 \\ 12-12 \end{pmatrix} = \begin{pmatrix} -18 \\ 0 \end{pmatrix}$

$\therefore$ distance $= \sqrt{(-18)^2 + 0^2} = 18$ km

(c) when Q is due north of P, i components the same

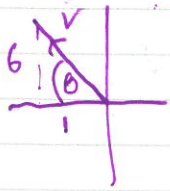
$\therefore$ $6 - 8t = 0$

$t = 0.75$

M1 – JAN 06

Q6 $r_0 = \begin{pmatrix} 2 \\ -10 \end{pmatrix}$ $V = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

(a) Speed $= \sqrt{(-1)^2 + 6^2} = \sqrt{37} = 6.1 \text{ ms}^{-1}$

(b) 

~~TAN~~ $\theta = \tan^{-1}\left(\frac{6}{1}\right) = 80.5°$

bearing $= 270 + 80.5 = 350°$

(c) ~~Ship~~ Boat B $r_0 = \begin{pmatrix} -26 \\ 4 \end{pmatrix}$ $V = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

at time t, Boat A $r_A = \begin{pmatrix} 2 \\ -10 \end{pmatrix} + \begin{pmatrix} -1 \\ 6 \end{pmatrix} t = \begin{pmatrix} 2-t \\ -10+6t \end{pmatrix}$

Boat B $r_B = \begin{pmatrix} -26 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} t = \begin{pmatrix} 3t-26 \\ 4+4t \end{pmatrix}$

If ships collide then $r_A = r_B$ for some value of t

$\therefore$ $2 - t = 3t - 26$ and $-10 + 6t = 4 + 4t$

$28 = 4t$ $\quad$ $2t = 14$

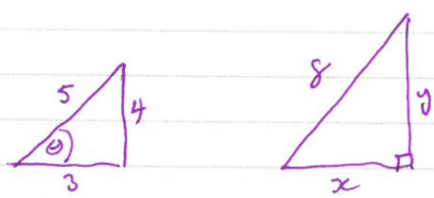
$t = 7$ $\quad$ $t = 7$

$\therefore$ Ships collide after 7 seconds at pos vector $\begin{pmatrix} 2-7 \\ -10+6\times7 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 32 \end{pmatrix}$

(d) If Boat B ~~[illegible]~~ Speed 8 in direction $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Need velocity vector



Sim Δ's $\frac{x}{3} = \frac{8}{5} \Rightarrow x = \frac{24}{5}$

$\frac{y}{4} = \frac{8}{5} \Rightarrow y = \frac{32}{5}$

$\therefore$ Velocity Vector is now $V = \frac{24}{5} i + \frac{32}{5} j$

M1 – Jan 06

Q6, (d) $\therefore$ for boat B, $r_B = \begin{pmatrix}-26\\4\end{pmatrix} + \begin{pmatrix}24/5\\32/5\end{pmatrix}t = \begin{pmatrix}-26 + 4.8t\\4 + 6.4t\end{pmatrix}$

Now position of B relative to A $= -r_A + r_B$

$$= -\begin{pmatrix}2 - t\\-10 + 6t\end{pmatrix} + \begin{pmatrix}-26 + 4.8t\\4 + 6.4t\end{pmatrix}$$

$$= \begin{pmatrix}5.8t - 28\\0.4t + 14\end{pmatrix}$$

when $t = 7$ $= \begin{pmatrix}5.8(7) - 28\\0.4(7) + 14\end{pmatrix} = \begin{pmatrix}12.6\\16.8\end{pmatrix}$

dist $= \sqrt{12.6^2 + 16.8^2} = \sqrt{441} = 21$ metres.

M1 - Jan 05

Q7(a) <u>Ship P</u> $r_0 = \begin{pmatrix} 20 \\ 10 \end{pmatrix}$ $t = 3$ $r = \begin{pmatrix} 29 \\ 34 \end{pmatrix}$

$$r = r_0 + vt$$

$$\begin{pmatrix} 29 \\ 34 \end{pmatrix} = \begin{pmatrix} 20 \\ 10 \end{pmatrix} + 3v$$

$$3v = \begin{pmatrix} 9 \\ 24 \end{pmatrix}$$

$$v = \begin{pmatrix} 3 \\ 8 \end{pmatrix} = 3\underline{i} + 8\underline{j}$$

(b) $$p = \begin{pmatrix} 20 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ 8 \end{pmatrix} t = \begin{pmatrix} 20+3t \\ 10+8t \end{pmatrix}$$

<u>Ship Q</u> $r_0 = \begin{pmatrix} 14 \\ -6 \end{pmatrix}$ $v = \begin{pmatrix} 0 \\ 12 \end{pmatrix}$

$$q = \begin{pmatrix} 14 \\ -6 \end{pmatrix} + \begin{pmatrix} 0 \\ 12 \end{pmatrix} t = \begin{pmatrix} 14 \\ -6+12t \end{pmatrix}$$

(c) $$\overrightarrow{PQ} = -p + q$$

$$= -\begin{pmatrix} 20+3t \\ 10+8t \end{pmatrix} + \begin{pmatrix} 14 \\ -6+12t \end{pmatrix}$$

$$= \begin{pmatrix} -6-3t \\ -16+4t \end{pmatrix}.$$

Now $d = \sqrt{(-6-3t)^2 + (-16+4t)^2}$

$$d^2 = 36 + 36t + 9t^2 + 256 - 128t + 16t^2$$

$d^2 = 25t^2 - 92t + 292$ As required.

M1 - JAN07

Q7 (d) $25t^2 - 92t + 292 \leq (15)^2$

$25t^2 - 92t + \cancel{292}\ 67 \leq 0.$

Use calc $t = 1, 2.68$

So ships move into sight when $t = 1$
and move out of sight when $t = 2.68$

Now 2.68 is 2 hours + $0.68 \times 60 = 40.8 \approx 41$ minutes

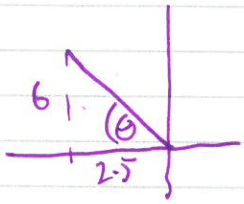
$\therefore$ ships move out of sight at 0241 or 2:41 am.

M1 – June 06

Q7. Ship S $\quad r_0 \begin{pmatrix} 16 \\ 5 \end{pmatrix} \quad V = \begin{pmatrix} -2.5 \\ 6 \end{pmatrix}$

(a) Speed of S $= \sqrt{(-2.5)^2 + 6^2} = 6.5 \text{ km h}^{-1}$

(b)



$\theta = \tan^{-1}\left(\frac{6}{2.5}\right) = 67.4°$

bearing $= 270 + 67.4 = 337°$.

(c) will hit rock when $t = 3$

$r = \begin{pmatrix} 16 \\ 5 \end{pmatrix} + \begin{pmatrix} -2.5 \\ 6 \end{pmatrix} t = \begin{pmatrix} 16 - 2.5t \\ 5 + 6t \end{pmatrix}$

$R = \begin{pmatrix} 16 - 7.5 \\ 5 + 18 \end{pmatrix} = \begin{pmatrix} 8.5 \\ 23 \end{pmatrix}$

(d) position of ship when $t = 2$

$r = \begin{pmatrix} 16 - 5 \\ 5 + 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 17 \end{pmatrix}$

Now $r_0 = \begin{pmatrix} 11 \\ 17 \end{pmatrix} \quad V = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

so t hours after 1400 $\quad r = \begin{pmatrix} 11 \\ 17 \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix} t = \begin{pmatrix} 11 \\ 17 + 5t \end{pmatrix}$

(e) when due east of R, $\underline{j}$ compont the same

$\therefore \quad 17 + 5t = 23$

$5t = 6$

$t = \frac{6}{5} = 1.2$ hours $=$ 1 hour 12 min after 1400

$\therefore$ time 1512

M1 - June 06

Q7 (f) position of S from R = $-R+S$

$$= -\begin{pmatrix} 8.5 \\ 23 \end{pmatrix} + \begin{pmatrix} 11 \\ 17+5t \end{pmatrix}$$

$$= \begin{pmatrix} 2.5 \\ 5t-6 \end{pmatrix}$$

when $t=2$ $\quad r_{RS} = \begin{pmatrix} 2.5 \\ 4 \end{pmatrix}$

$\therefore$ dist $= \sqrt{2.5^2+16} = 4.7$ km.