M1 - Vectors & UVAST

M1 Jun 10

1. A particle P is moving with constant velocity $(-3\mathbf{i}+2\mathbf{j})\ \mathrm{m\,s^{-1}}$. At time $t=6$ s P is at the point with position vector $(-4\mathbf{i}-7\mathbf{j})$ m. Find the distance of P from the origin at time $t=2$ s.

(5)

M1 Jan 09

1. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})\ \mathrm{m\,s^{-2}}$. At time $t = 0$, P has speed $u\ \mathrm{m\,s^{-1}}$. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})\ \mathrm{m\,s^{-1}}$.

Find the value of u.

(5)

M1 Jan 08

6. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})\ \mathrm{m\,s^{-1}}$. Find

(a) the speed of P,

(2)

(b) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})\ \mathrm{m\,s^{-1}}$, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})\ \mathrm{m\,s^{-1}}$.

(c) Find the values of u and v.

(5)

(d) Find the total time taken for P to move from A to a position which is due south of A.

(3)

M1 Jan 10

7. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is $\mathbf{s}$ km. When $t=0$, $\mathbf{s}=9\mathbf{i}-6\mathbf{j}$. When $t=4$, $\mathbf{s}=21\mathbf{i}+10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s}=(3t+9)\,\mathbf{i}+(4t-6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i}+6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

M1 June 05

8. *[In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.]*

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(*a*) the speed of the ball,

(2)

(*b*) the position vector of the ball after t seconds.

(2)

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(*c*) Find the time when the ball is due north of B.

(2)

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(*d*) find the value of v.

(6)

(*e*) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

(1)

Leave blank

M1 June 09

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)

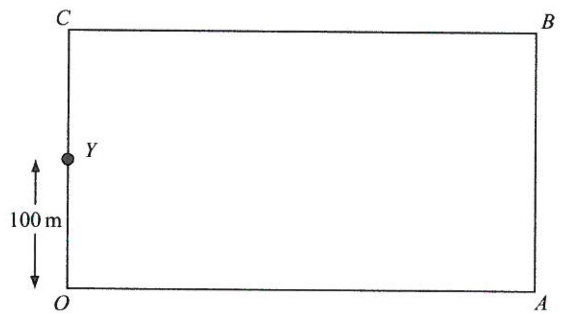


Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector $100\,\mathbf{j}$ m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = \left[(9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}\right] \text{ metres.}$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

M1 - June 10

Q1. $r = vt + r_0$

@ $t=6$ $\begin{pmatrix}-4\\-7\end{pmatrix} = \begin{pmatrix}-3\\2\end{pmatrix}.6 + r_0$

$r_0 = \begin{pmatrix}-4\\-7\end{pmatrix} - \begin{pmatrix}-18\\12\end{pmatrix}$

$r_0 = \begin{pmatrix}14\\-19\end{pmatrix}$

Now @ $t=2$ $r = \begin{pmatrix}-3\\2\end{pmatrix}.2 + \begin{pmatrix}14\\-19\end{pmatrix}$

$r = \begin{pmatrix}14-6\\-19+4\end{pmatrix}$

$r = \begin{pmatrix}8\\-15\end{pmatrix}$

distance $= \sqrt{64+225} = 17$ metres

M1 – JAN 09

Q1.
$$V = u + at$$

@ $t=3$
$$\begin{pmatrix}-6\\1\end{pmatrix} = u + \begin{pmatrix}2\\-5\end{pmatrix}.3$$

$$u = \begin{pmatrix}-6\\1\end{pmatrix} - \begin{pmatrix}6\\-15\end{pmatrix}$$

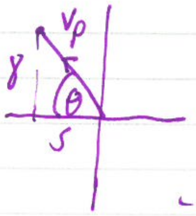
$$u = \begin{pmatrix}-12\\16\end{pmatrix}$$

$$u = \sqrt{12^2+16^2} = 20ms^{-1}$$

M1 JAN 08

Q6. $V_p = \begin{pmatrix}-5\\8\end{pmatrix}$

(a) $|V_p| = \sqrt{25+64} = \sqrt{89} = 9.43\,ms^{-1}$

(b) 

$\theta = \tan^{-1}\left(\frac{8}{5}\right) = 58^\circ$

$\therefore$ direction of P $= 270 + 58 = 328^\circ$

(c) position of P when $t=3$ $\quad r = \begin{pmatrix}7\\-10\end{pmatrix} + \begin{pmatrix}-5\\8\end{pmatrix}.3 = \begin{pmatrix}-8\\14\end{pmatrix}$

whilst new velocity $\quad r = \begin{pmatrix}-8\\14\end{pmatrix} + \begin{pmatrix}u\\v\end{pmatrix}t$

when $t=4$ $\quad r = \begin{pmatrix}0\\0\end{pmatrix}$

$\therefore \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}-8\\14\end{pmatrix} + \begin{pmatrix}u\\v\end{pmatrix}4$

$\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}4u-8\\4v+14\end{pmatrix}$

$\therefore u = 2$

$v = -\frac{14}{4} = -3.5$

(d) when P is due south of A, i component $= 7$

$\therefore -8 + 2t = 7$

$t = \frac{15}{2} = 7.5$

$\therefore$ total time $= 3 + 7.5 = 10.5$ secs

M1 - JAN 10

Q7. (a) $r_t = r_0 + vt$

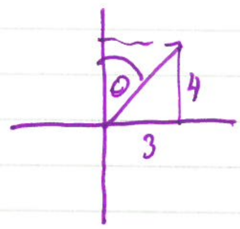
@ $t=4$ $\begin{pmatrix}21\\10\end{pmatrix} = \begin{pmatrix}9\\-6\end{pmatrix} + 4v$

$4v = \begin{pmatrix}12\\16\end{pmatrix}$

$v = \begin{pmatrix}3\\4\end{pmatrix}$

Speed = 5 kmh

(b)



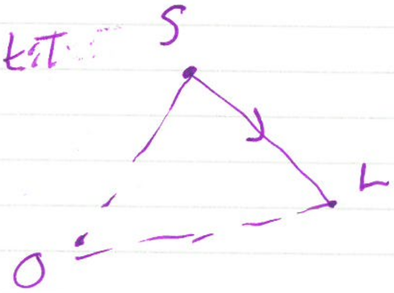
$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$

$\therefore$ bearing 037°

(c) $r_t = S = \begin{pmatrix}9\\-6\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix}t$

$S = \begin{pmatrix}3t+9\\4t-6\end{pmatrix}$ As required.

(d) at $t=T$



$r_{SL} = -\vec{OS} + \vec{OL}$

$= -\begin{pmatrix}3T+9\\4T-6\end{pmatrix} + \begin{pmatrix}18\\6\end{pmatrix}$

$r_{SL} = \begin{pmatrix}9-3T\\12-4T\end{pmatrix}$

Now $|r_{SL}| = 10$ $10 = \sqrt{(9-3T)^2 + (12-4T)^2}$

$100 = 81 - 54T + 9T^2 + 144 - 96T + 16T^2$

$25T^2 - 150T + 125 = 0$

$T^2 - 6T + 5 = 0$

$T = 1, 5$

M1 - JUNE07

Q8. (a) $|v| = \sqrt{25+64} = \sqrt{89} = 9.43\,ms^{-1}$

(b) $r_t = r_0 + vt$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 8 \end{pmatrix} t = \begin{pmatrix} 5t+2 \\ 8t+1 \end{pmatrix}$$

(c) ball due north of B when i comp = 10

$5t+2=10$

$t = \frac{8}{5} = 1.6\,sec$

(d) position vector of intersept $\begin{pmatrix} 10 \\ 8(1.6)+1 \end{pmatrix} = \begin{pmatrix} 10 \\ 13.8 \end{pmatrix}$

for player $r = r_0 + Vt$

$$\begin{pmatrix} 10 \\ 13.8 \end{pmatrix} = \begin{pmatrix} 10 \\ 7 \end{pmatrix} + 1.6V$$

$$1.6V = \begin{pmatrix} 0 \\ 6.8 \end{pmatrix}$$

$$V = \begin{pmatrix} 0 \\ 4.25 \end{pmatrix}$$

$\therefore$ speed $V = 4.25\,ms^{-1}$

(e) ball unlikely to only move horizontally.

M1 - June 09

Q8 (a) $|V_H| = \sqrt{1.2^2 + 0.9^2} = 1.5\,ms^{-1}$

(b) $r_H = r_0 + vt$

$r_H = \begin{pmatrix} 0 \\ 100 \end{pmatrix} + \begin{pmatrix} 1.2 \\ -0.9 \end{pmatrix} t = \begin{pmatrix} 1.2t \\ 100 - 0.9t \end{pmatrix}$ — 1

(c) $r_K = \begin{pmatrix} 9 \\ 46 \end{pmatrix} + \begin{pmatrix} 0.75 \\ 1.8 \end{pmatrix} t = \begin{pmatrix} 0.75t + 9 \\ 1.8t + 46 \end{pmatrix}$

$\overrightarrow{HK} = -\overrightarrow{OH} + \overrightarrow{OK}$

$= -\begin{pmatrix} 1.2t \\ 100 - 0.9t \end{pmatrix} + \begin{pmatrix} 0.75t + 9 \\ 1.8t + 46 \end{pmatrix}$

$= \begin{pmatrix} 9 - 0.45t \\ 2.7t - 54 \end{pmatrix}$

(d) If hikers meet then $\overrightarrow{HK} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$9 - 0.45t = 0$
$t = 20$

$2.7t - 54 = 0$
$t = 20$

$\therefore$ hikers meet after 20 seconds.

In 1 $r = \begin{pmatrix} 1.2(20) \\ 100 - 0.9(20) \end{pmatrix} = \begin{pmatrix} 24 \\ 82 \end{pmatrix}$